Modelling the Core Magnetic Field Secular Variations using a Maximum Entropy Regularisation

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Usual Inverse Problem

Given a data set γ and a forward prediction function f…

… find the best solution of the problem $f(m) = \gamma$

… pb is ill-posed → add prior information (damping)

… minimize $\Theta = \|\text{misfit}\| + \lambda_S R_S(m) + \lambda_T R_T(m)$

Usually: quadratic regularisations in space & time:

$$R_S = \int_{t_s}^{t_e} \int_{CMB} \left|\nabla_h^{n_1} B_r\right|^2 d\Omega dt$$

$$R_T = \int_{t_s}^{t_e} \int_{CMB} (\partial_t^{n_2} B_r)^2 d\Omega dt$$

n_1 is 0 or 1
n_2 is 1 or 2

- Suppose *a priori* property of the final model
- Adds artificial correlation
- Decrease the power at small scales → loss of contrast

use instead entropy: another measure of the complexity that minimises the *a priori* on the final model (*Gull & Skilling [1990]*)

Instead of minimising a quadratic norm…

… maximise the entropy S

$$S[B_r, m_0] = -\int_\Omega \Phi[B_r, m_0]\, d\Omega$$

(Hobson & Lasenby [1998])

$$\Phi[B_r, m_0] = \psi - 2m_0 - B_r \log\left(\frac{\psi + B_r}{2m_0}\right), \text{with } \psi = \sqrt{B_r^{\,2} + 4m_0^{\,2}}$$

m_0 : the default parameter… corresponds to a flat map

Quadratic *versus* Max. Ent.

$$S[h, m_0] \xrightarrow[m_0 >> |h|]{} \frac{1}{4m_0} \int_\Omega |h(x)|^2 dx$$

… quadratic $|h|^2$ damping is used for the comparison…

… finally the regularisation functions become:

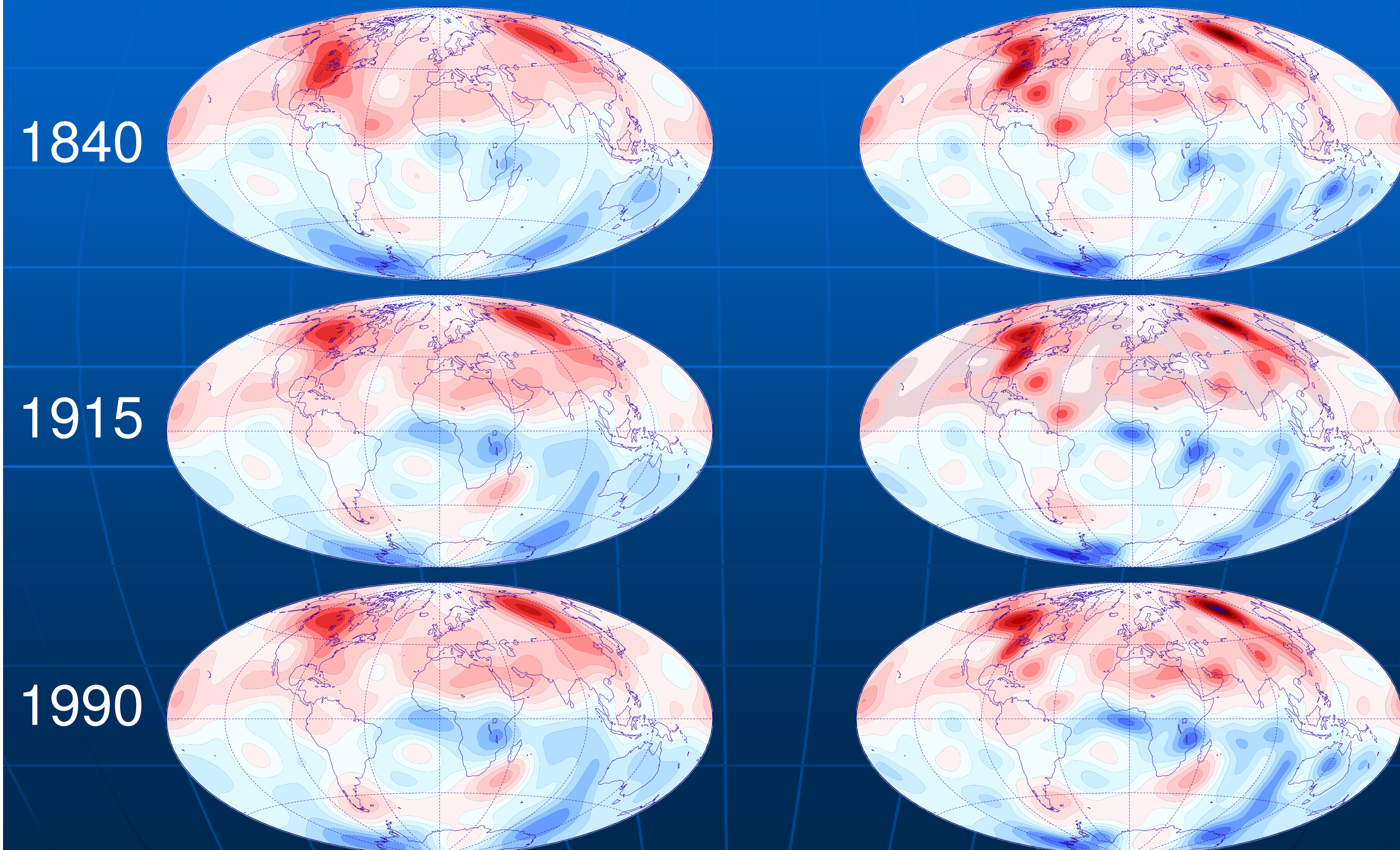
$$R_S = 4m_S \int_{ts}^{te} \int_{CMB} \Phi[B_r, m_S] d\Omega dt$$

