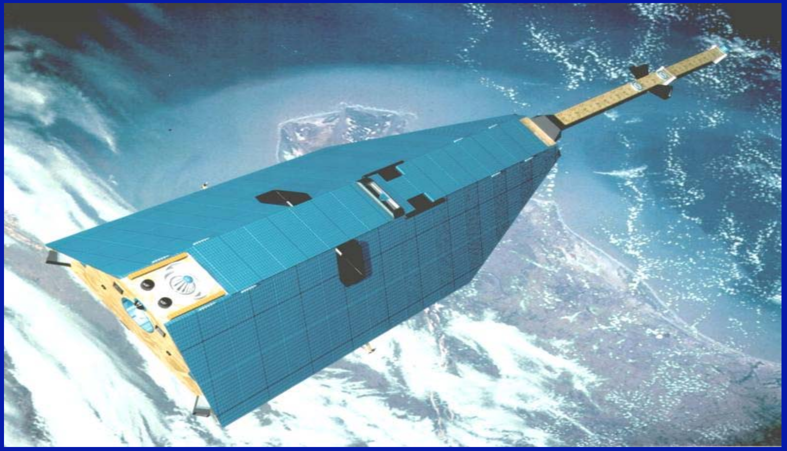
From ships to satellites: Constraining geomagnetic secular variation

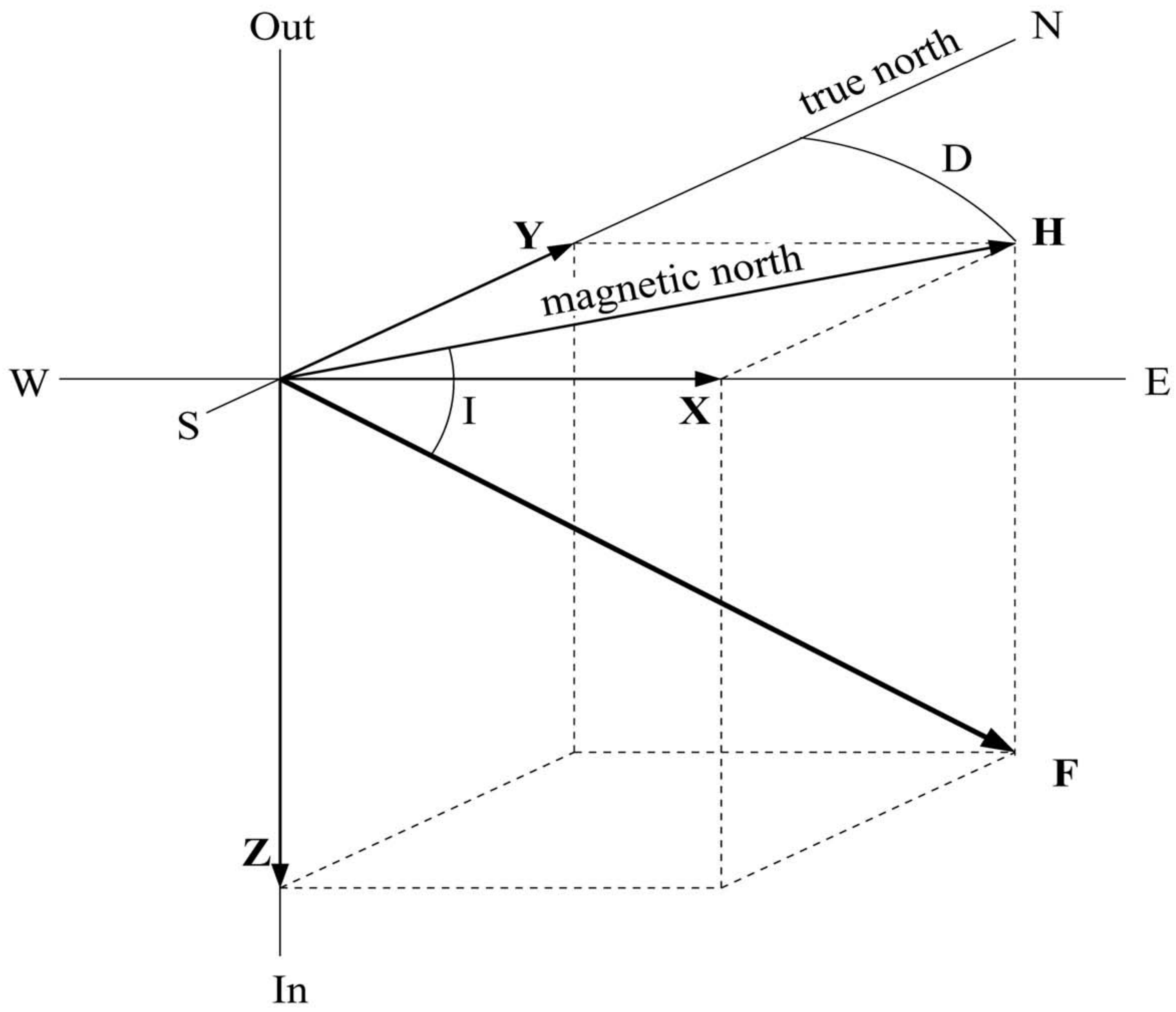
Andy Jackson
Institute for Geophysics
ETH Zurich

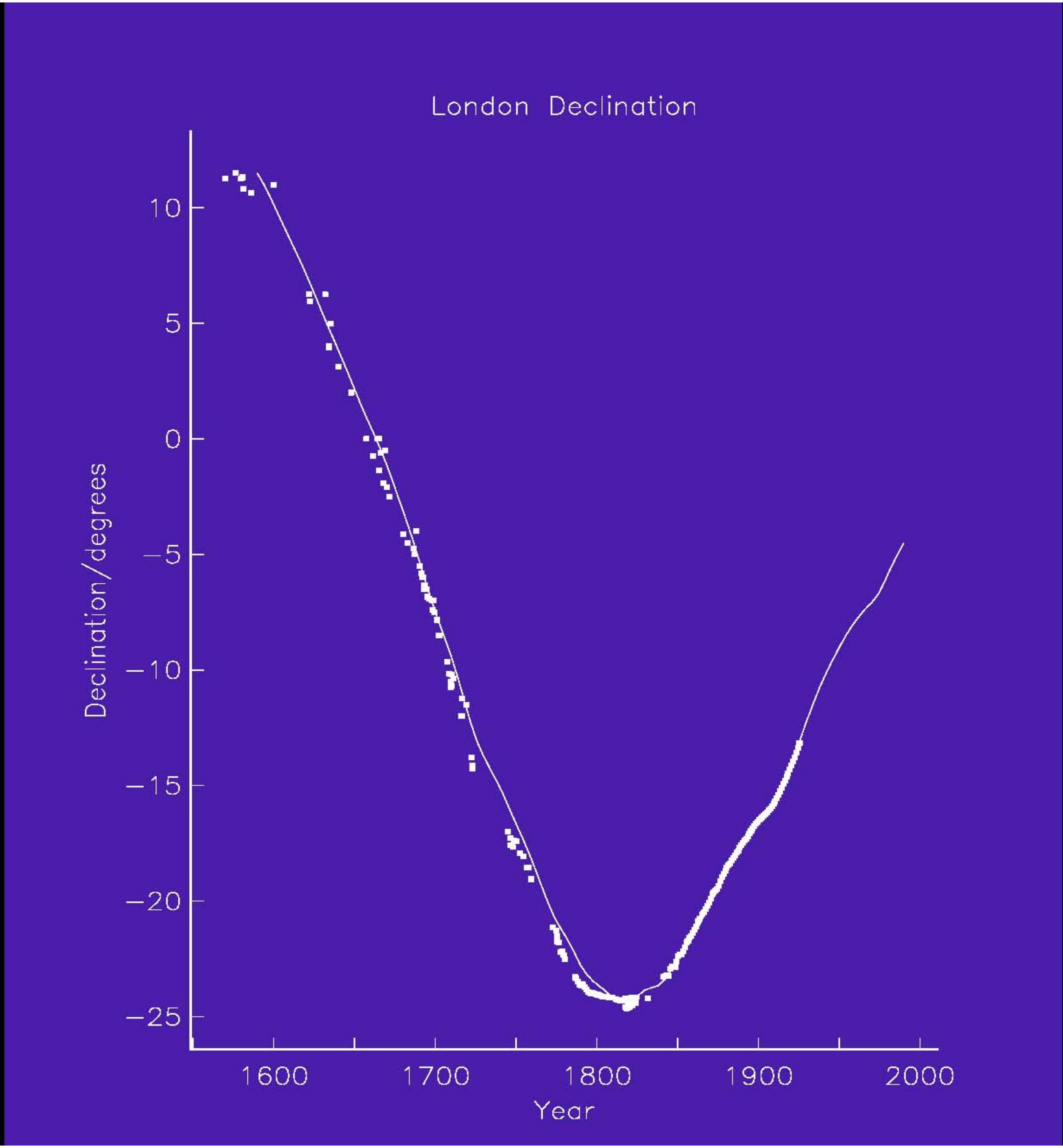
Acknowledgements: Art Jonkers, Chris Finlay, Nicolas Gillet, Cathy Constable, Bob Parker

Plan

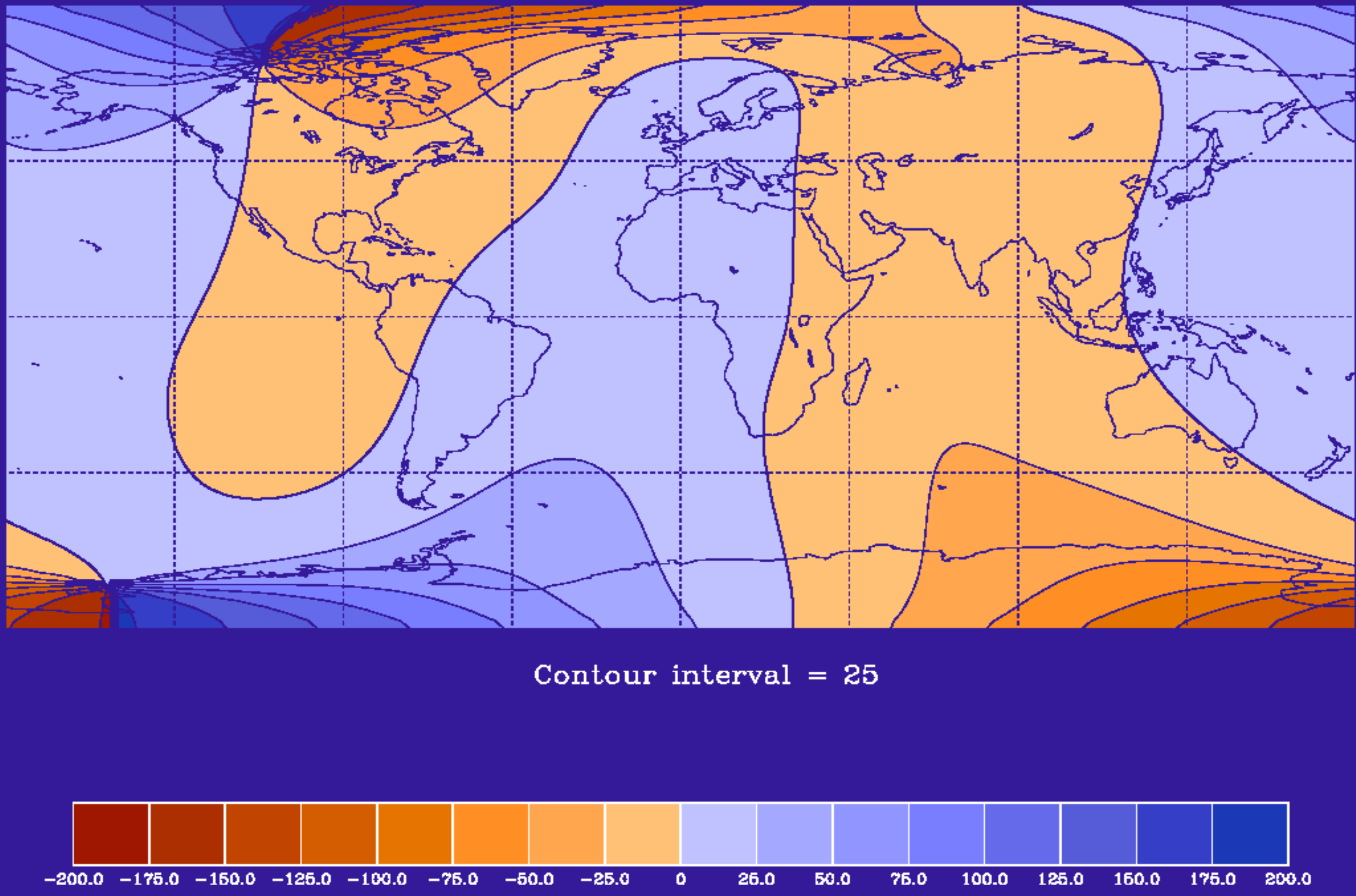
- Historical core field
  - General morphology (400 years)
- Core fluid motions
  - Effect on Earth's spin rate
- Retrieval of core motions
  - The frozen flux hypothesis




