Swarm and Gravity

Possibilities and Expectations for Gravity Field Recovery

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- **Representation of the Gravity Field and Spaceborne Observations**

- **Methodology for Error Analysis**
  - error propagation using transfer coefficients
  - full simulation based on …
  - energy integral method

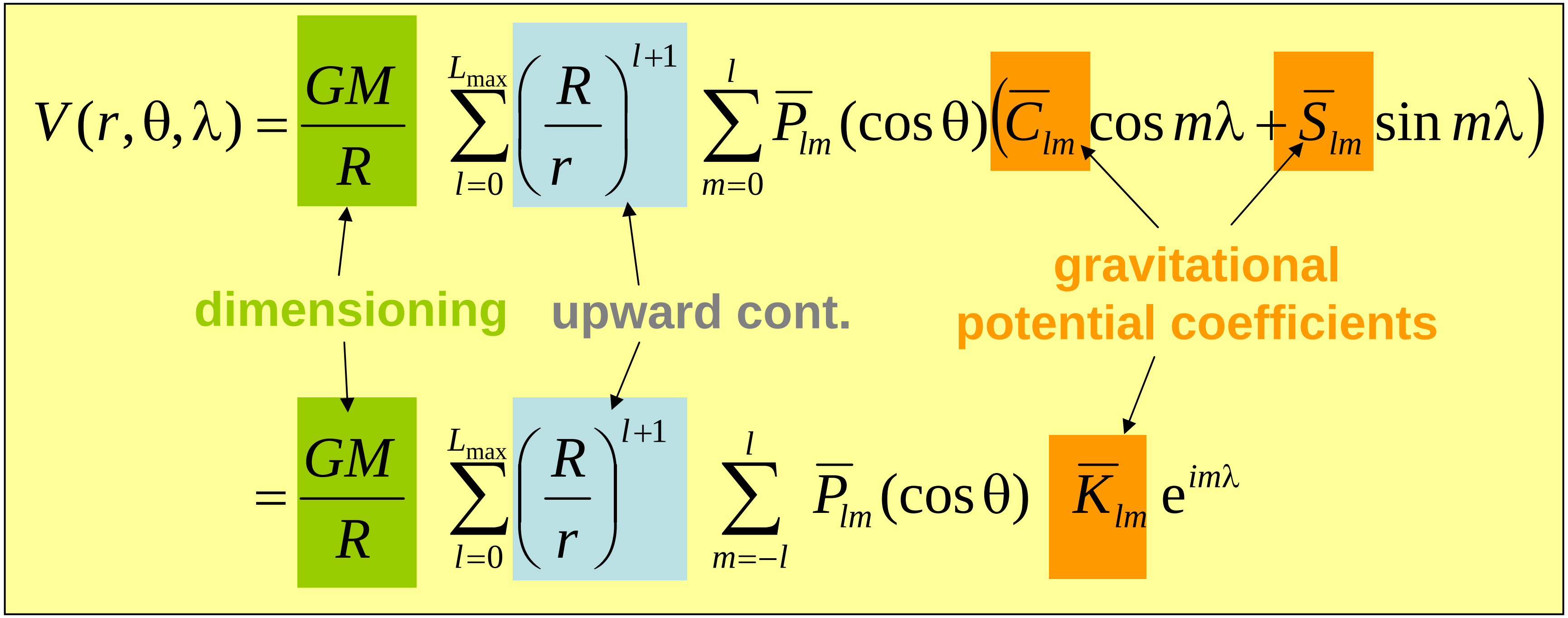
- **Use of Swarm for Gravity Field Recovery**

- **Simulation Results**

Gravity Field and Spaceborne Observations

Representation of the gravity field in terms of **spherical harmonics (sh)**

***On the sphere*:**



$$V(r,\theta,\lambda) = \frac{GM}{R}\sum_{l=0}^{L_{\max}}\left(\frac{R}{r}\right)^{l+1}\sum_{m=0}^{l}\bar{P}_{lm}(\cos\theta)\left(\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right)$$

$$= \frac{GM}{R}\sum_{l=0}^{L_{\max}}\left(\frac{R}{r}\right)^{l+1}\sum_{m=-l}^{l}\bar{P}_{lm}(\cos\theta)\,\bar{K}_{lm}\,\mathrm{e}^{im\lambda}$$

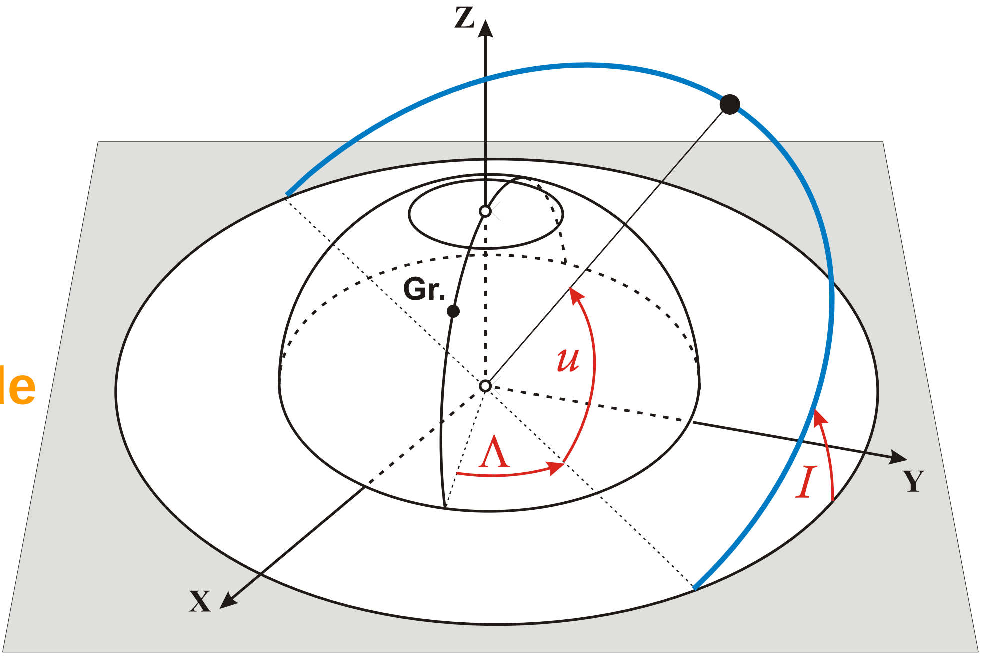
Gravity Field and Spaceborne Observations