$$R_T = 4m_T \int_{ts}^{te} \int_{CMB} \Phi[\partial_t B_r, m_T] d\Omega dt$$

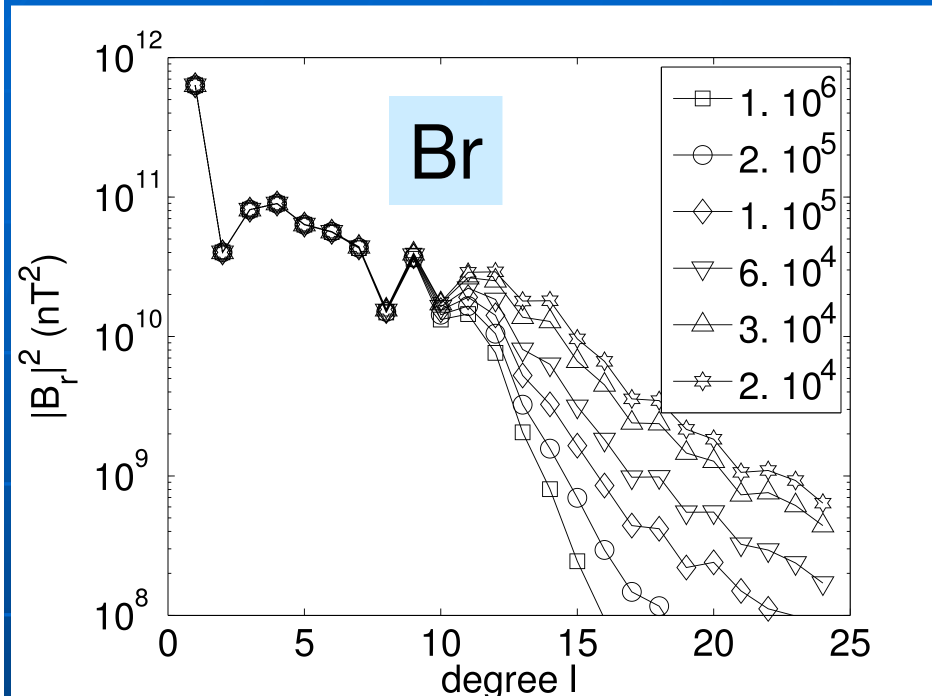
- Test on periode 1840-1990: L=24, N=63
  39 312 free parameters
- Data sets: (from *gufm1, Jackson et al. [2000]*):
  observatories, surveys, satellites
  ~ 262 400 data
- We fix $\lambda_S = 10^{-10}$ & $\lambda_T = 5.10^{-6}$, such as $\chi^2 \sim 0.94\ N_{dat}$

$m_S=10^7$ $m_T=10^6$
rms=0.9479
N_{dat}=262410

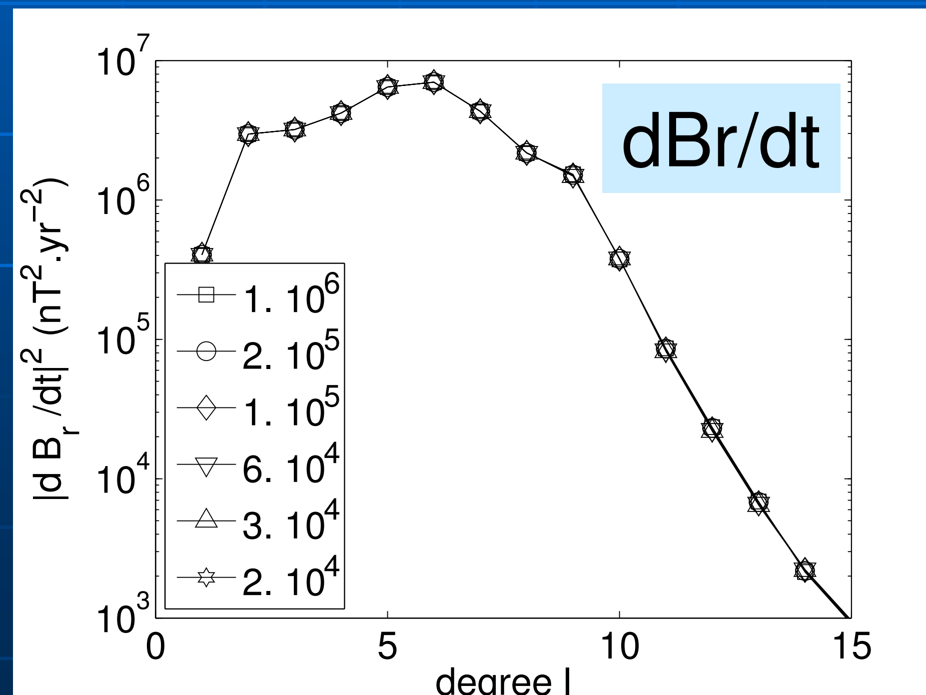
$m_S=3.10^4$ $m_T=10^6$
rms=0.9470
Ndat=262407