Out
N
true north
D
Y
H
magnetic north
W
E
S
I
X
F
Z
In

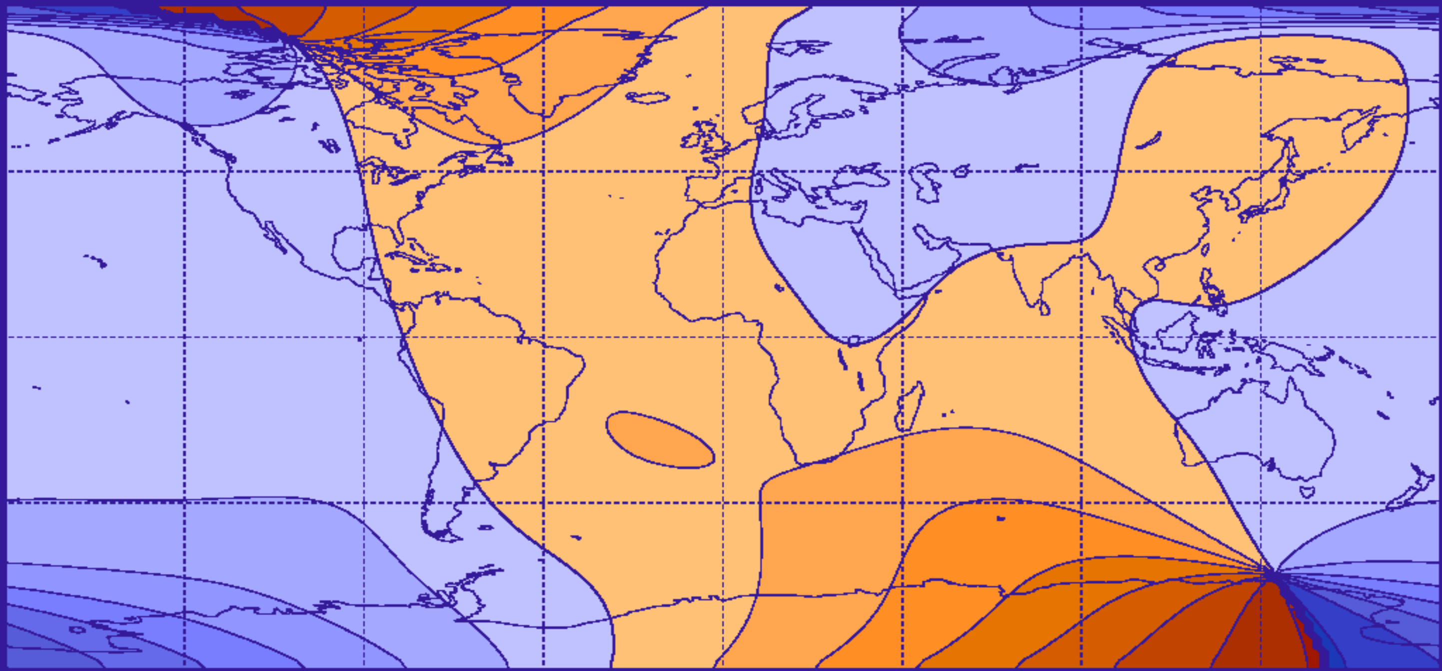

London Declination
Declination/degrees
10
5
0
−5
−10
−15
−20
−25
1600
1700
1800
1900
2000
Year

Declination 1590

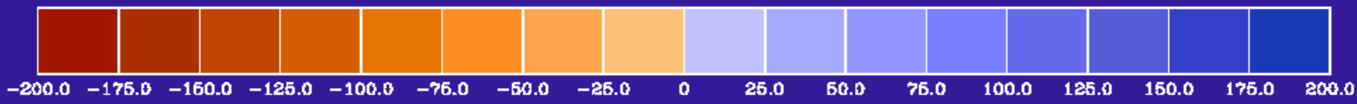


Contour interval = 25

Declination 2000

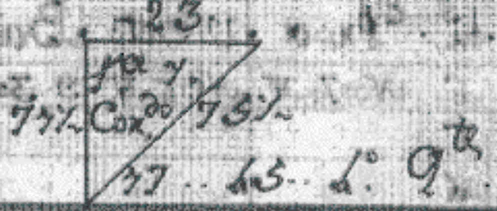


Contour interval = 25

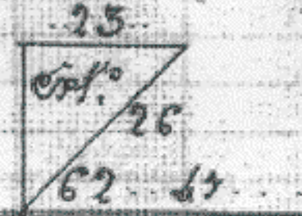


De el dia 12 Sabado al medio dia al Domingo 13 a la misma ora de este mes.
Lat.d P. N. 26..44.. y de la Long.d de 345 33 1/4

Hor.s	m.s	br.s	Proa	Viento	Bar.on	Ab.to	R.s Corr.s	G.os	D.s
1	1	..	N.3.N.	N1/4NE	9.30.No.	9 30 N:	45..30.	3.°	.18.
2	1	1							
3	1	1							
4	1	3							
5	1	3							
6	1	2							
7	1	..							
8	.0.	5							
9	1	..							
10	.0.	3½							
11	.0.	3½							
12	.0.	[illegible]							
1	.0.	..							
2	1	[illegible]							
3	.0.	3½							
4	.0.	3½							
5	.0.	..							
6	.0.	3½	NNO.6.O.	NE:	- - -	Nos O.	47..00.	4.°	.06.
7	.0.	3½							
8	.0.	3½							
9	1	..							
10	1	..							
11	1	2							
12	1	1½							



Lat.d	P	N. 26..44
Diferen.a	al	N. 01..11
Lat.d de Fant.a		N. 27..55
Lat.d obser.a		N. 27..54



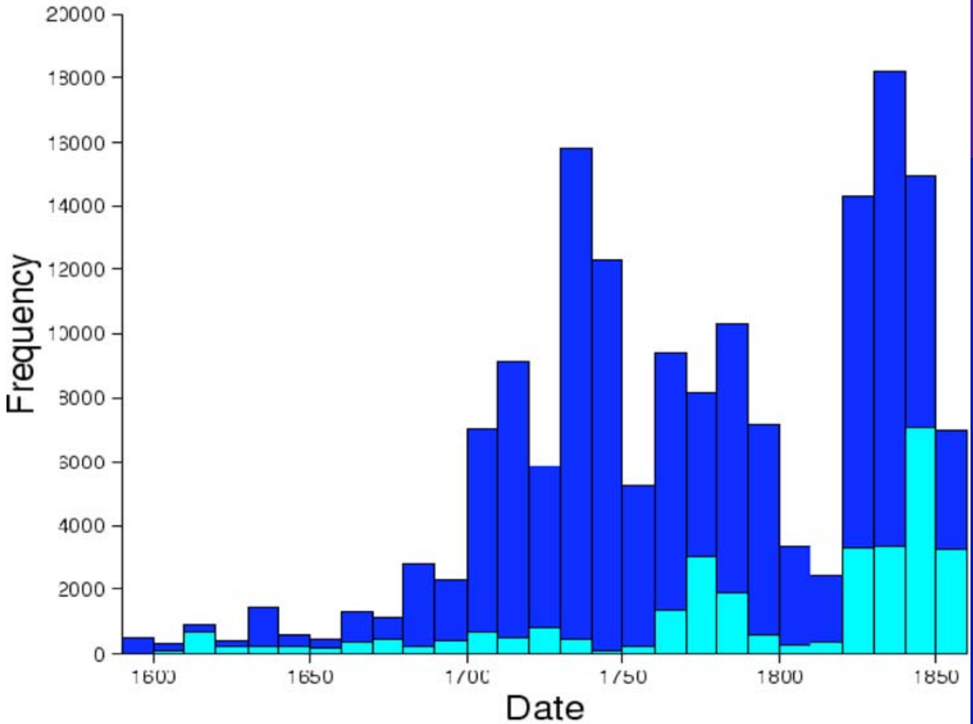
Long.d	P	de 345..33¼
Diferen.a	al	O. ..0..26
Long.d Arr.a		O. 345..05¼

Historical database

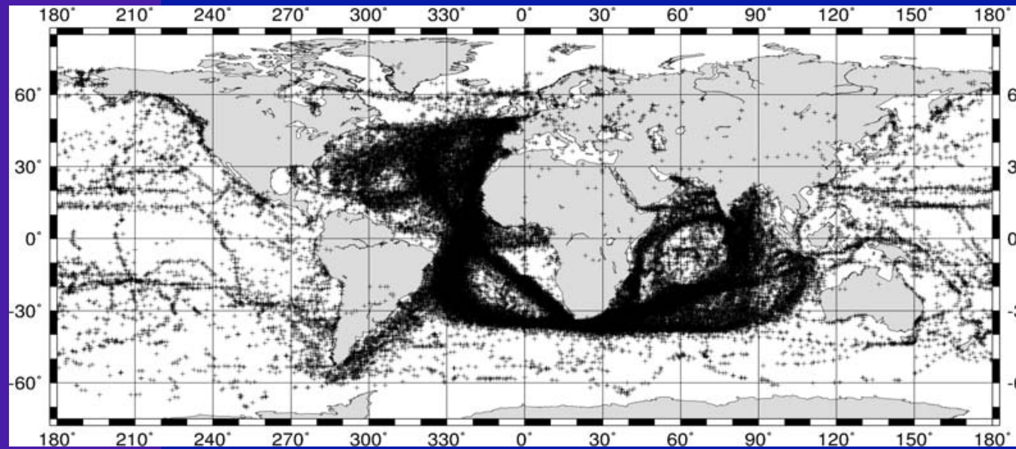
Over 80,000 data from 17th & 18th centuries

Trading Companies used in data compilation

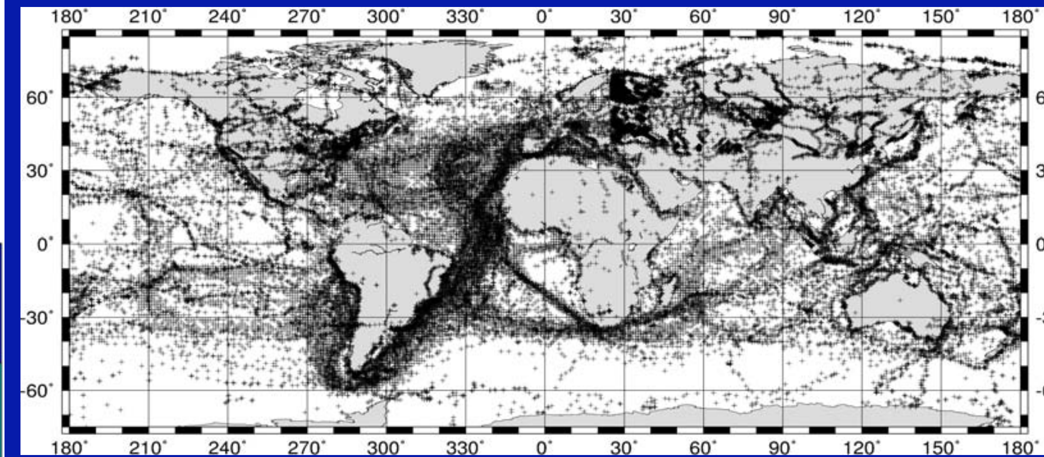
Compagnie des Indes 1664-1795
Danske Asiatiske Compagnie 1732-1844
East India Company 1600-1834
Hudson's Bay Company 1670-present
Real Companañía de Filipinas 1785-1834
Vereenigde Oost-Indische Compagnie 1602-1799
Manila Galleon ~1571-1813