Along the orbit:

$$V(r,u,\Lambda)=\frac{GM}{R}\sum_{l=0}^{L_{\max}}\left(\frac{R}{r}\right)^{l+1}\sum_{m=-l}^{l}\sum_{k=-l}^{l}\overline{K}_{lm}\overline{F}_{lmk}(I)\,\mathrm{e}^{i(ku+m\Lambda)}$$

orbit coordinates:

- ***r*** - geocentric radius
- ***u*** - argument of latitude
- ***Λ*** - longitude of asc. node
- ***I*** - inclination



Gravity Field and Spaceborne Observations

Representation along the orbit for arbitrary functionals $f^{\#}$:

$$f^{\#}(r,u,\Lambda) = \sum_{m} \sum_{k} A^{\#}_{mk} \, \mathrm{e}^{i(ku+m\Lambda)}$$

with lumped coefficients $A^{\#}_{mk} = \sum_{l} H^{\#}_{lmk} \bar{K}_{lm}$

and transfer coefficients $H^{\#}_{lmk}$

for example:

$$H^{V}_{lmk} = \frac{GM}{R} \left(\frac{R}{r} \right)^{l+1} \bar{F}_{lmk}(I)$$

where # can be, e.g., gravitational potential V

gravity gradients V_{ij} ***(GOCE)***

orbit perturbations $\Delta \boldsymbol{x}$ ***(CHAMP)***

range, range rate $\rho, \dot{\rho}$ ***(GRACE)***

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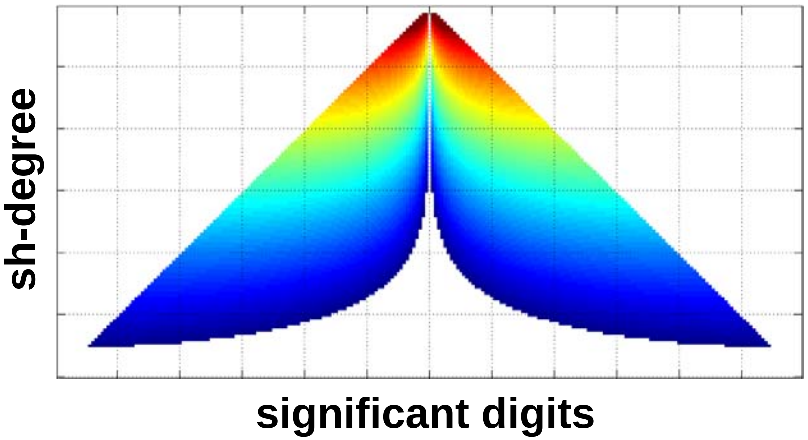
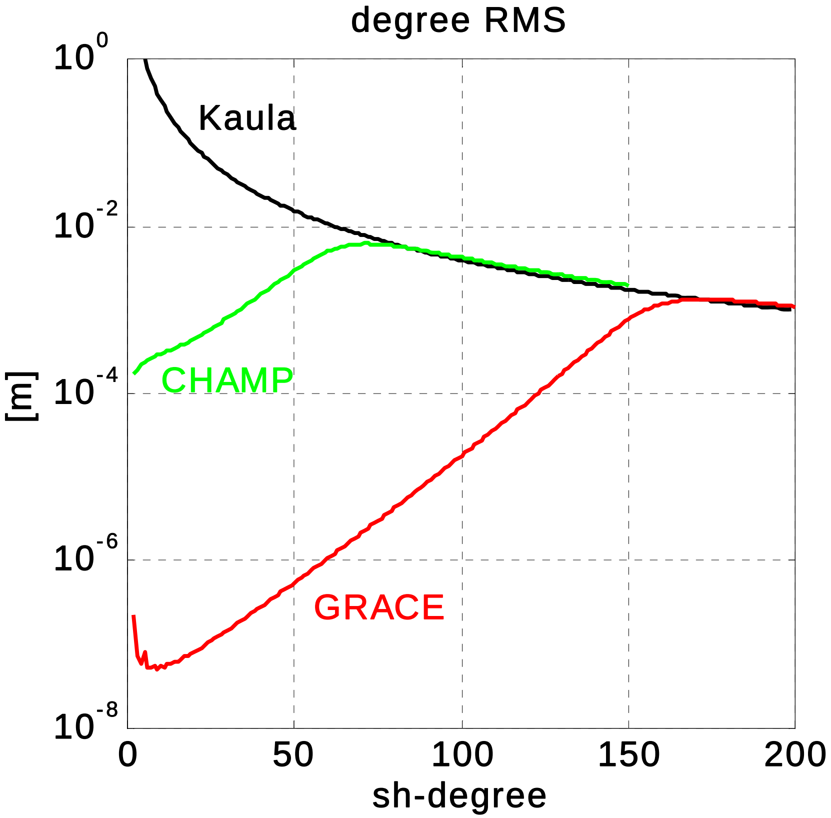
Methodology – Error Propagation

- based on error models of the observations
- using specific transfer coefficients $H_{lmk}^{\#}$