Effect of m_S on spectrums ($m_T=10^6$)



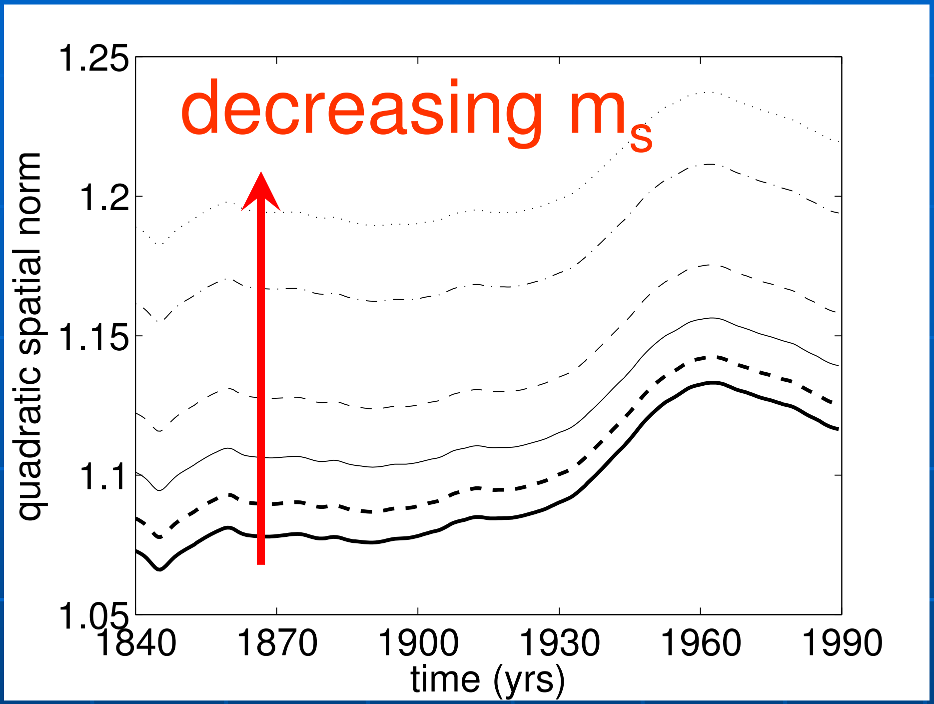
Temporal spectrum unchanged

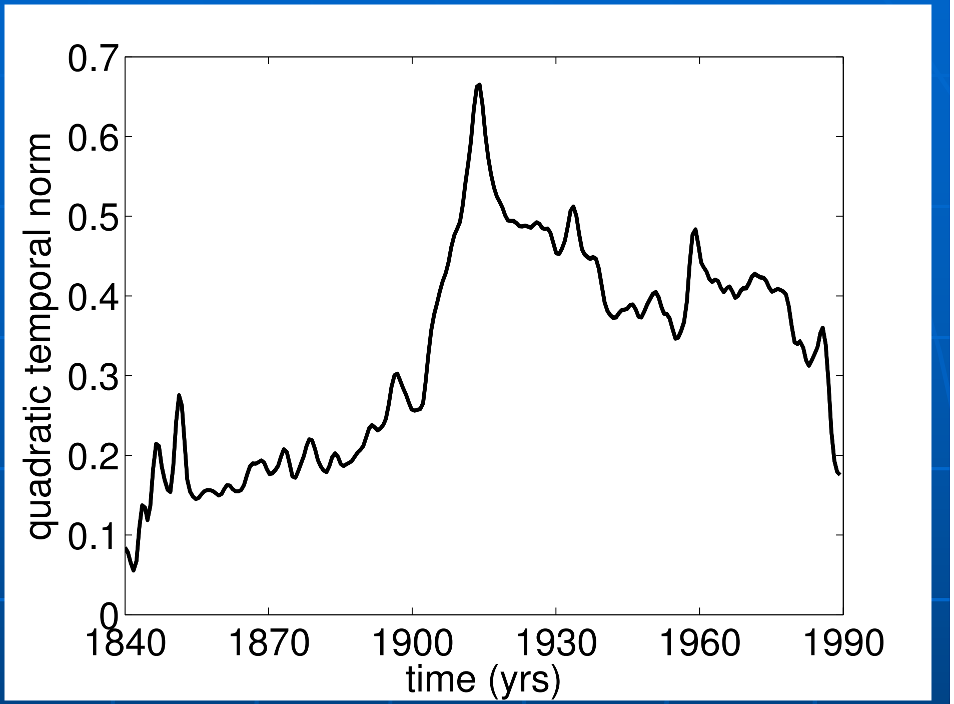


Spatial spectrum fed at large wave numbers…

… to give sharper patches (but not on smaller length-scales!)