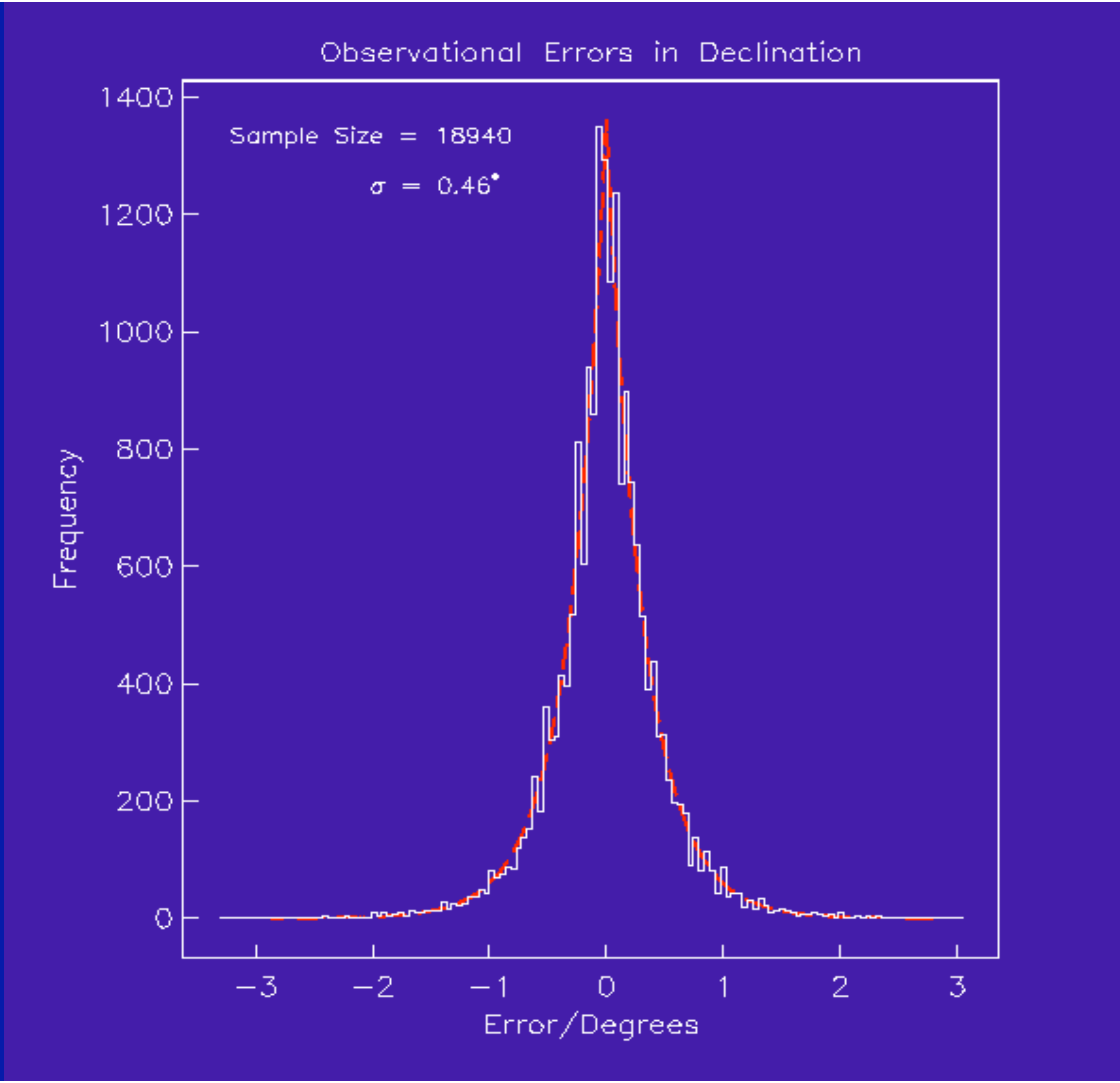
Declinations 1700-1799



Declinations 1800-1899



Jonkers *et al* 2003



Observational Errors in Declination
Sample Size = 18940
σ = 0.46°
Frequency
1400
1200
1000
800
600
400
200
0
−3
−2
−1
0
1
2
3
Error/Degrees

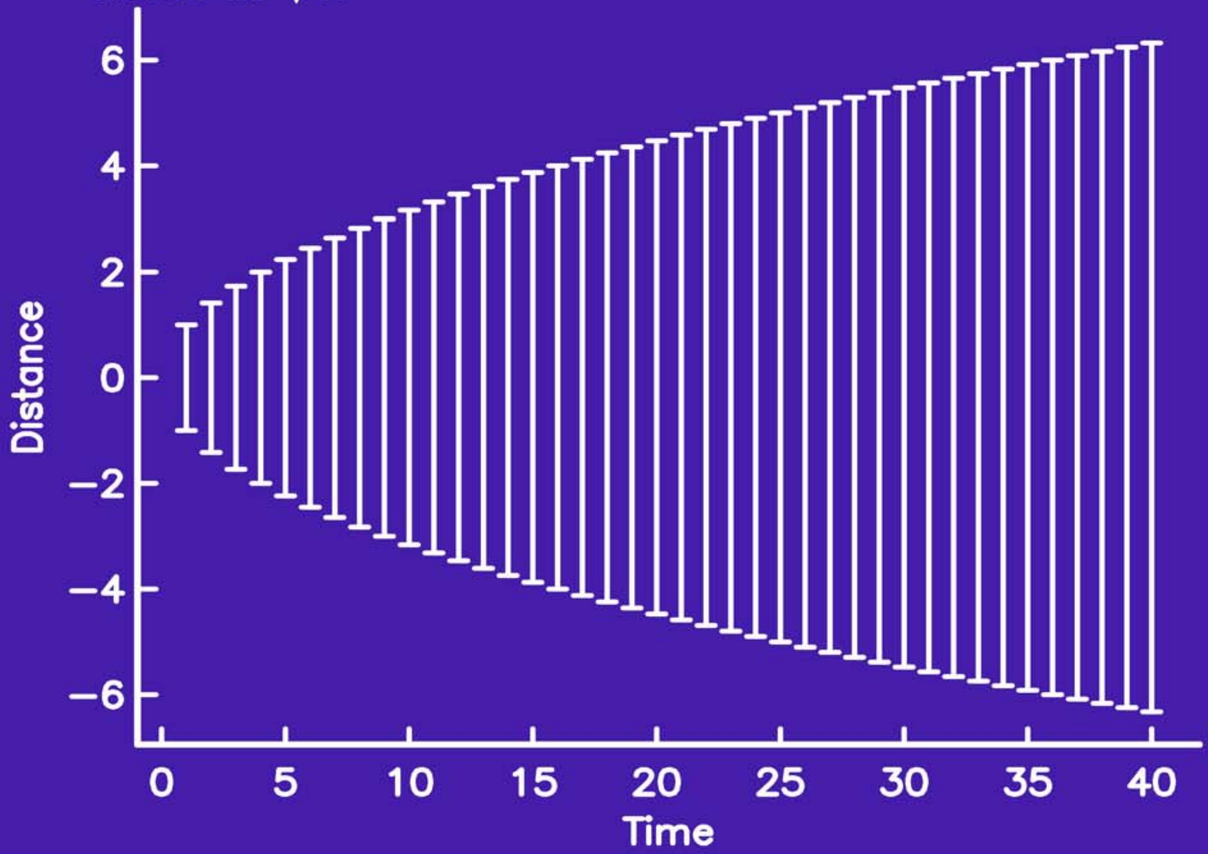
Navigation

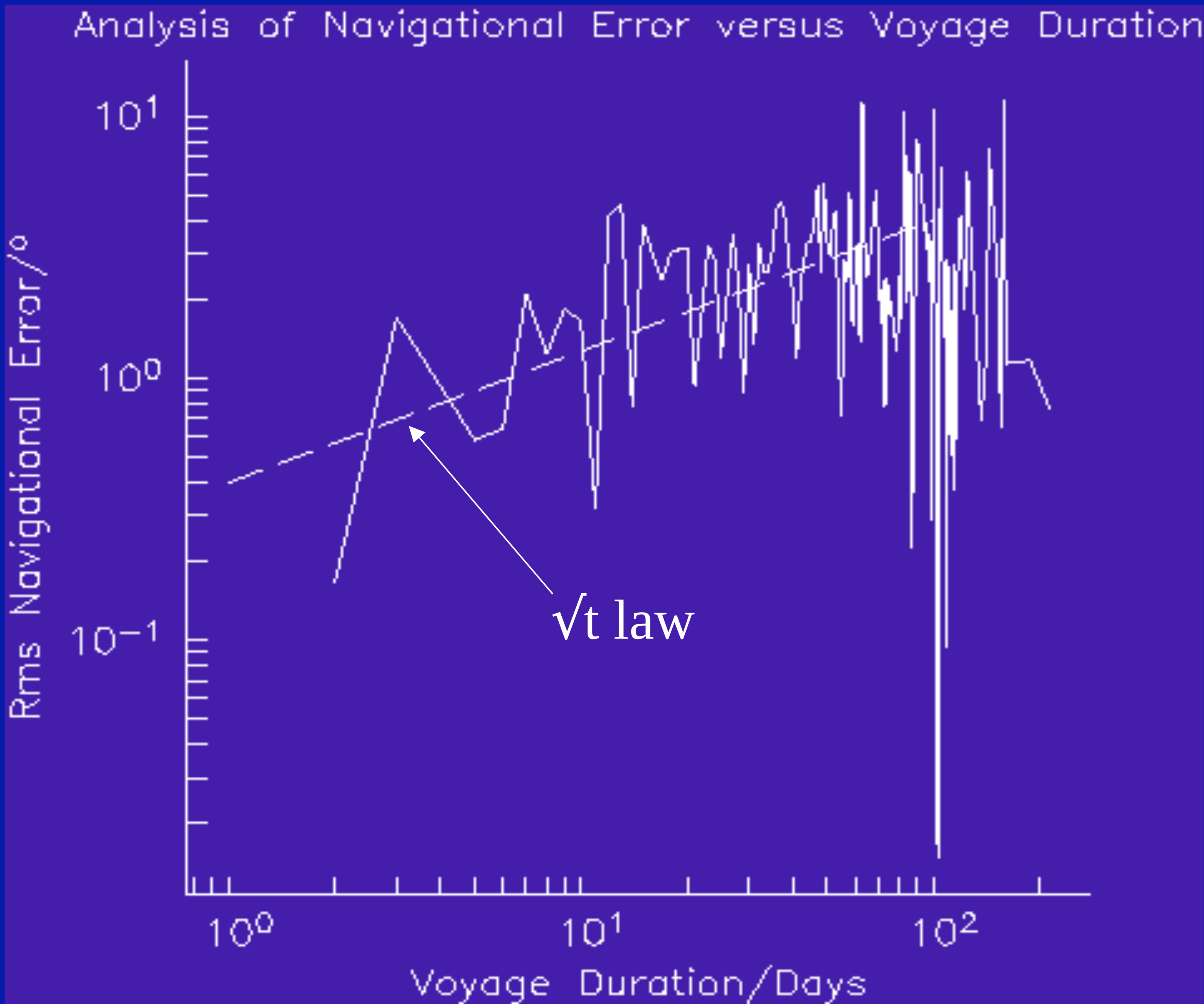
- Prior to the introduction of the marine chronometer by Harrison, longitude determination was by a process of “dead-reckoning”
- Relied on estimation of velocity and heading

Navigational Corrections

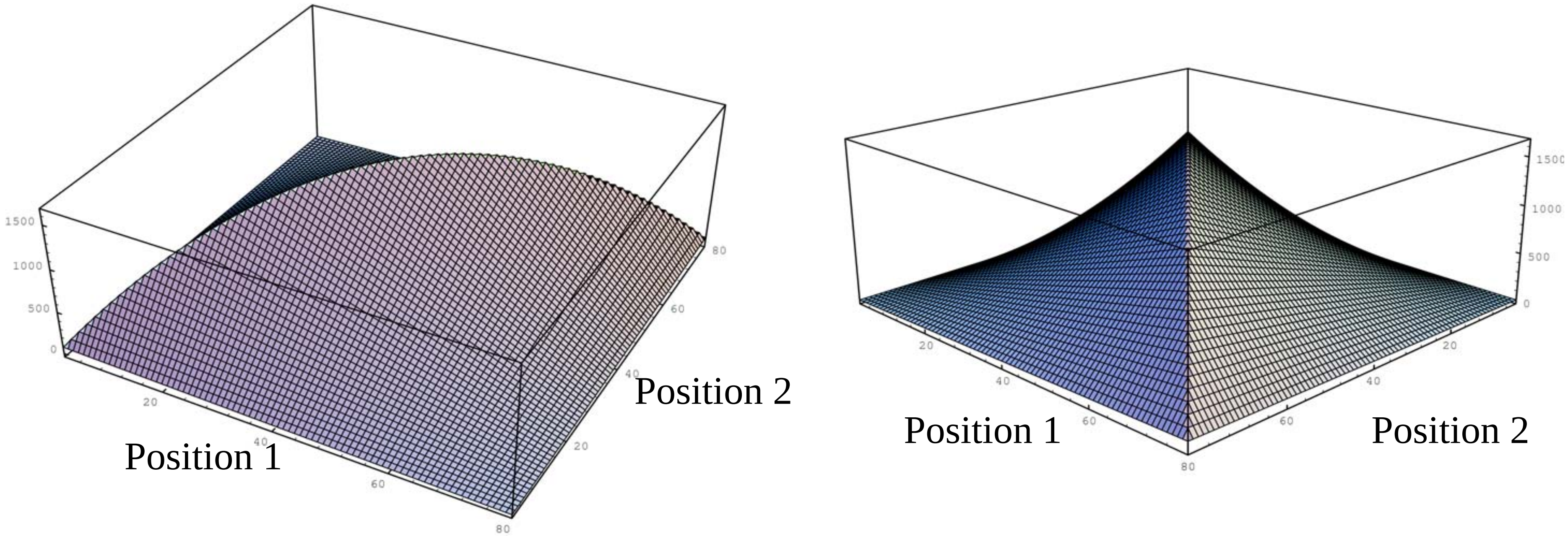
Drunkard (Bud) takes same number of random steps per minute.

Generally Bud's root mean square deviation increases with time t as $\sqrt{t}$.