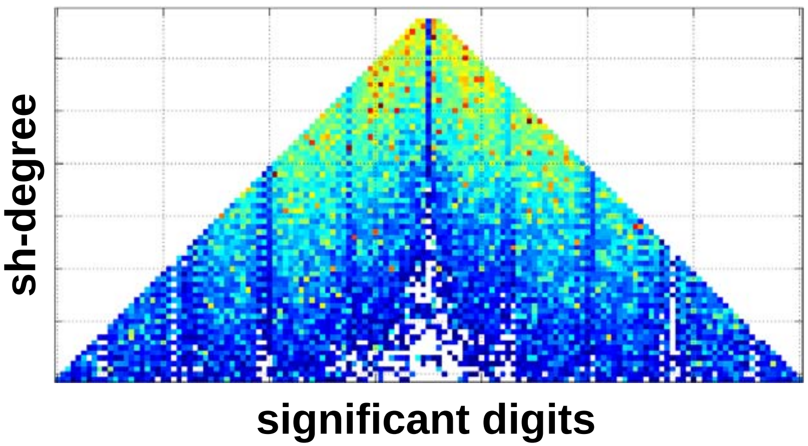


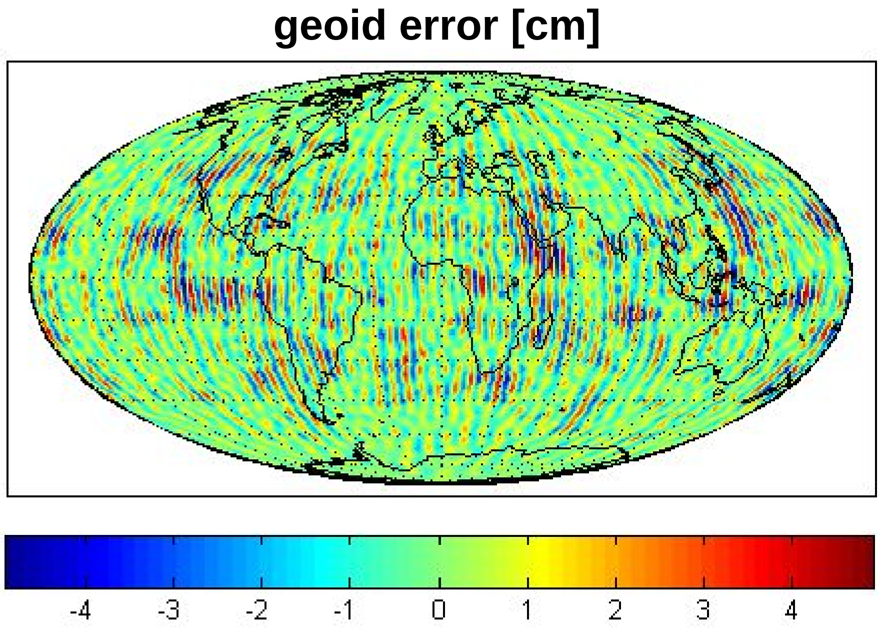


⇨ **formal error estimates (variances)**

Methodology – Full Simulation

- based on **energy integral** ⇒ ***pseudo observation*** $V, \Delta V$
- using time series of observation errors





⇒ **empirical errors**
(differences to „truth“)

Methodology - Energy Integral (one satellite)

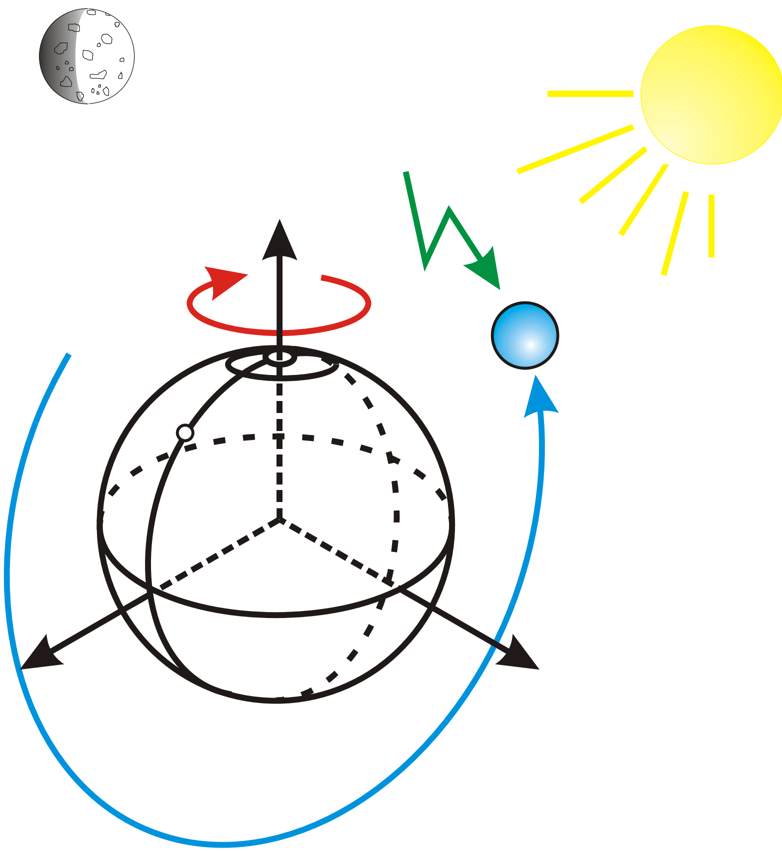
Energy conservation law $$\boxed{E_{kin} + E_{pot} = const.}$$

$$\Rightarrow \quad \boxed{V = \frac{\dot{\boldsymbol{x}}^2}{2} - Z + \int_{\boldsymbol{x}} \boldsymbol{a} \cdot \mathrm{d}\boldsymbol{x} + C}$$