Effect of m_S on norms ($m_T=10^6$)





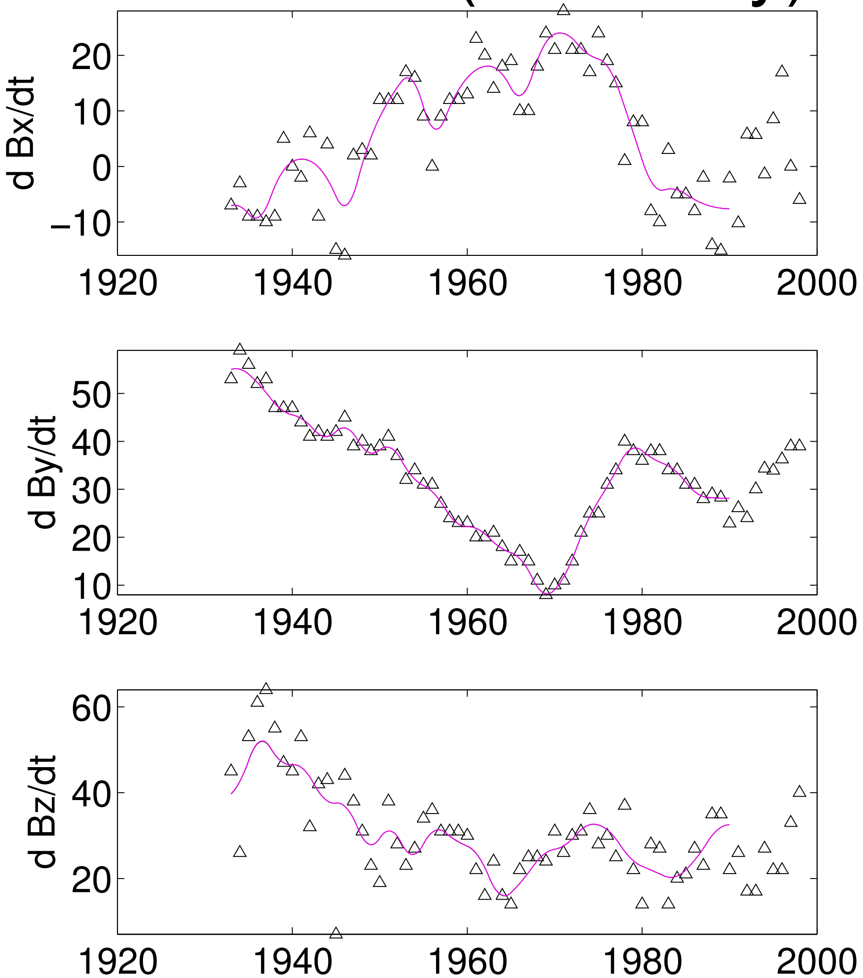
By the same time

$$\oint_{CMB} |B_r| d\Omega \approx cte$$

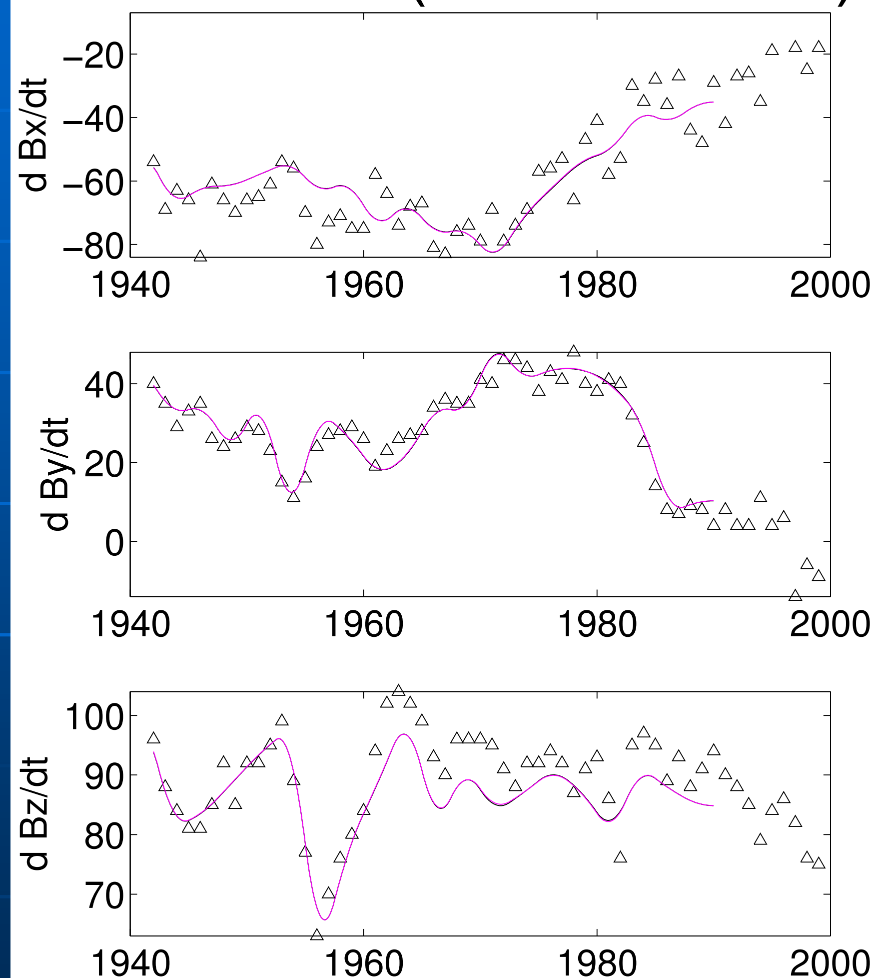
…≠ decreasing λ_s!!
But: reorganize field lines.