Analysis of Navigational Error versus Voyage Duration
Rms Navigational Error/°
10^1
10^0
10^{-1}
10^0
10^1
10^2
Voyage Duration/Days
√t law

Brownian Bridge Covariance Matrix



Simple Error Budget

Major Contributions:

- **Observational Errors:**

  In a declination measurement, $\sigma = 0.46^\circ$

- **Positional Errors:**

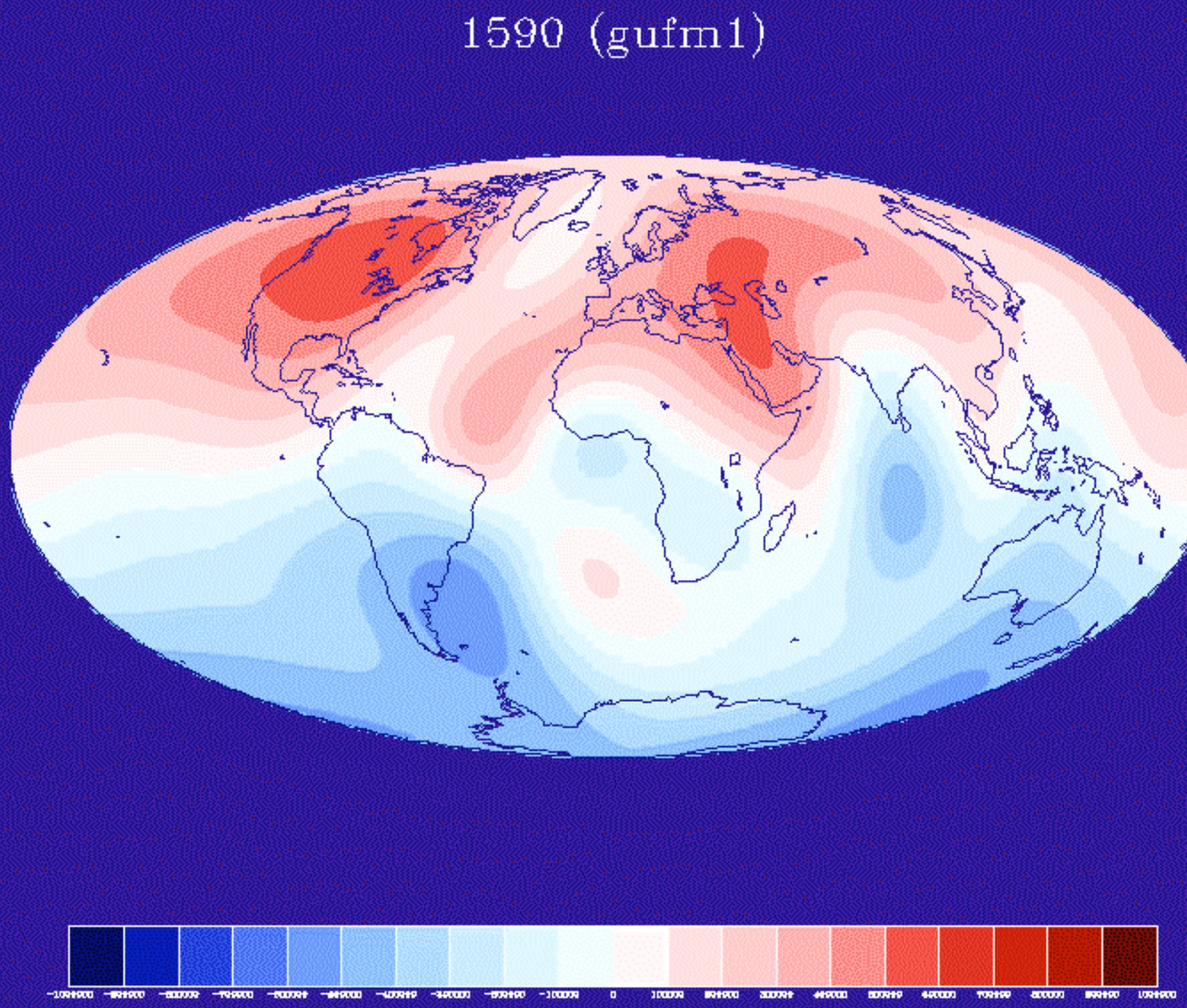
  One day increment in longitude has $\sigma \approx 0.4^\circ$

  Effect on declination varies locally

- **Errors due to magnetized crust:**

  Typically causes $\sigma \approx 0.5^\circ$

400 year animation of radial magnetic field

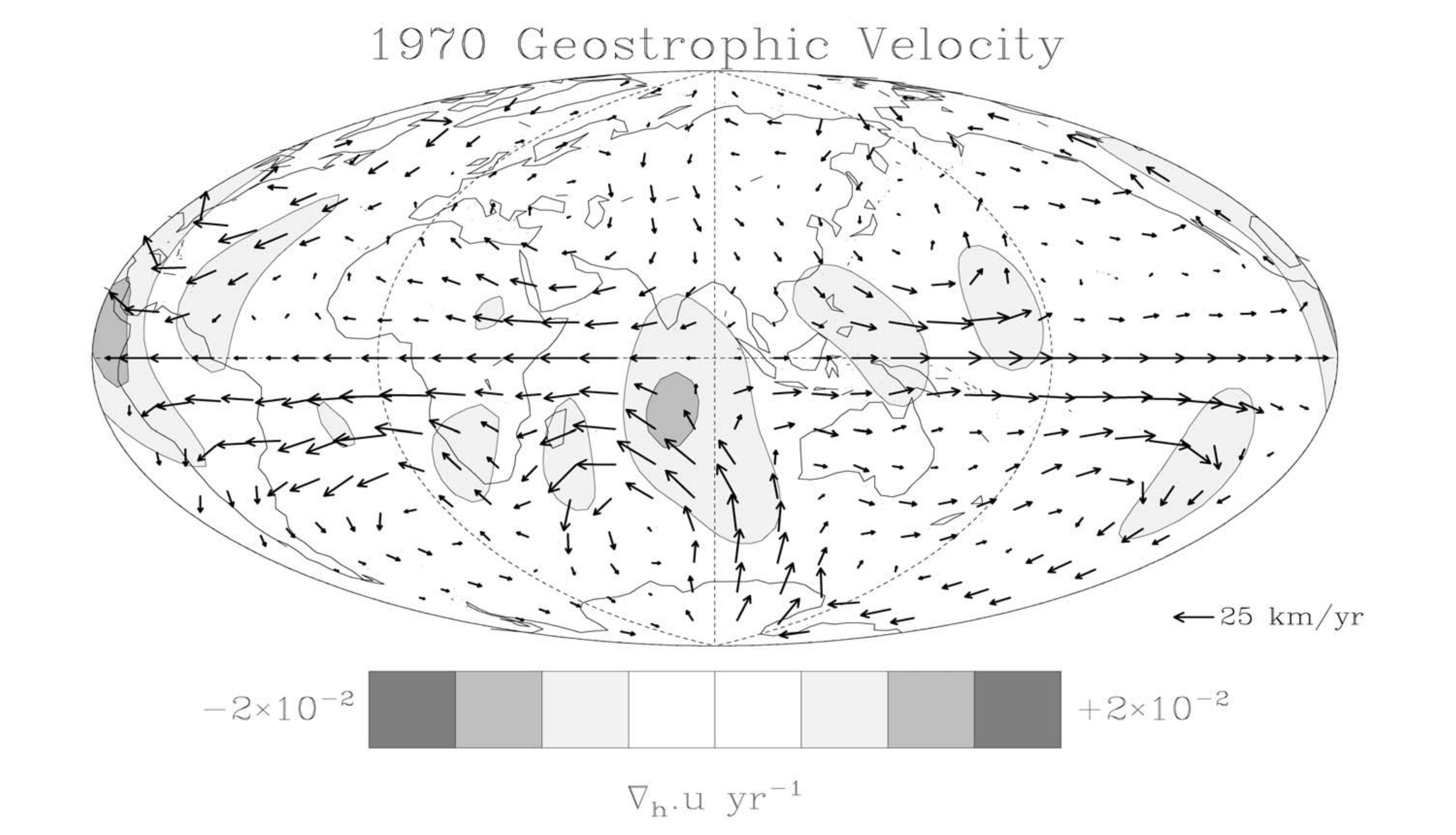


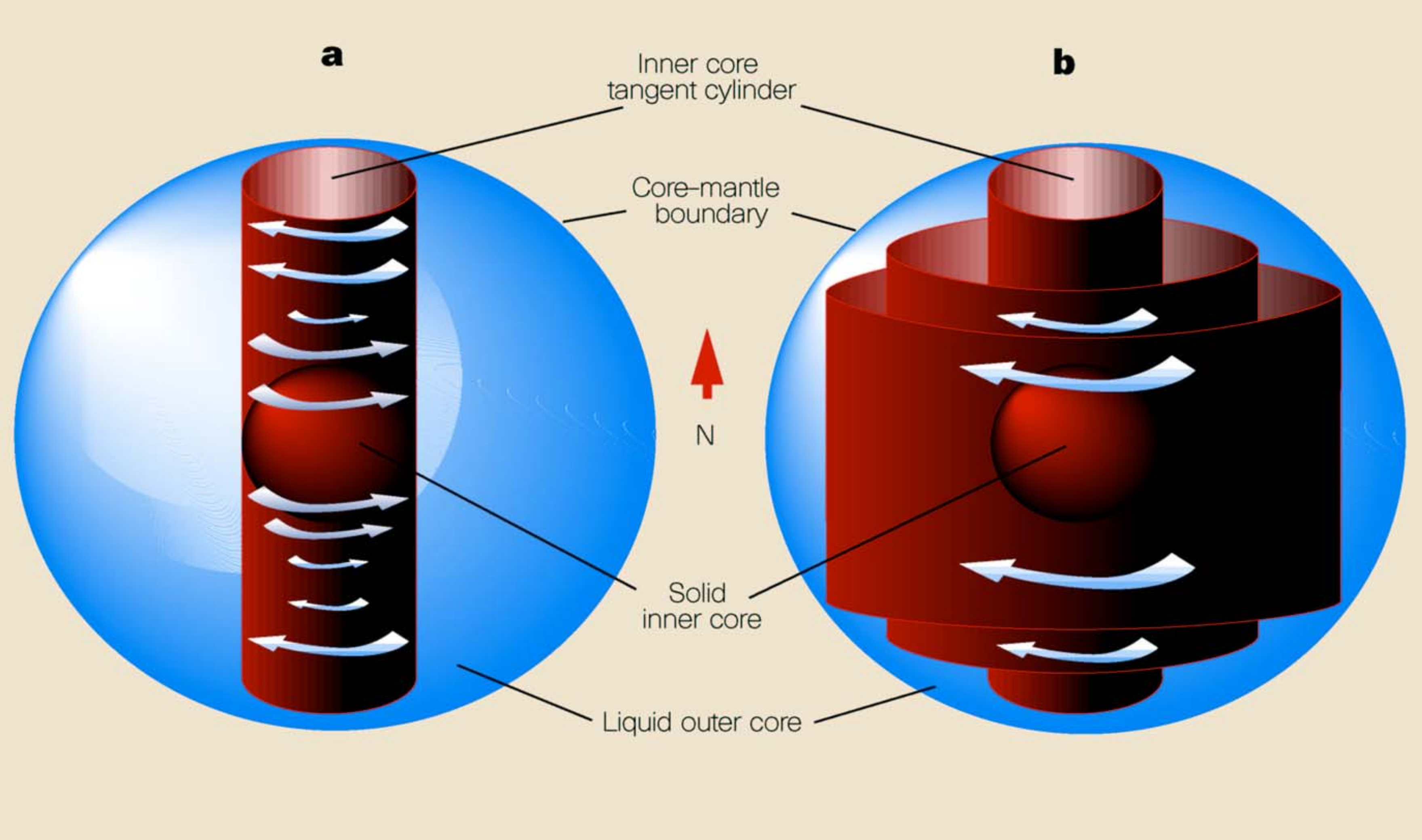
Current work (see poster/talk)

- Update of gufm1 to 1590-2005 (Finlay et al)

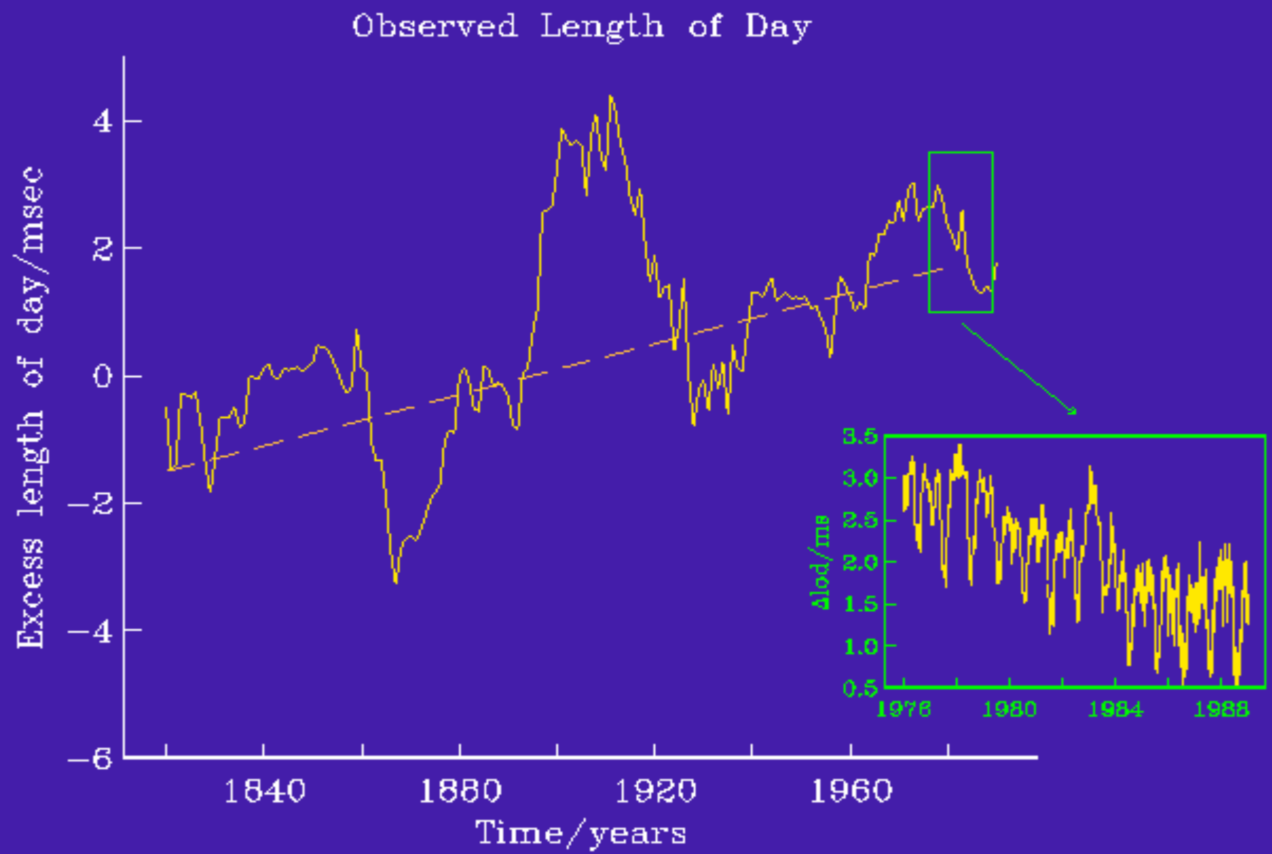
- New type of regularisation by Maximum Entropy (Gillet et al)