centrifugal potential

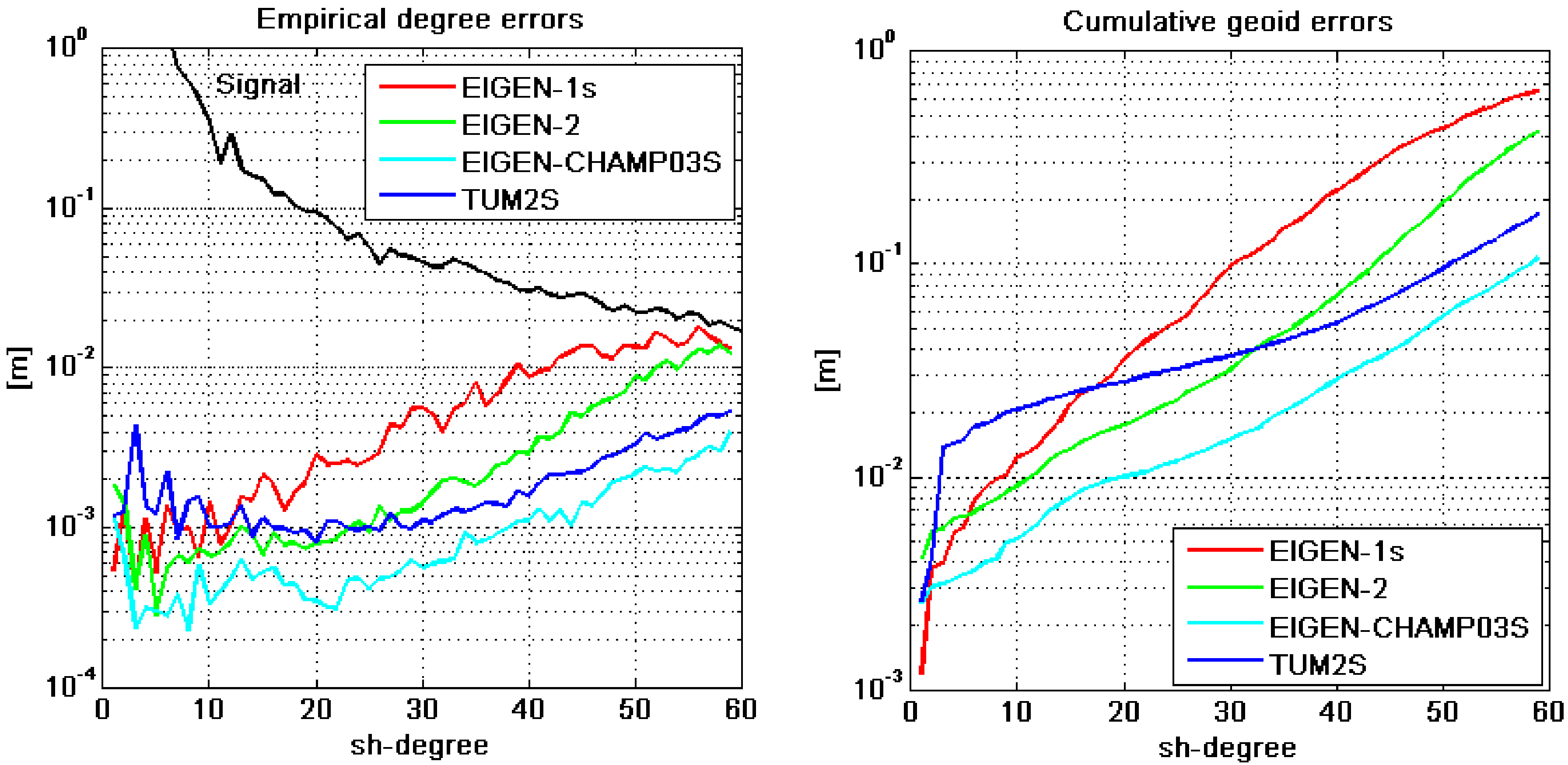
non-conservative forces

$$\boxed{V \overset{\wedge}{=} \frac{\dot{\boldsymbol{x}}^2}{2}}$$



Methodology - Energy Integral (proof of concept)

Global gravity models by IAPG using CHAMP data
(empirical errors wrt. EIGEN-GRACE02S)



Methodology - Energy Integral (two satellites)

Relative motion between satellites

$$V \triangleq \frac{\dot{\boldsymbol{x}}^2}{2} \quad \Rightarrow \quad \Delta V \triangleq \frac{\Delta\dot{\boldsymbol{x}}^2}{2} + \dot{\boldsymbol{x}}_1 \cdot \Delta\dot{\boldsymbol{x}}$$

absolute velocity vector

relative velocity vector

For **along-track** separation

GRACE: scalar KBR-observation

$$\Delta V \approx \| \dot{\boldsymbol{x}}_1 \| \dot{\rho}$$

range rate

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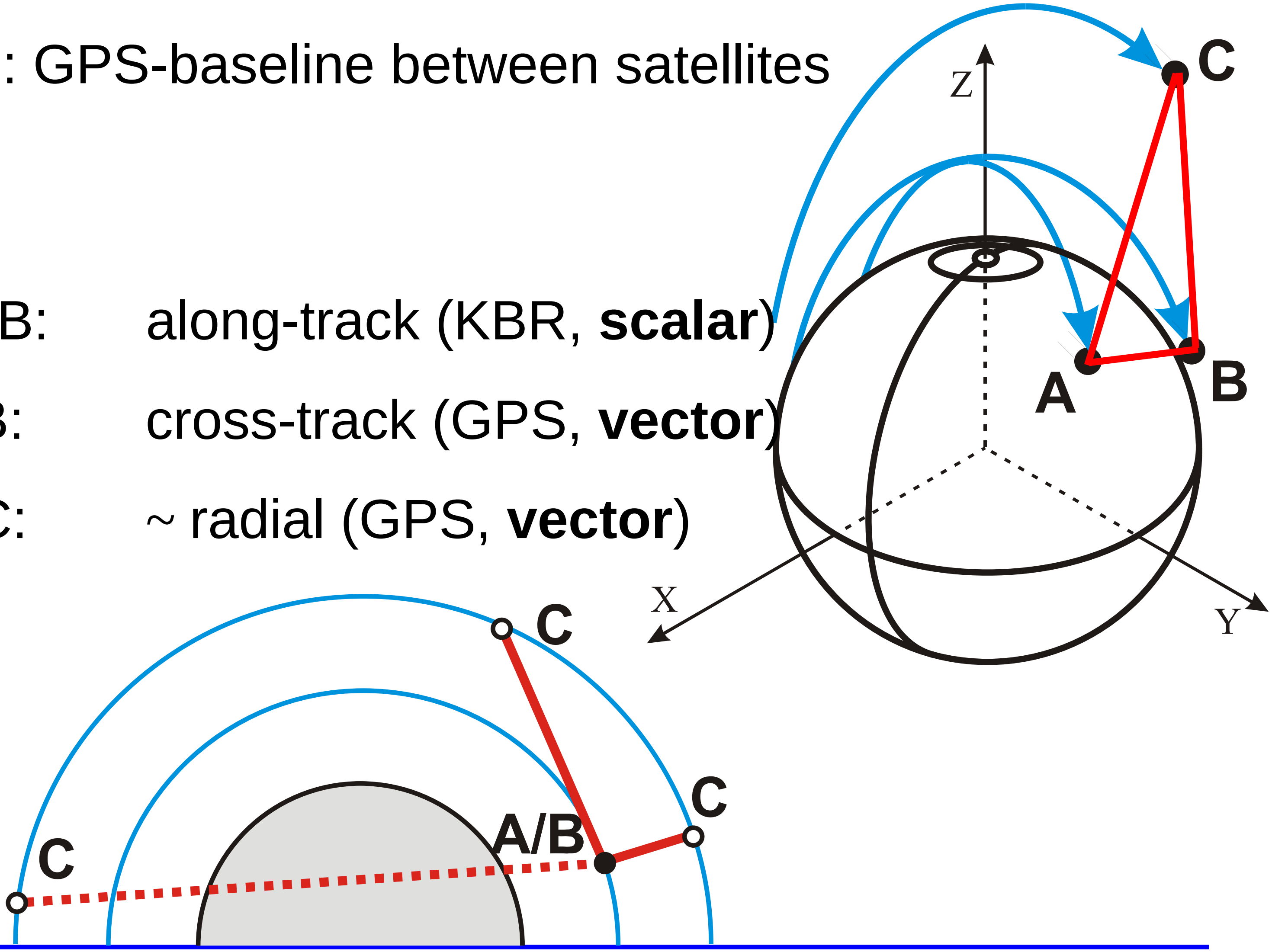
Swarm and Gravity - Constellation