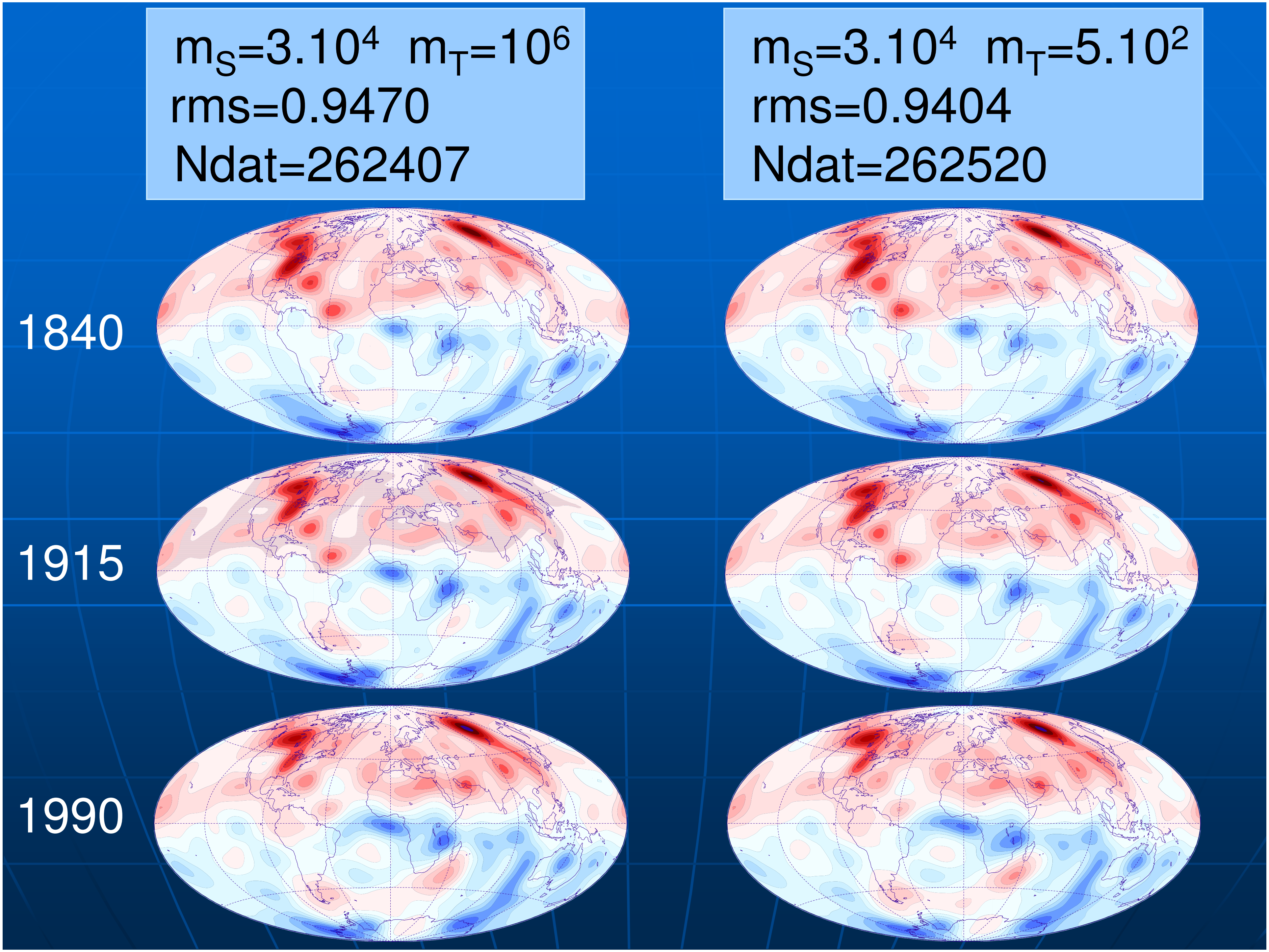
Models *vs* observatories

NGK (Germany)