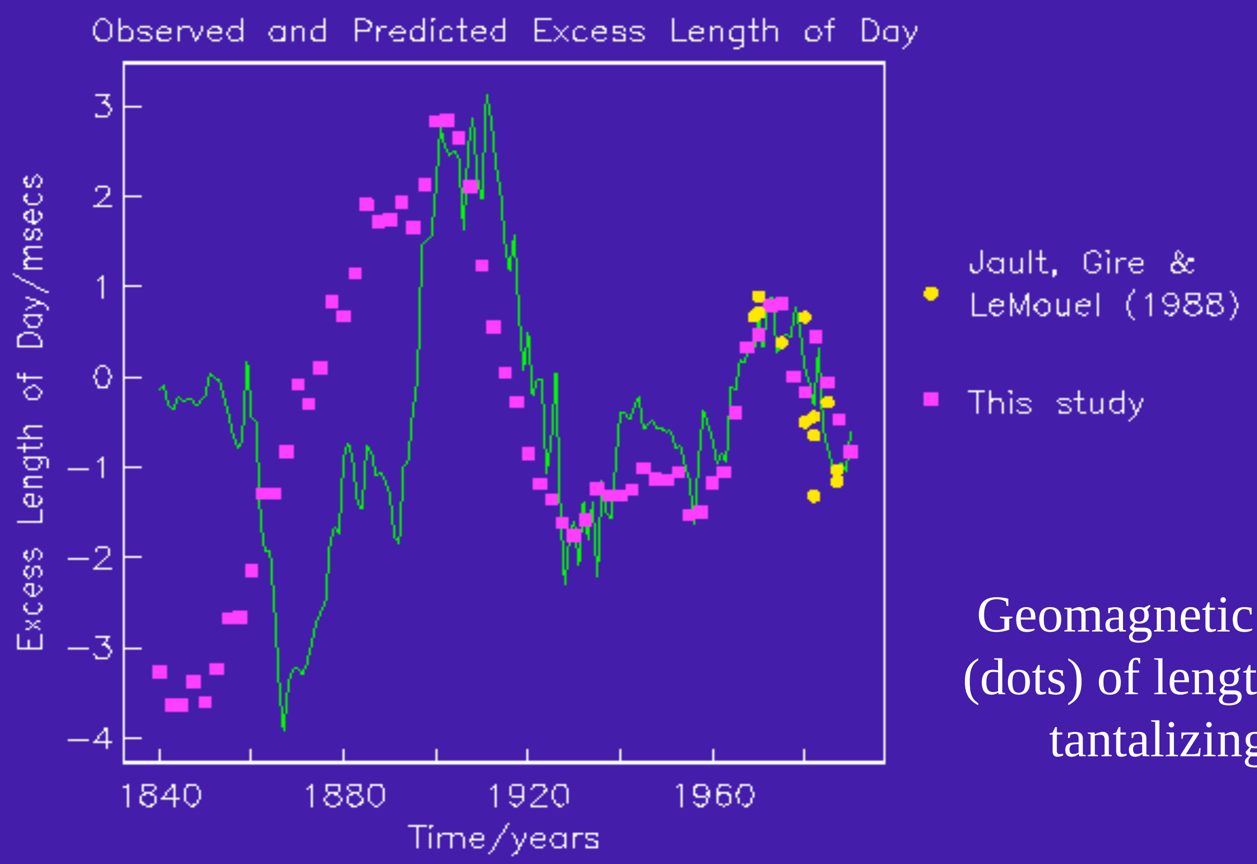
Example of flow retrieved for 1970





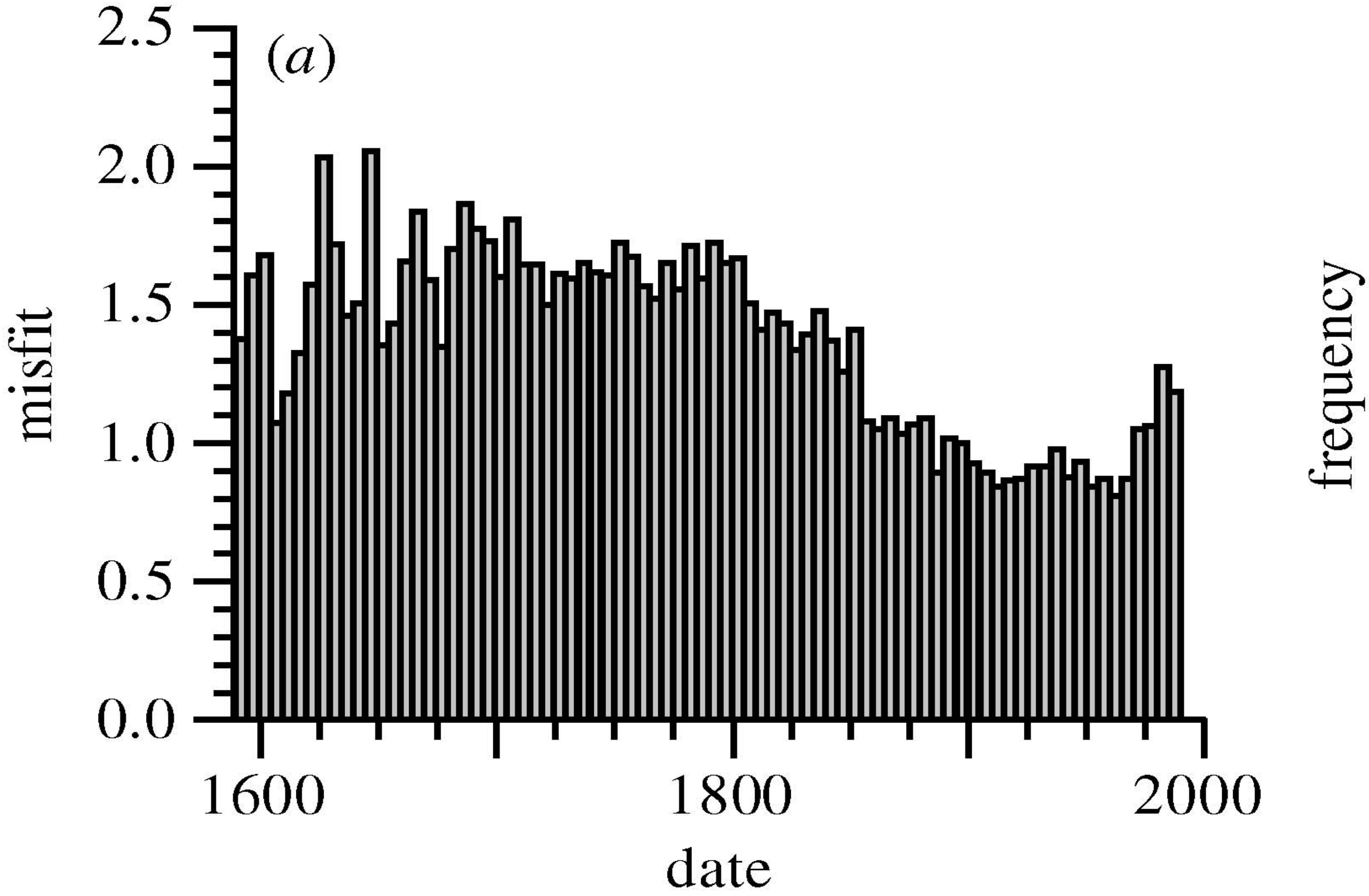
a
b
Inner core
tangent cylinder
Core-mantle
boundary
N
Solid
inner core
Liquid outer core


Observed Length of Day
Excess length of day/msec
4
2
0
−2
−4
−6
1840
1880
1920
1960
Time/years
3.5
3.0
2.5
2.0
1.5
1.0
0.5
Δlod/ms
1976
1980
1984
1988

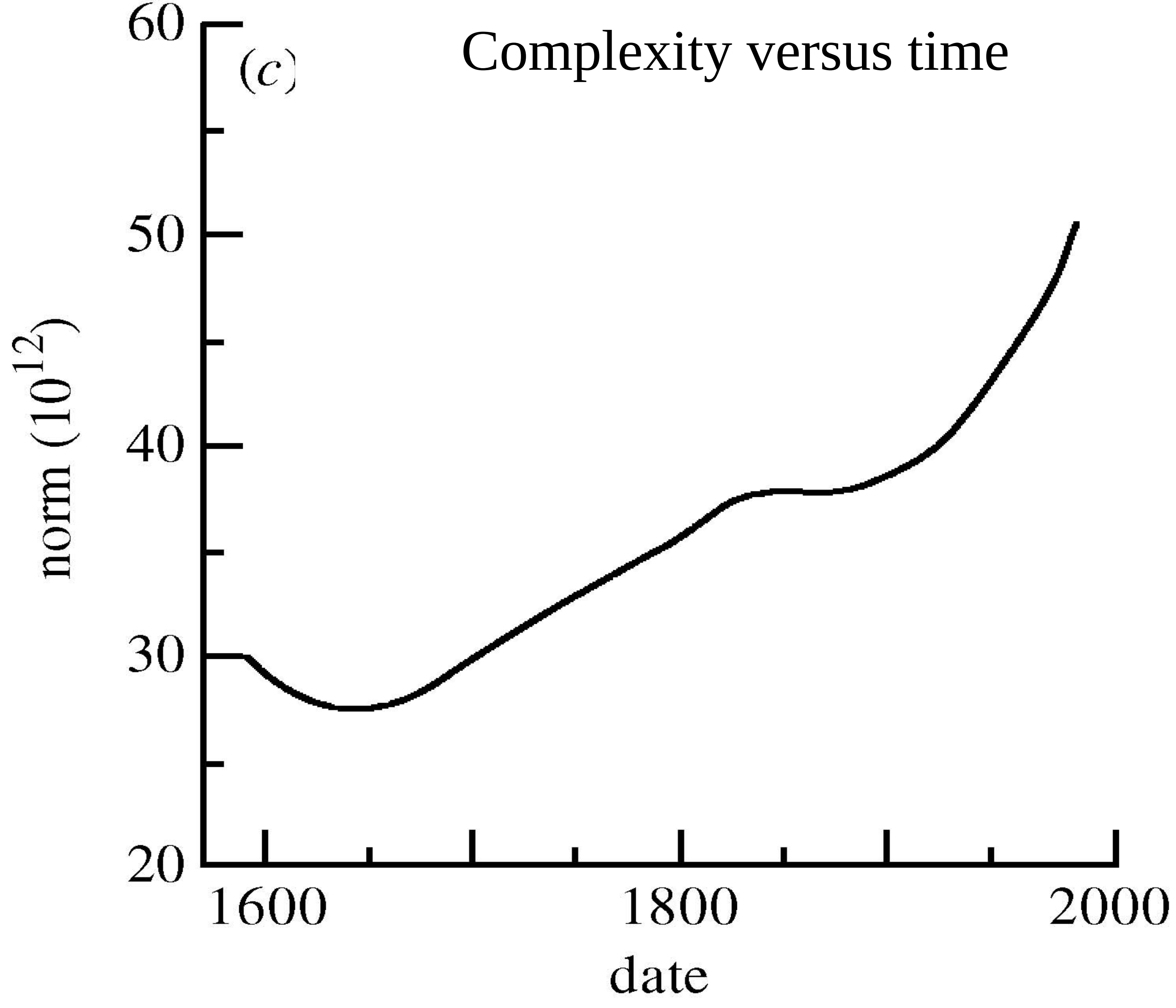


Geomagnetic predictions (dots) of length-of-day are tantalizingly good

Can we do better?