Observation: GPS-baseline between satellites

Geometry:

GRACE A-B:	along-track (KBR, **scalar**)
Swarm A-B:	cross-track (GPS, **vector**)
Swarm A-C:	~ radial (GPS, **vector**)



Swarm and Gravity

Use of GPS-Baseline Observations

(1) Error propagation using transfer coefficients for potential gradients $V_x, \ V_y, \ V_z$

For short baselines: $\boxed{\Delta V \,/\, \|\Delta x\| \approx V_x}$

(2) Full simulation using energy integral for potential differences

$\Delta V_{\text{along-track}}$

$\Delta V_{\text{cross-track}}$

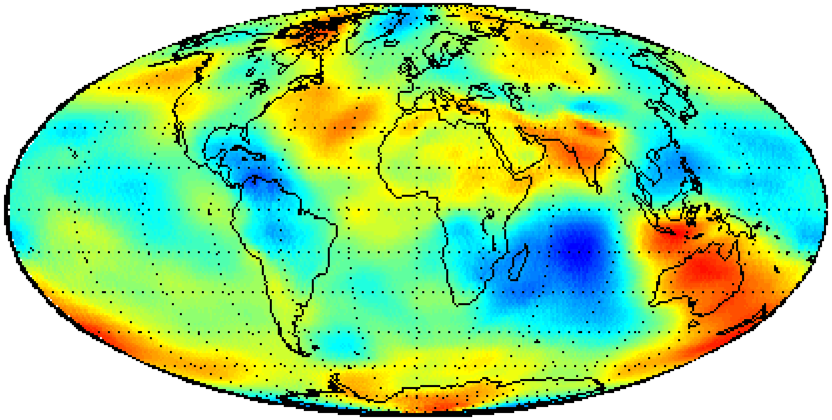
ΔV_{radial}

Swarm and Gravity

Along-Track Gradient vs. Potential Differences

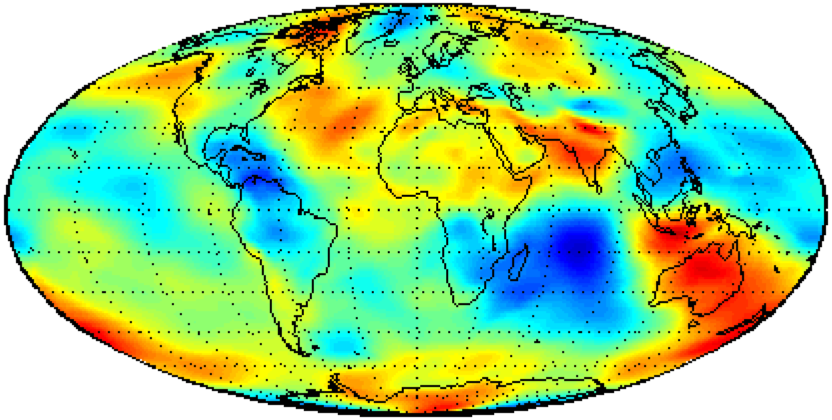
Approximate gradient

potential difference along simulated orbit



Along-track gradient

sh-synthesis using transfer coefficients $H^{\#}_{lmk}$



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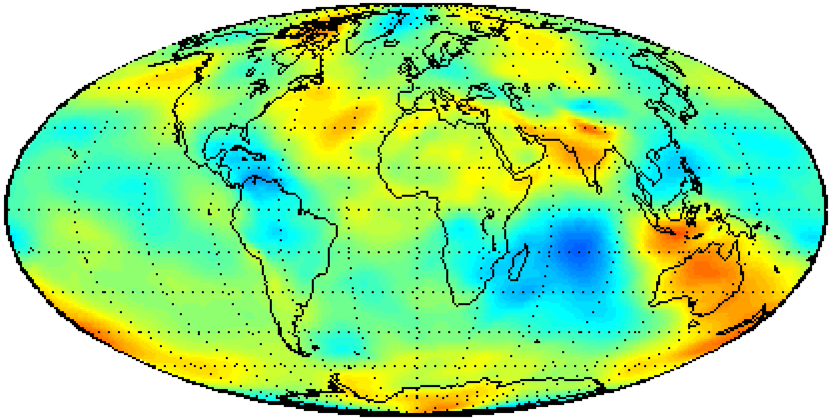
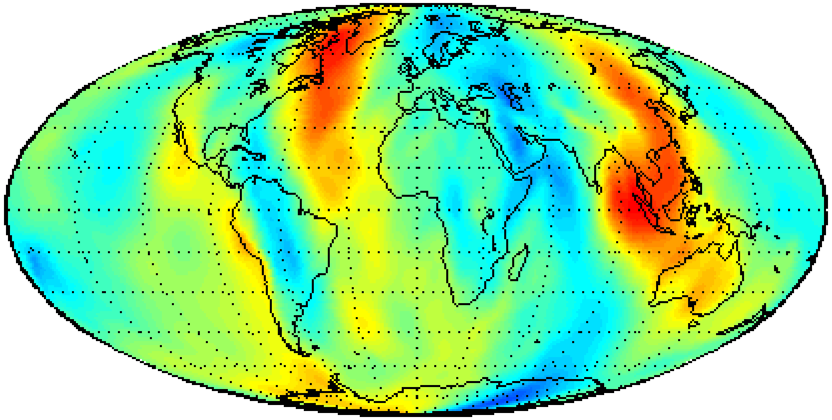
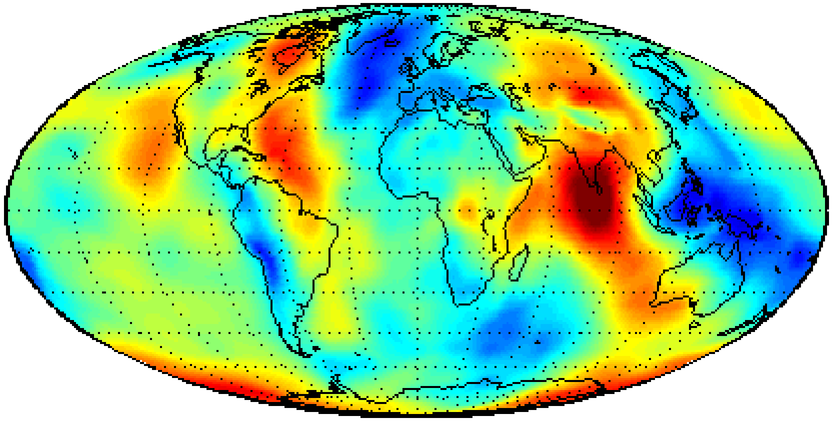
Gravity Field Gradients