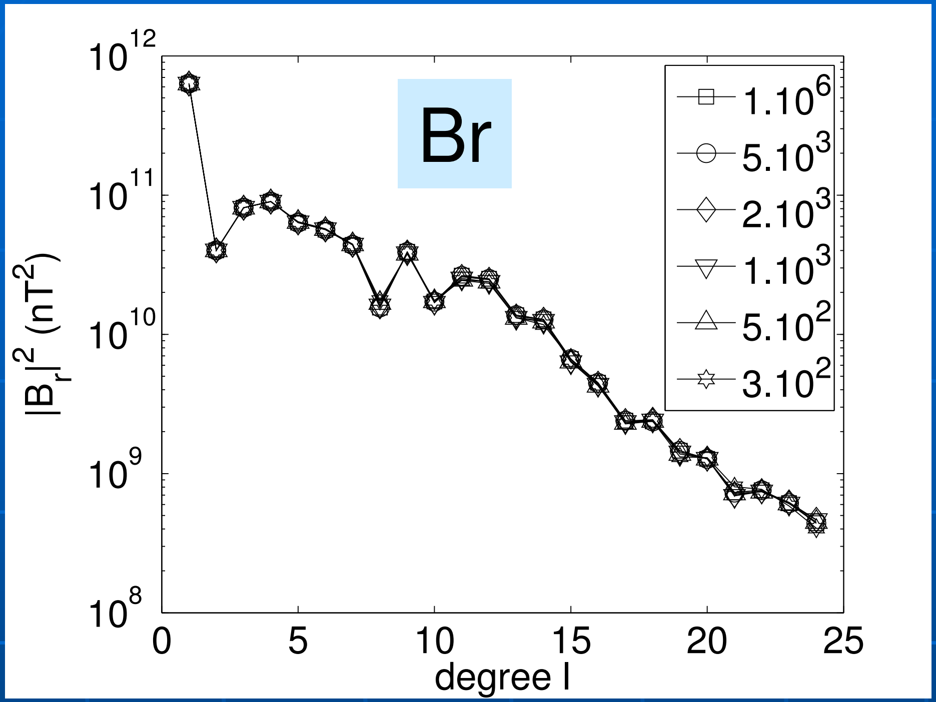


HER (South Africa)



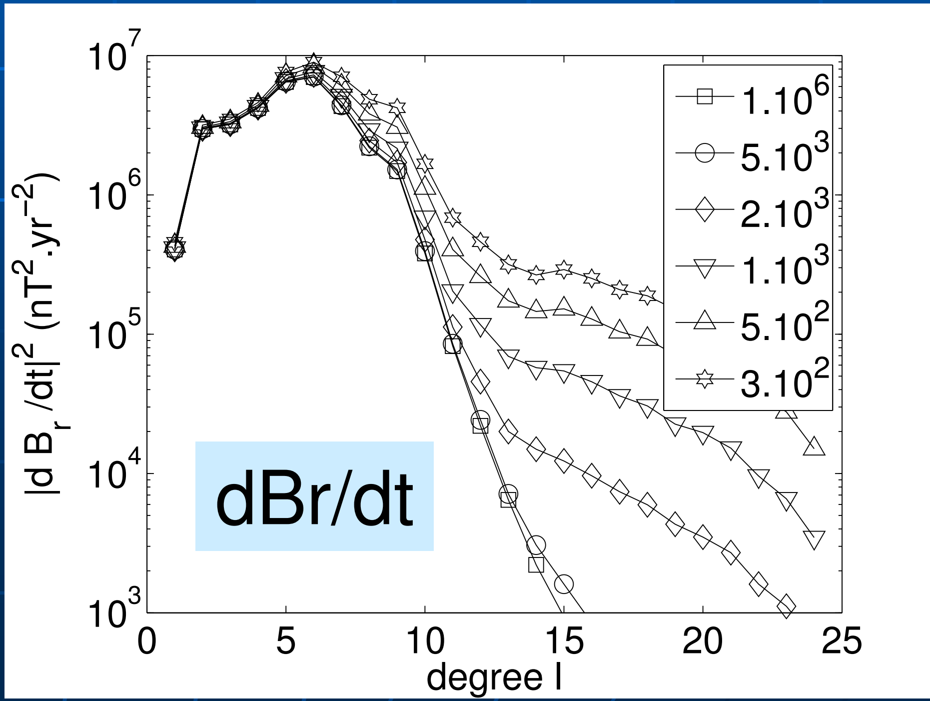

$m_S=3.10^4$ $m_T=10^6$
rms=0.9470
Ndat=262407
$m_S=3.10^4$ $m_T=5.10^2$
rms=0.9404
Ndat=262520
1840
1915
1990

Effect of m_T on spectrums (m_S=3.10^4)