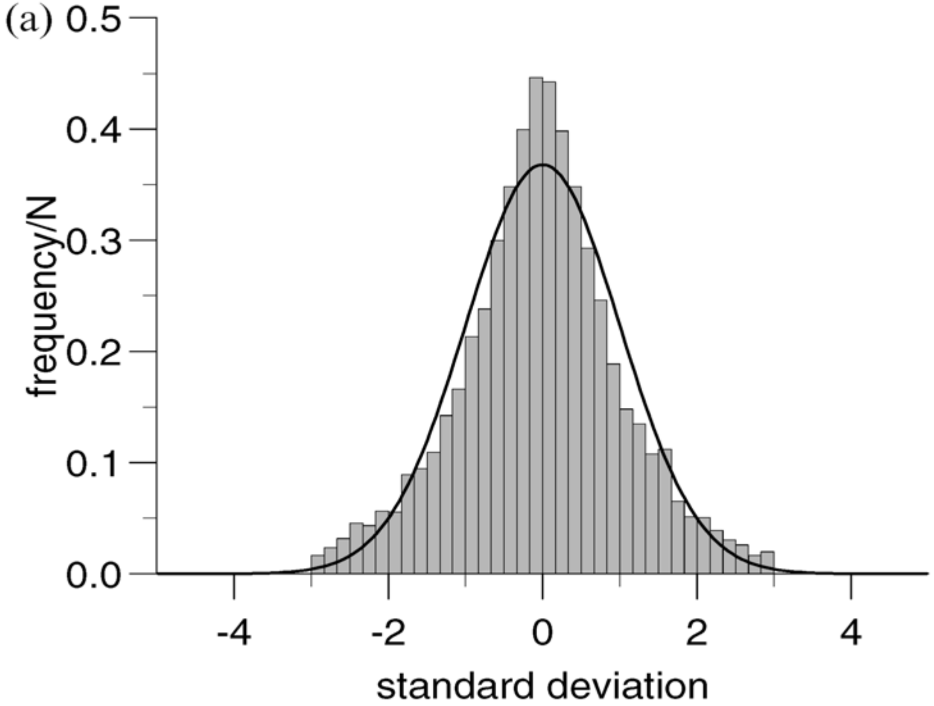


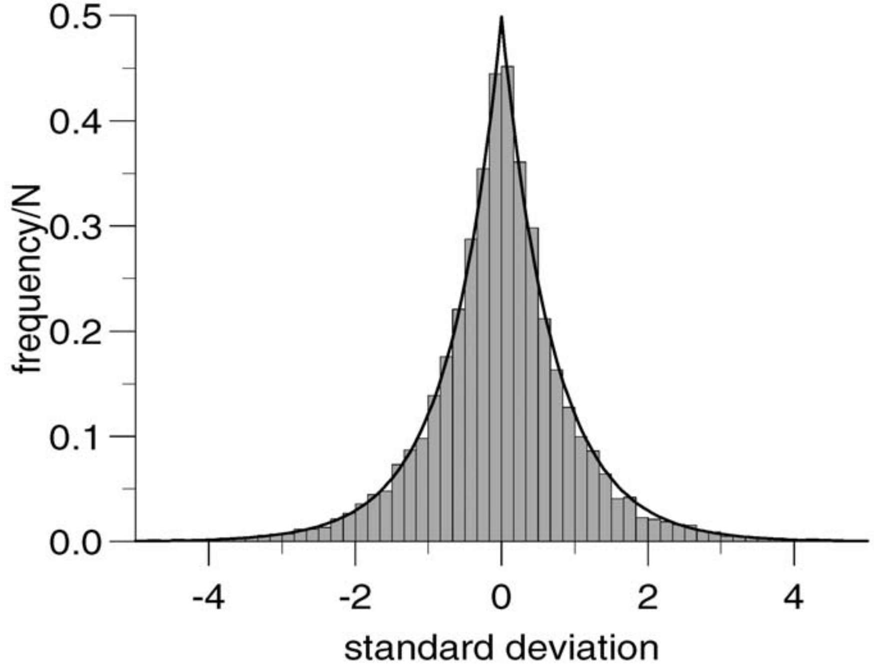
(a)
misfit
frequency
2.5
2.0
1.5
1.0
0.5
0.0
1600
1800
2000
date



Complexity versus time
(c)
60
50
40
30
20
norm (10^{12})
1600
1800
2000
date

Residuals to 1880 dataset





Two –norm fitting

One-norm fitting

Walker & Jackson (2000)

Calculation of core velocities

- To find core velocities $\mathbf{v}$, we use the frozen flux approximation and global models of $\mathbf{B}$ and $\partial_t \mathbf{B}$

$$\partial_t B_r = -\nabla_{\mathrm{h}} \cdot (\mathbf{v} B_r)$$

- But the models of $\mathbf{B}$ and $\partial_t \mathbf{B}$ don't satisfy constraints demanded by the physics
- There is a problem of a lack of self-consistency

Are observations back in time compatible with necessary conditions for self-consistency?

Consequence of Frozen Flux

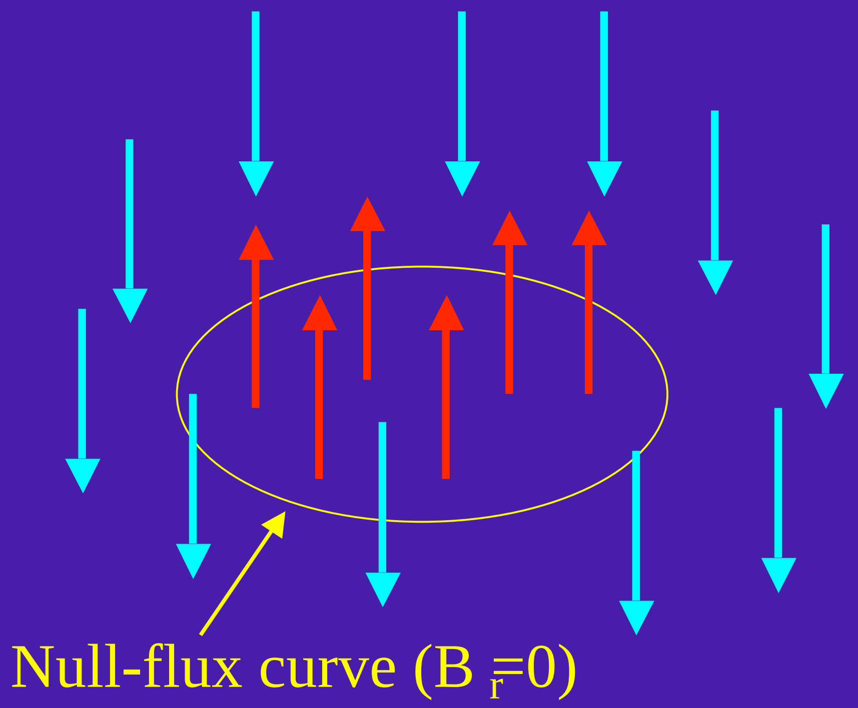
Integral constraints on field

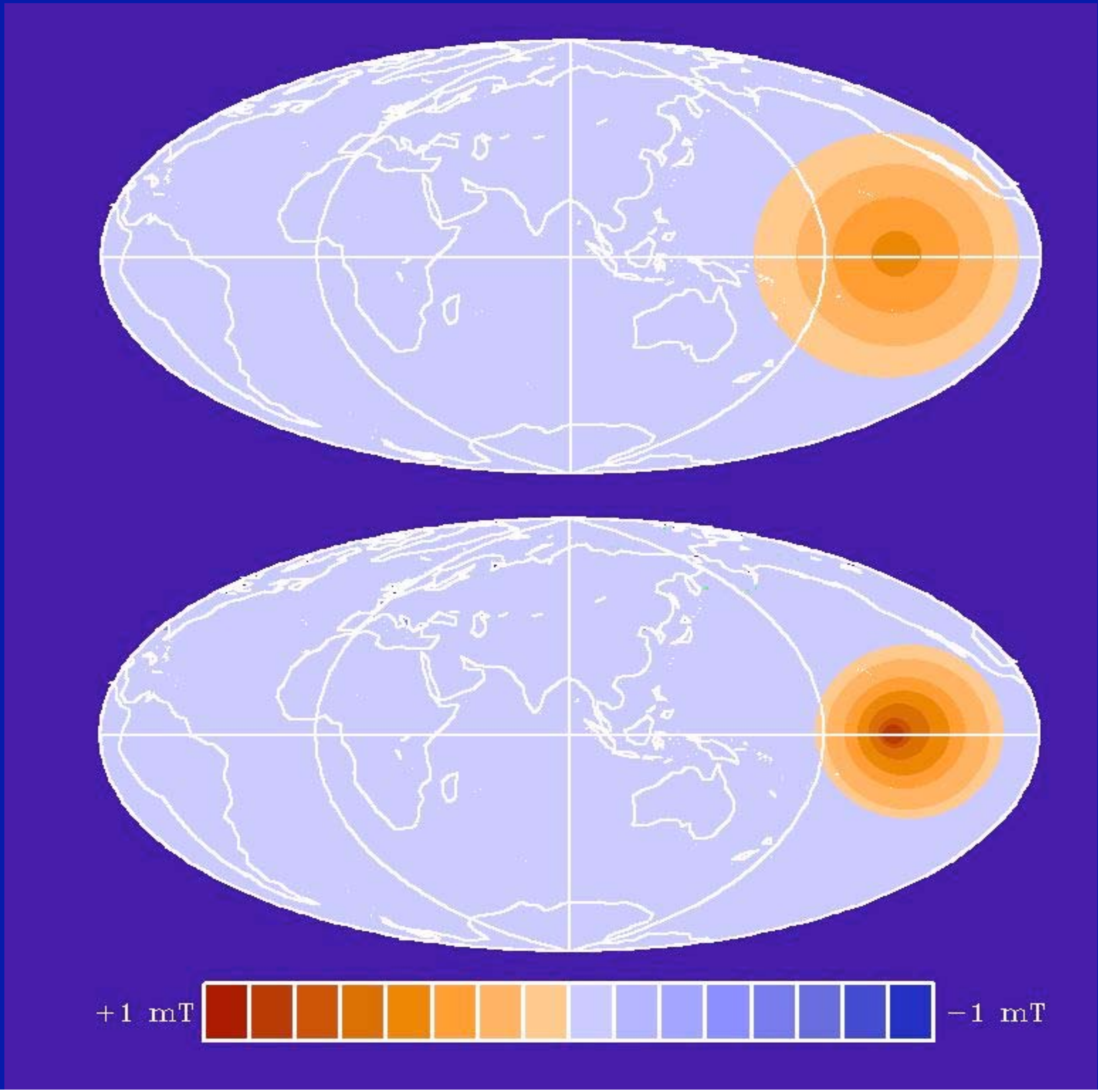
$$\mathcal{F}_i = \frac{d}{dt} \int_{S_i} B_r \, d\Omega = 0$$

and

$$B_r = 0 \text{ on } \partial S_i$$

Backus (1968)



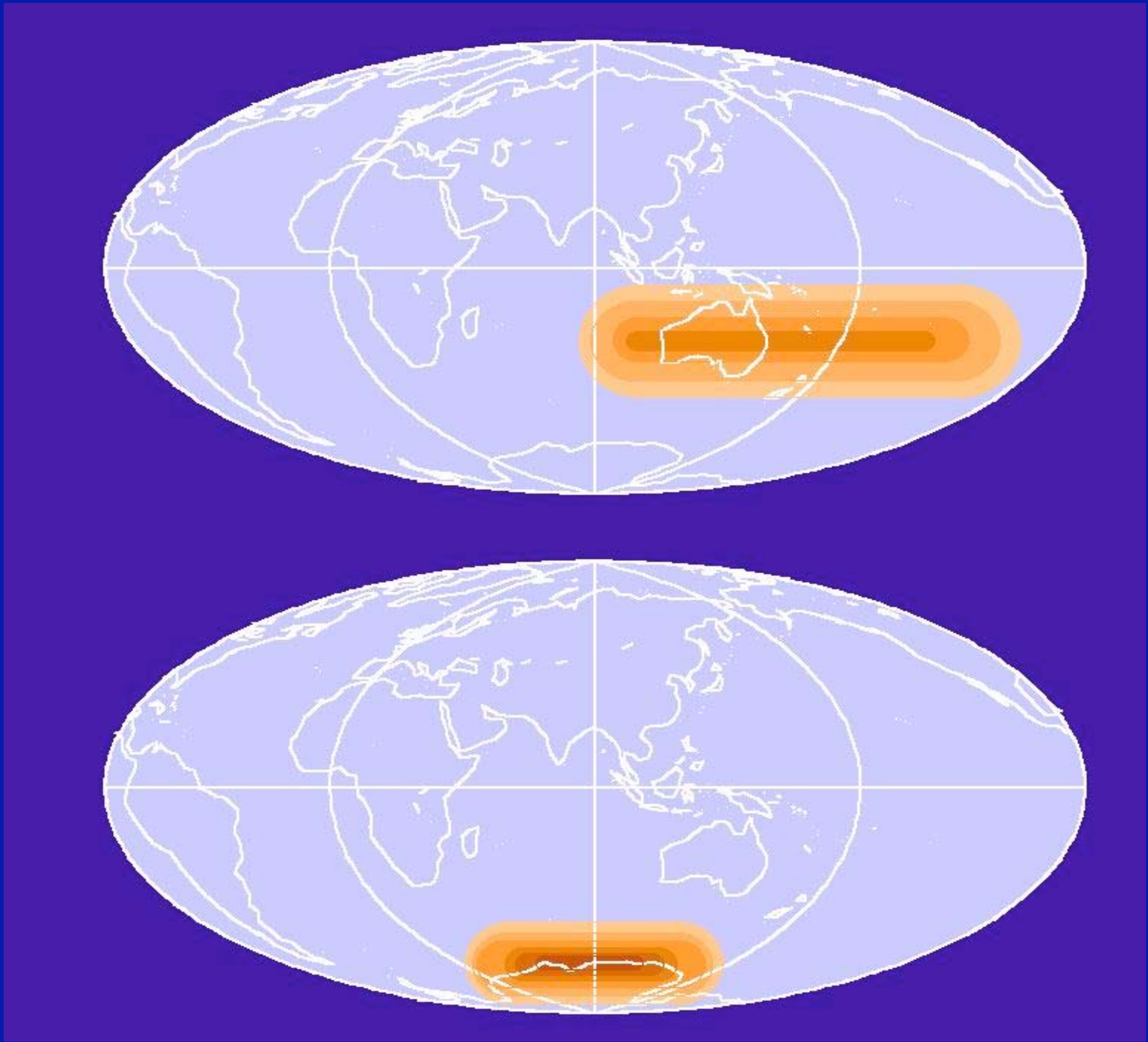


Flux conservation: Two different times

- Horizontal Lorentz force vanishes on ∂S_i
- Therefore null-flux curves move geostrophically, even in the magnetostrophic limit
- Null flux curves are material curves