along-track



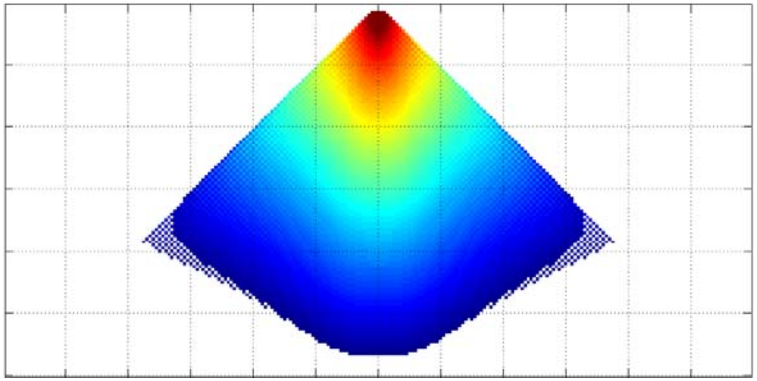
cross-track



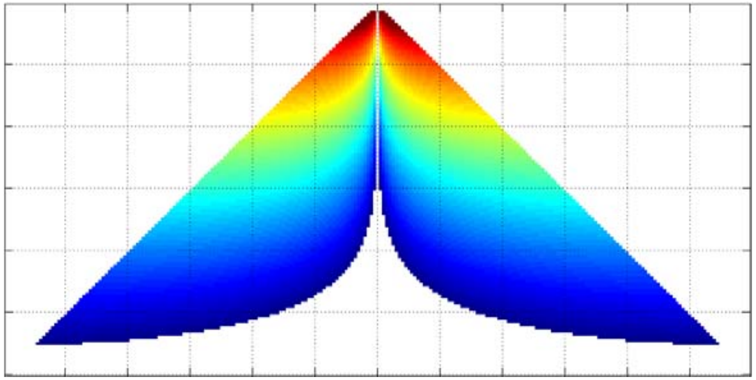
radial

SH-Error Characteristics from Different Observation Directions

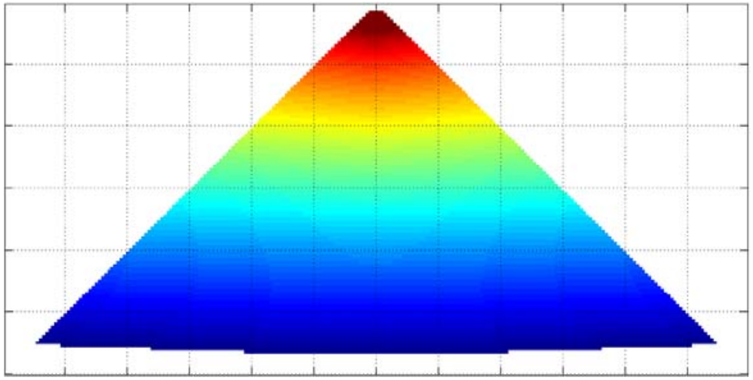
(1)