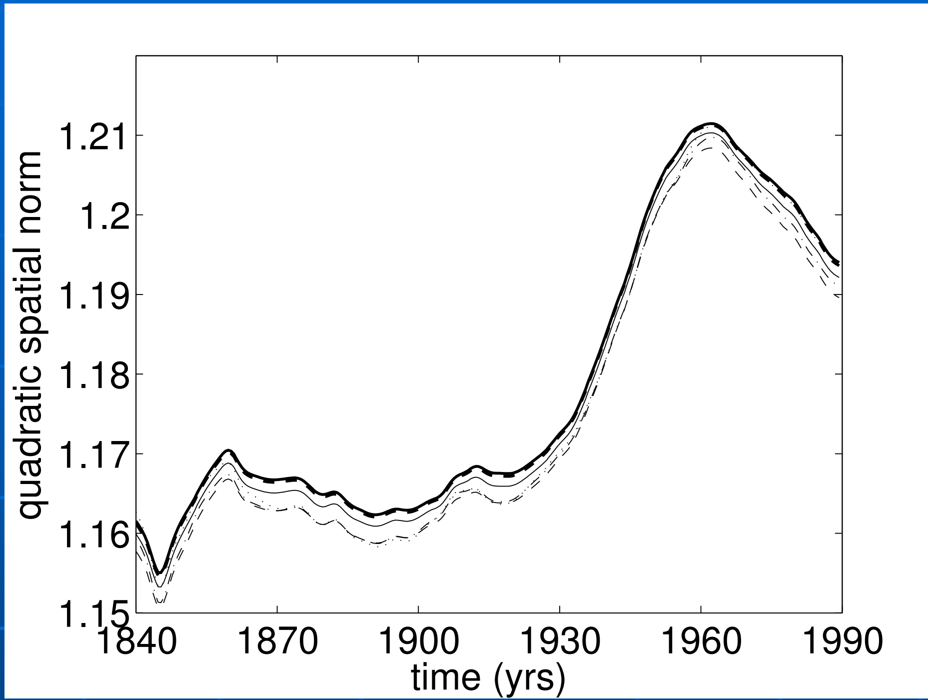


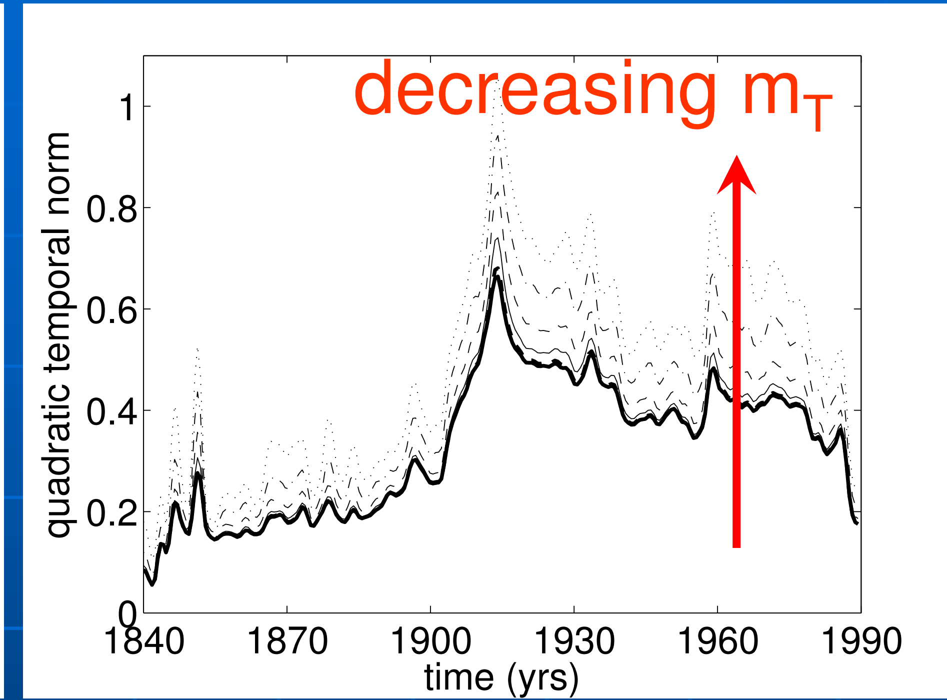
Temporal spectrum fed at large wave numbers…
… to give sharper changes in time (but not on shorter time scales!)

Spatial spectrum unchanged



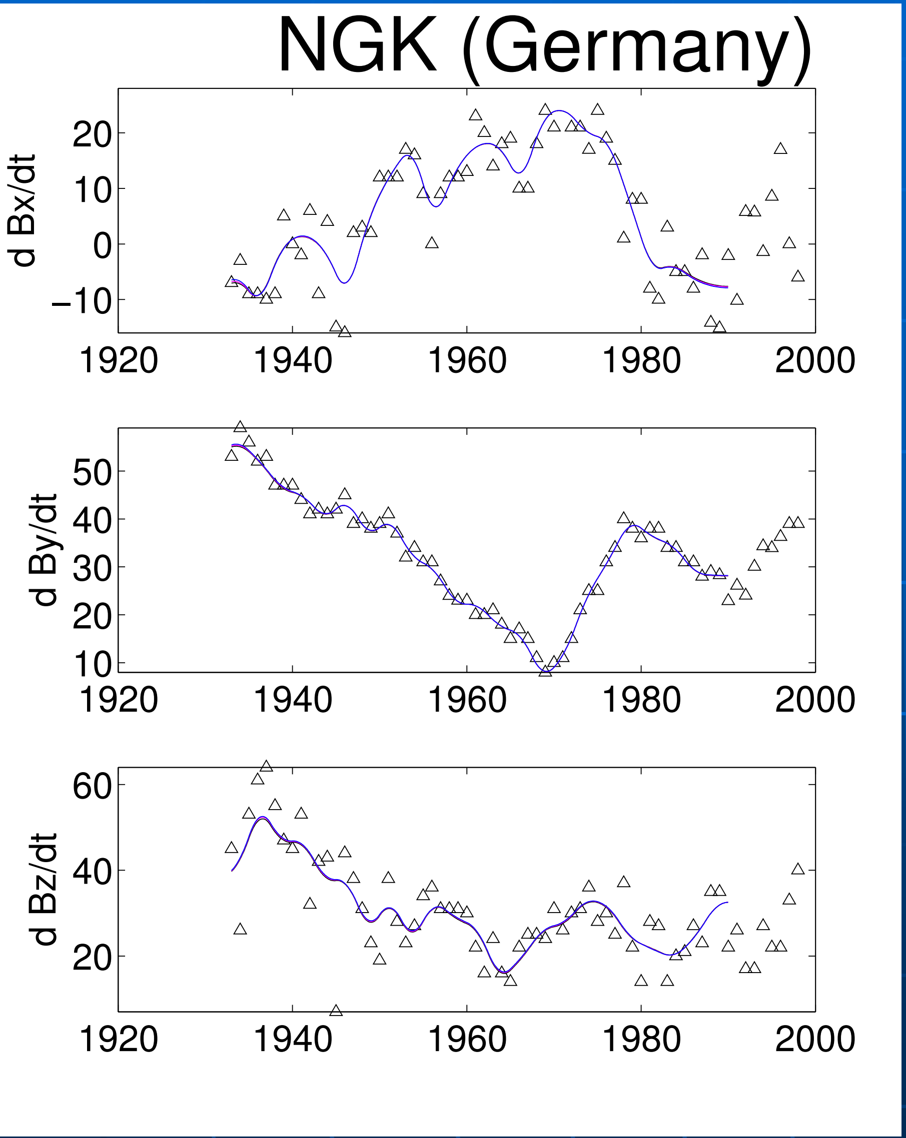
Effect of m_T on spectrums (m_S=3.10[4])