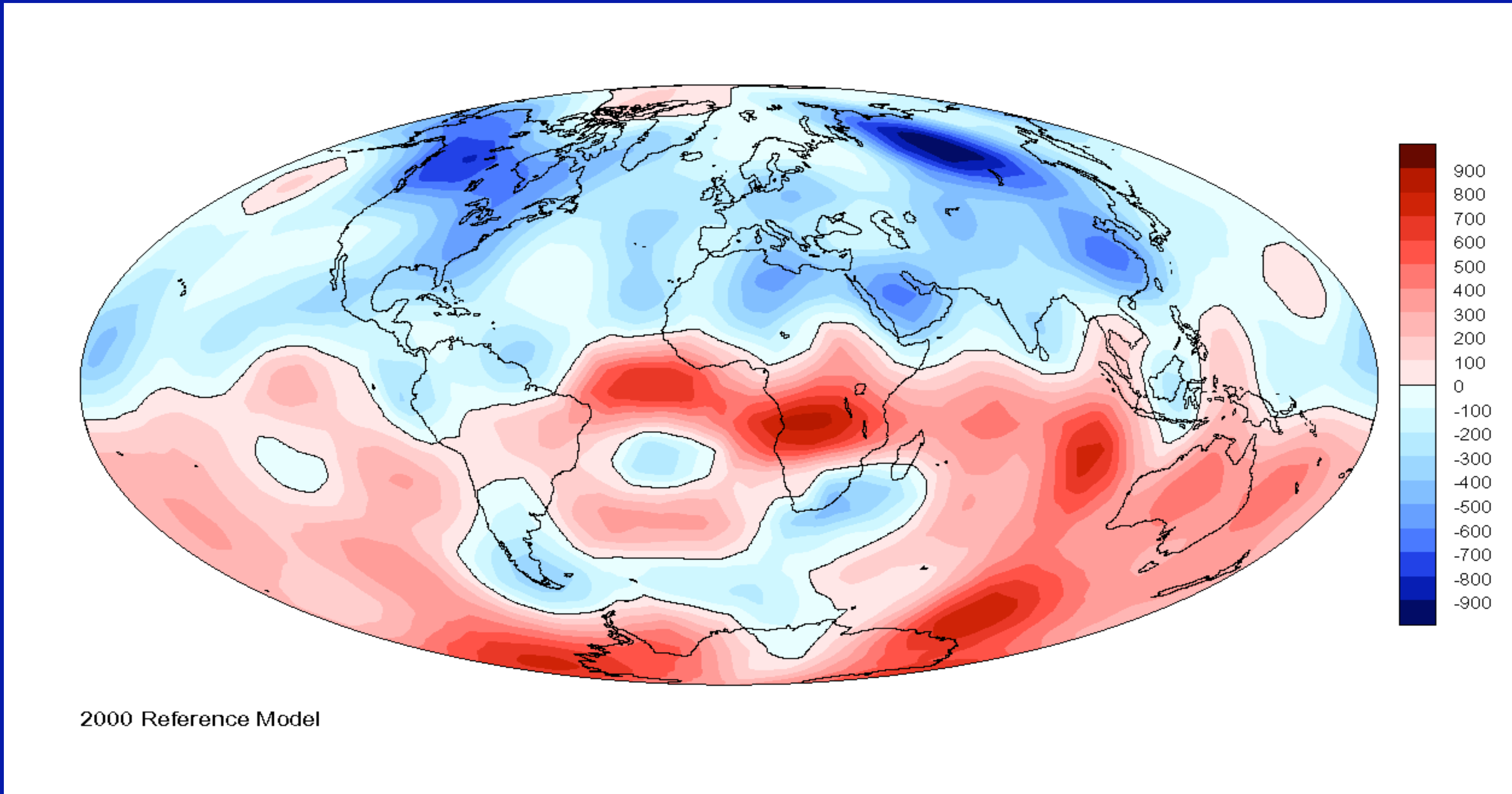
<u>Kelvin's theorem for inviscid fluids applies to patches bounded by null-flux curves</u>

i.e. Area of patch projected onto equatorial plane is invariant with time.



Radial vorticity conservation: Two different times

Reference Model: Oersted satellite data (2000)



Inverse Problem Methodology (1)

- Fit data using χ^2 criterion, which assumes each datum d_i is contaminated with Gaussian noise with variance σ_i^2.
- If e_i is the difference between observed and calculated values, we minimise

$$\chi^2 = \sum_{i=1}^{N} e_i^2/\sigma_i^2$$

  and aim to find $\chi^2/N = 1$.
- Perform nonlinear optimisation using Newton-type method to iteratively improve fit to data and constraints.
- Starting model supplies topology, which is automatically retained during the iterative improvement by solving bounded-value least-square problem using algorithm BVLS of Stark and Parker (1988).
- In areas where flux is required to remain positive we impose a lower bound on B_r of $|\epsilon|$, and in negative flux regions supply an upper bound of $-|\epsilon|$.

Inverse Problem Methodology (2)

- At each iteration the Frechet derivatives for the data and the constraints with respect to the model parameters are recomputed, until convergence is reached.
- We regularise the inversion by minimizing

  $$N = \int_S |\nabla_h B_r|^2 d^2\mathbf{s}$$

  This means we find the smoothest model (in the sense defined above) compatible with the data and constraints.
- Have to impose the additional no-monopole condition

  $$\int_S B_r d^2\mathbf{s} = 0$$

Model Calculations subject to constraints

- For each epoch 1980, 1945, 1915, 1882, attempt to fit datasets with same topology, fluxes and radial vorticities as in 2000 reference model
- Data taken from 10 year intervals; each dataset contains ~ 10-20,000 global observations
- Minimise

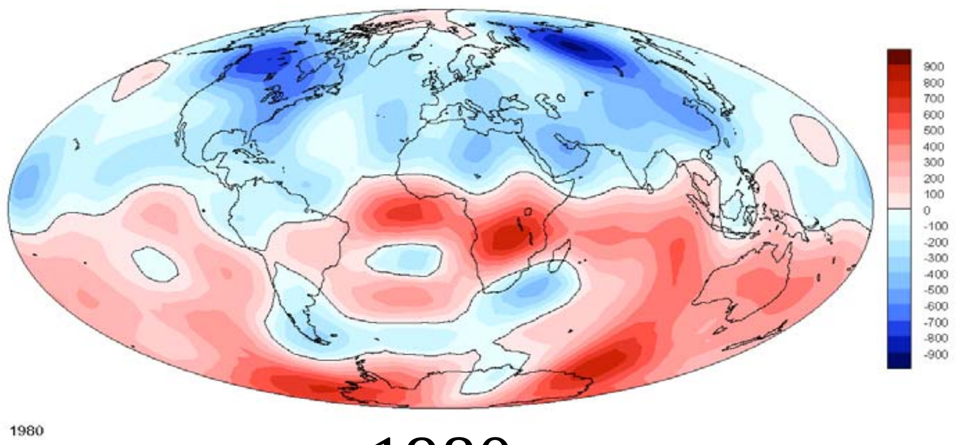
Data misfit — Roughness — Flux misfit

$$\Gamma(\mathbf{b}) = \sum_{j=1}^{M} \left(\frac{d_j - \mathcal{F}_j(\mathbf{b})}{\sigma_j} \right)^2 + \lambda_s \mathcal{R}(\mathbf{b}) + \lambda_f \sum_{j=1}^{P} \left(\mathcal{B}_j(\mathbf{b}) - \mathcal{B}_j(\mathbf{b}^*) \right)^2$$

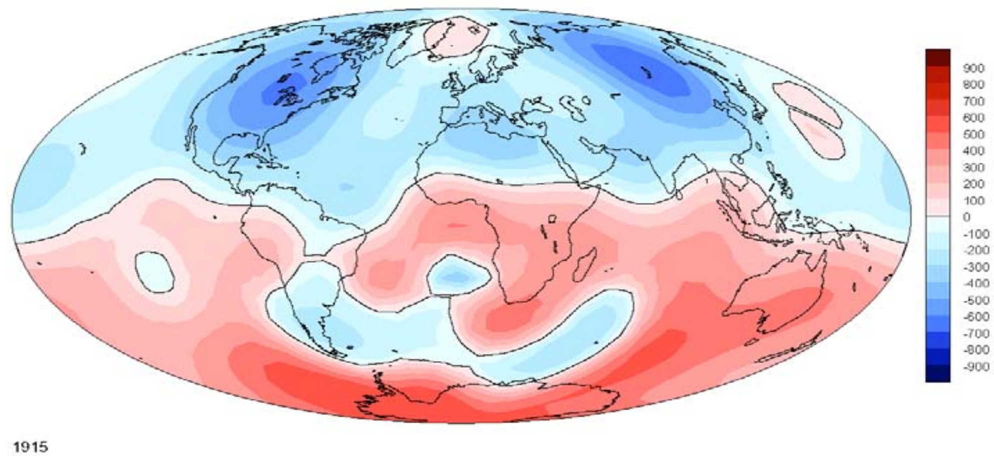
Radial vorticity misfit

$$+ \lambda_v \sum_{j=1}^{P} \left(\mathcal{V}_j(\mathbf{b}) - \mathcal{V}_j(\mathbf{b}^*) \right)^2$$

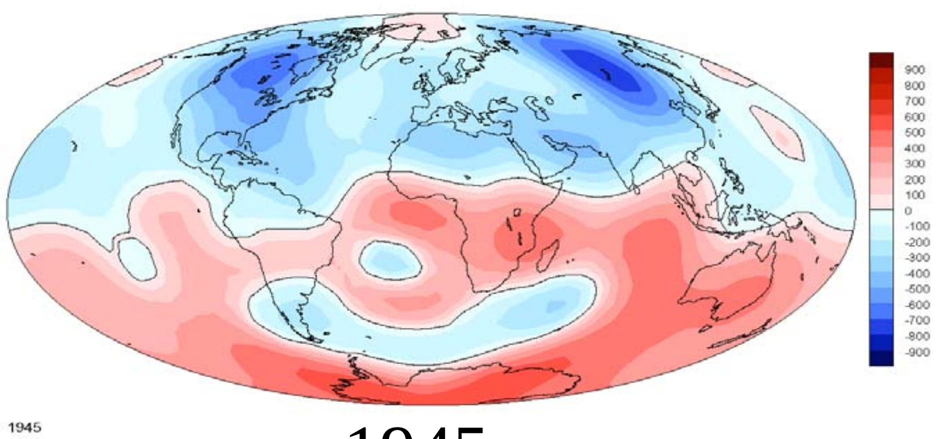
Flux and Radial Vorticity Constrained Models