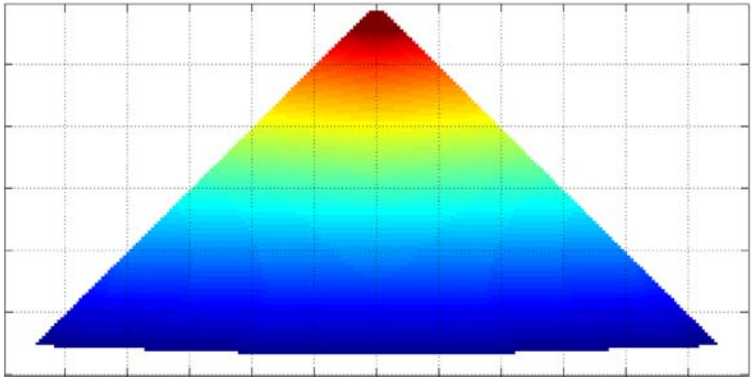
along-track



cross-track



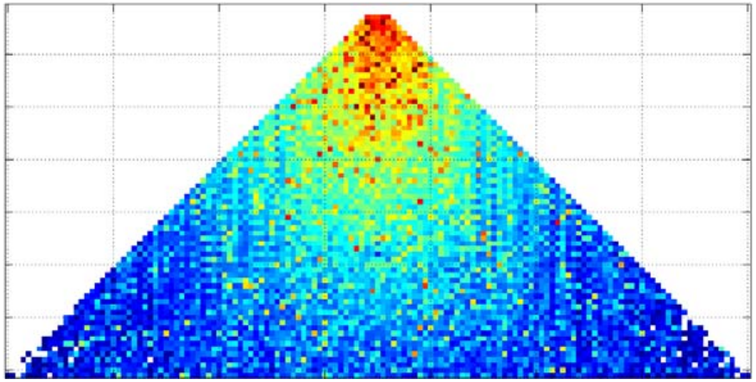
along + cross



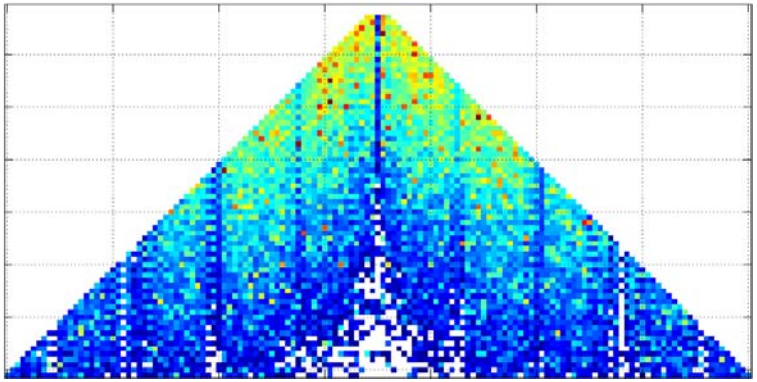
radial

SH-Error Characteristics from Different Observation Directions

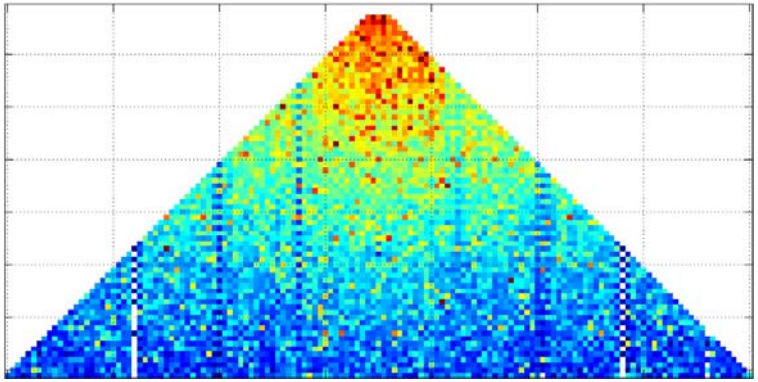
(2)



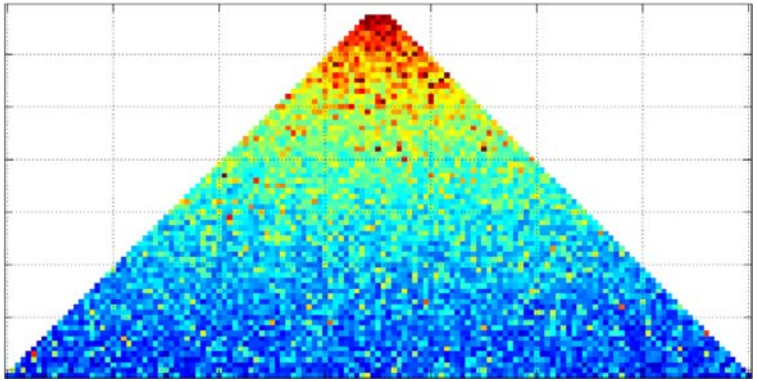
along-track



cross-track



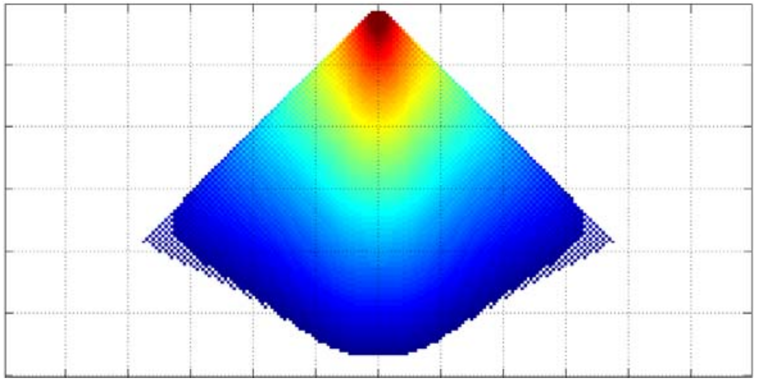
along + cross



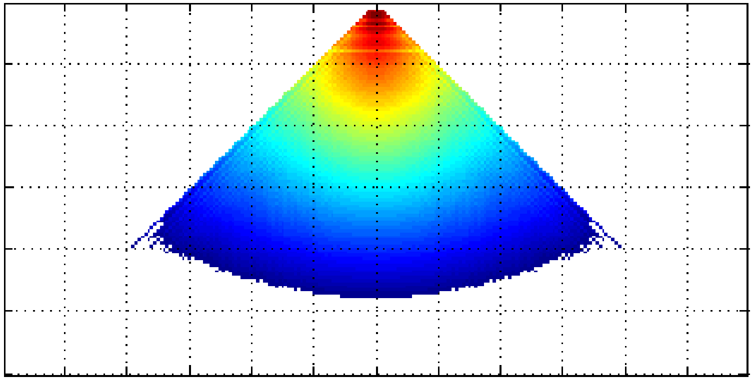
radial

SH-Error Characteristics
Along-track simulations vs. real data (GRACE)

(1)

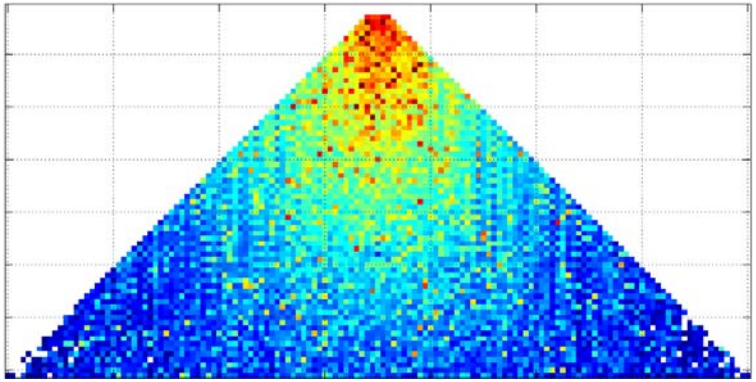


error propagation



EIGEN-GRACE02s (GFZ)

(2)