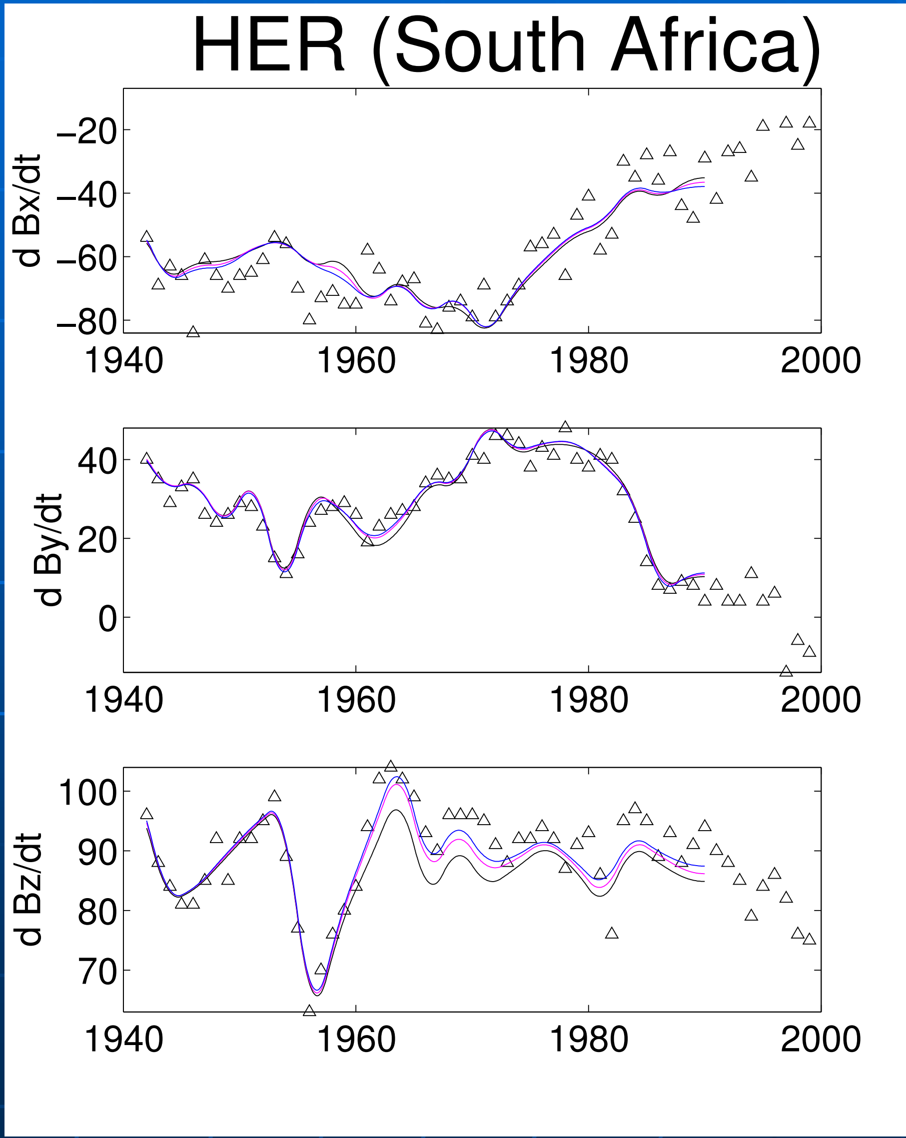




…≠ decreasing λ_T, that would introduce shorter time-scales fluctuations
But: allows sharper changes in time

Models *vs* observatories





Conclusions

- Usual quadratic dampings (space & time)
  - do not rely on physical assumptions;
  - introduce artificial correlation;
  - loss of contrast.
- Maximum Entropy spatial damping:
  - sharper flux patches
  - same misfit to the data
  - isolate more individuals patches
  - dissociate previously wide patches
- Maximum Entropy temporal damping:
  - allows sharper changes in time
  - similar fit to the data when good sampling
  - better fit to the data when needed

Perspectives

- Expand the data sets up to 2005

  (collaboration with C. Finlay and A. Jackson)

- Portable to the comprehensive models:

  - description of the crustal and core fields

  … push the limit of the SV robust description

- Test physical constraints that cannot be accounted for in the spectral domain:

- frozen flux limit (*Constable et al [1993], Jackson et al [2005]*)

- presence of diffusion ? *(Gubbins & Bloxham [1985])*

- drift of wave patterns … dispertion ? *(Finlay & Jackson [2003]*)

- Test impact on Core Flow inversions:

Max. Ent. Br → Quad. flow *vs* Quad. Br → Quad flow

MaxEnt Br → Quad flow *vs* MaxEnt Br → MaxEnt flow