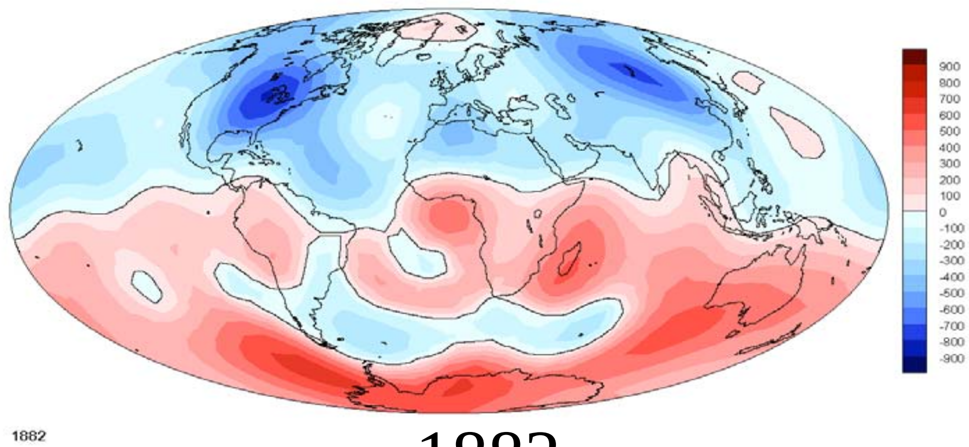
1980



1915

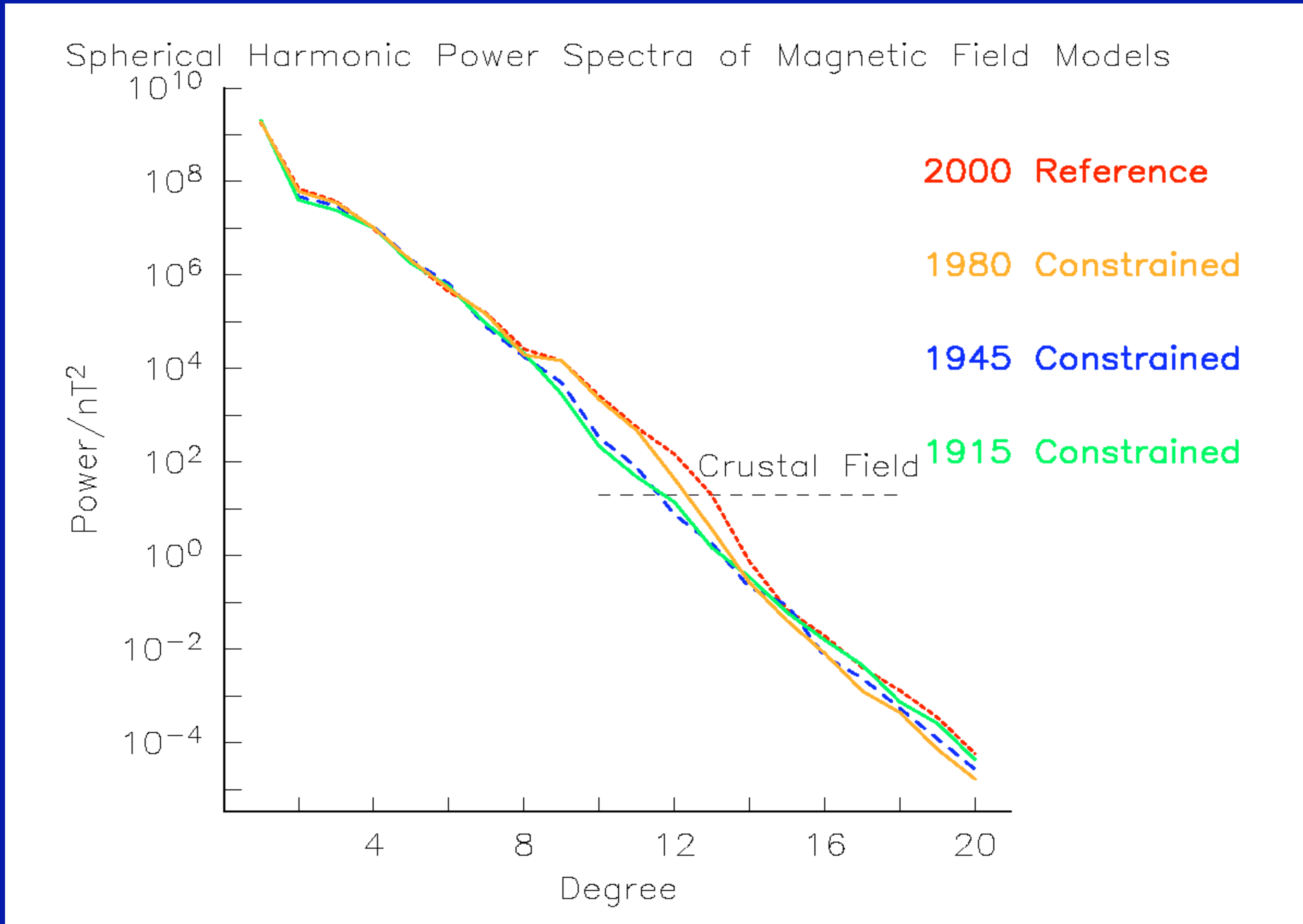


1945



1882

Model	Misfit $\mathcal{M}$	Roughness $\mathcal{R}$
2000	1.010	0.484 μT/km
1980	0.994	0.431 μT/km
1945	0.973	0.337 μT/km
1915	0.976	0.325 μT/km
1882	1.071	0.370 μT/km

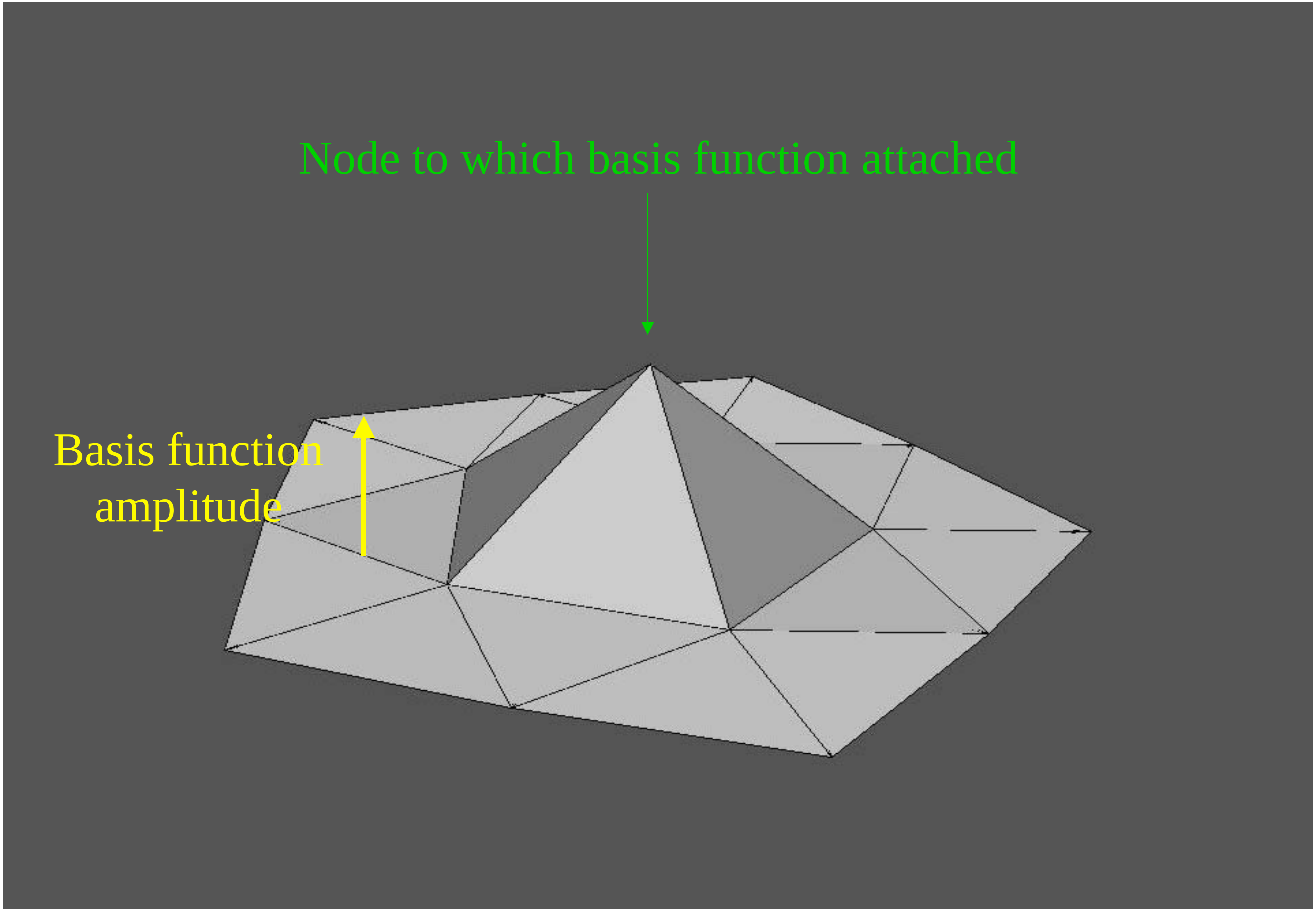


Spherical Harmonic Power Spectra of Magnetic Field Models
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
Power/nT^2
4
8
12
16
20
Degree
Crustal Field
2000 Reference
1980 Constrained
1945 Constrained
1915 Constrained

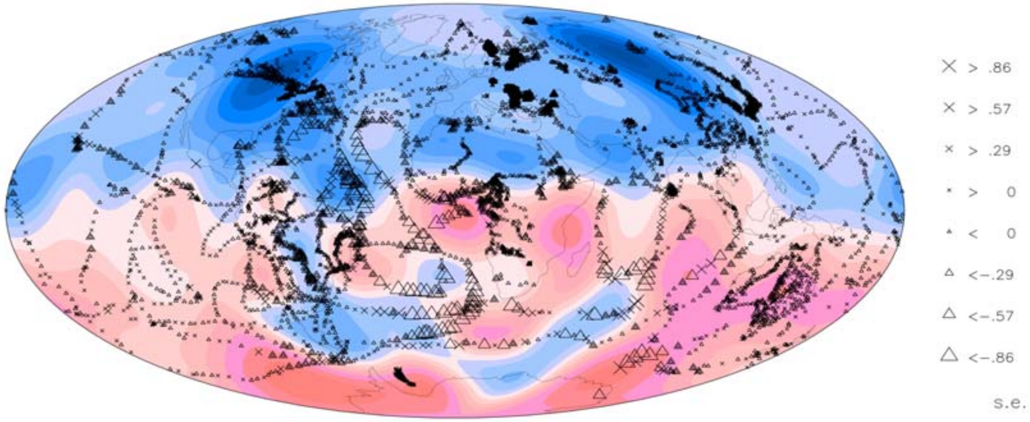
- No problem in satisfying the constraints back in time at individual epochs
- Next stage – develop time dependent model with constraints implemented

Summary

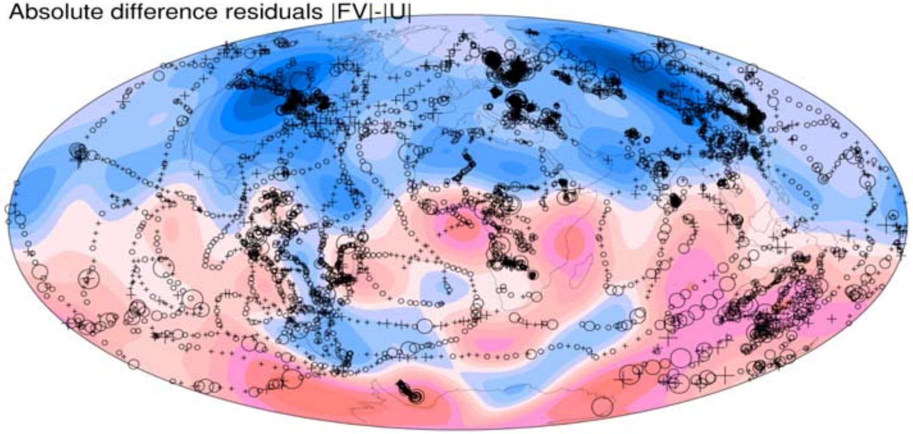
- Gufm1 and its extensions are good representations of the SV over the last 400 years
  - Large data set; sophisticated error budget
- Still some open questions
  - Effect of increase in complexity in the model
  - More accurate descriptions of error distributions?
  - Effective way to integrate with satellite data
- Current core motions results are encouraging
  - Real or ``apparent'' diffusion doesn't affect results too badly
  - Might we do better using self-consistent models of main field/SV?



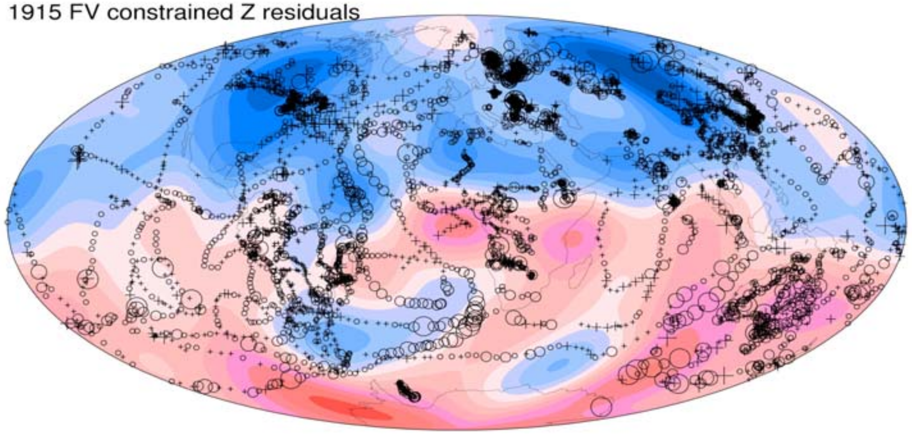
Node to which basis function attached
Basis function
amplitude



Absolute difference residuals |FV|-|U|



1915 FV constrained Z residuals



1915 Unconstrained Z residuals