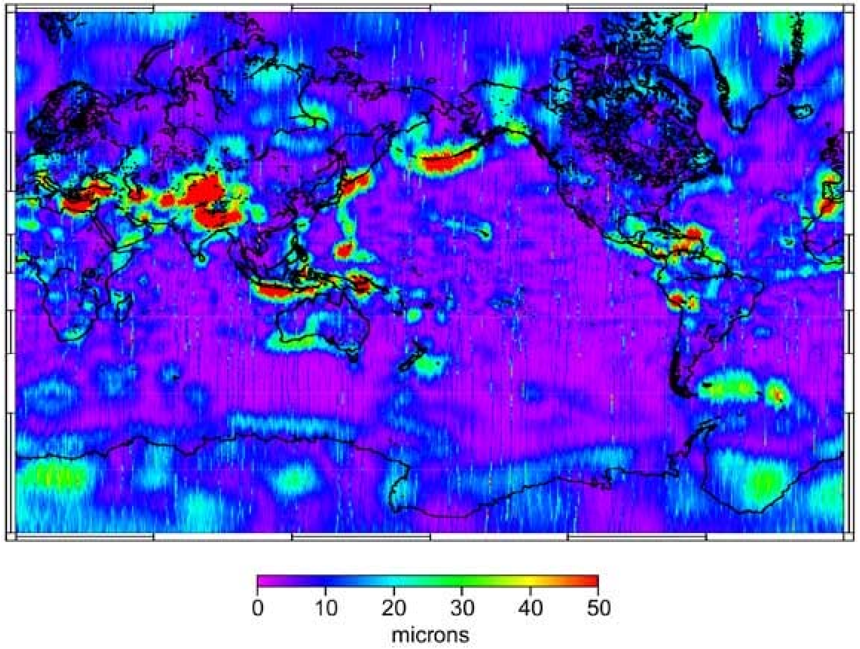


full simulation

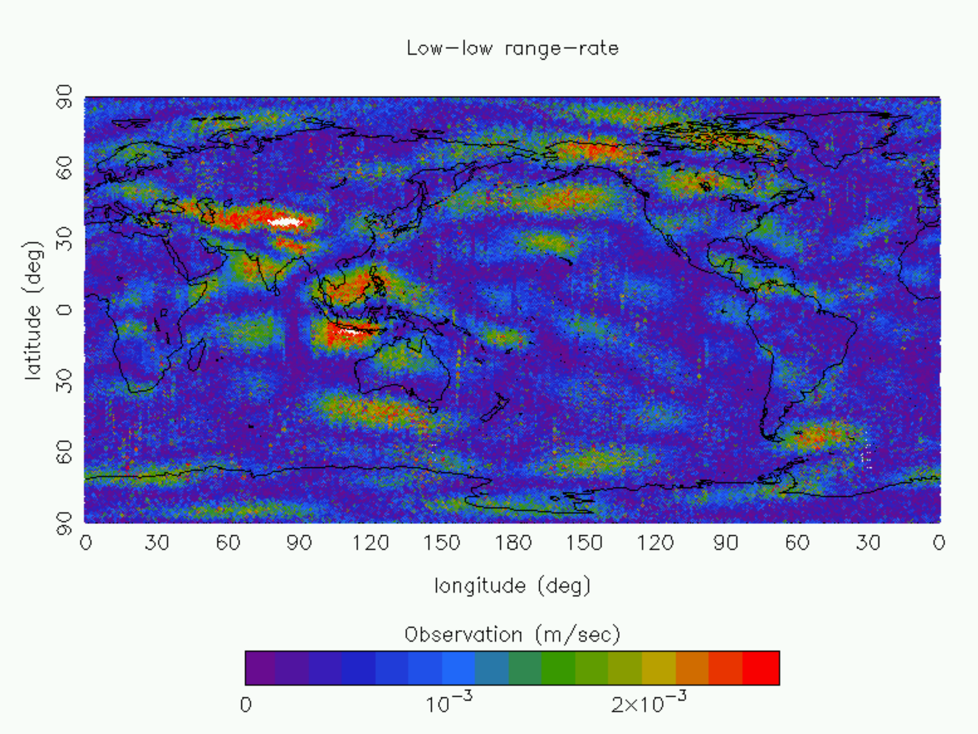
GPS inter-satellite baseline
Test case: GRACE

High-pass filtered GRACE inter-satellite ranging

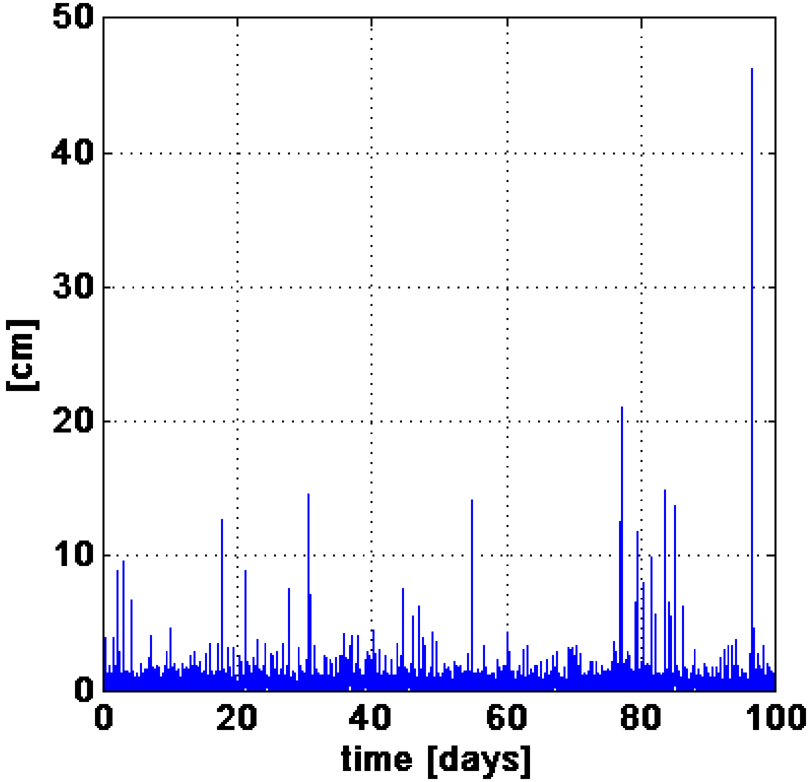
From KBR



From GPS



GPS inter-satellite baseline
Test case: GRACE



Empirical range errors (GPS vs. KBR)

$$\sigma_{\text{along}} = 2.1 \text{ mm}$$

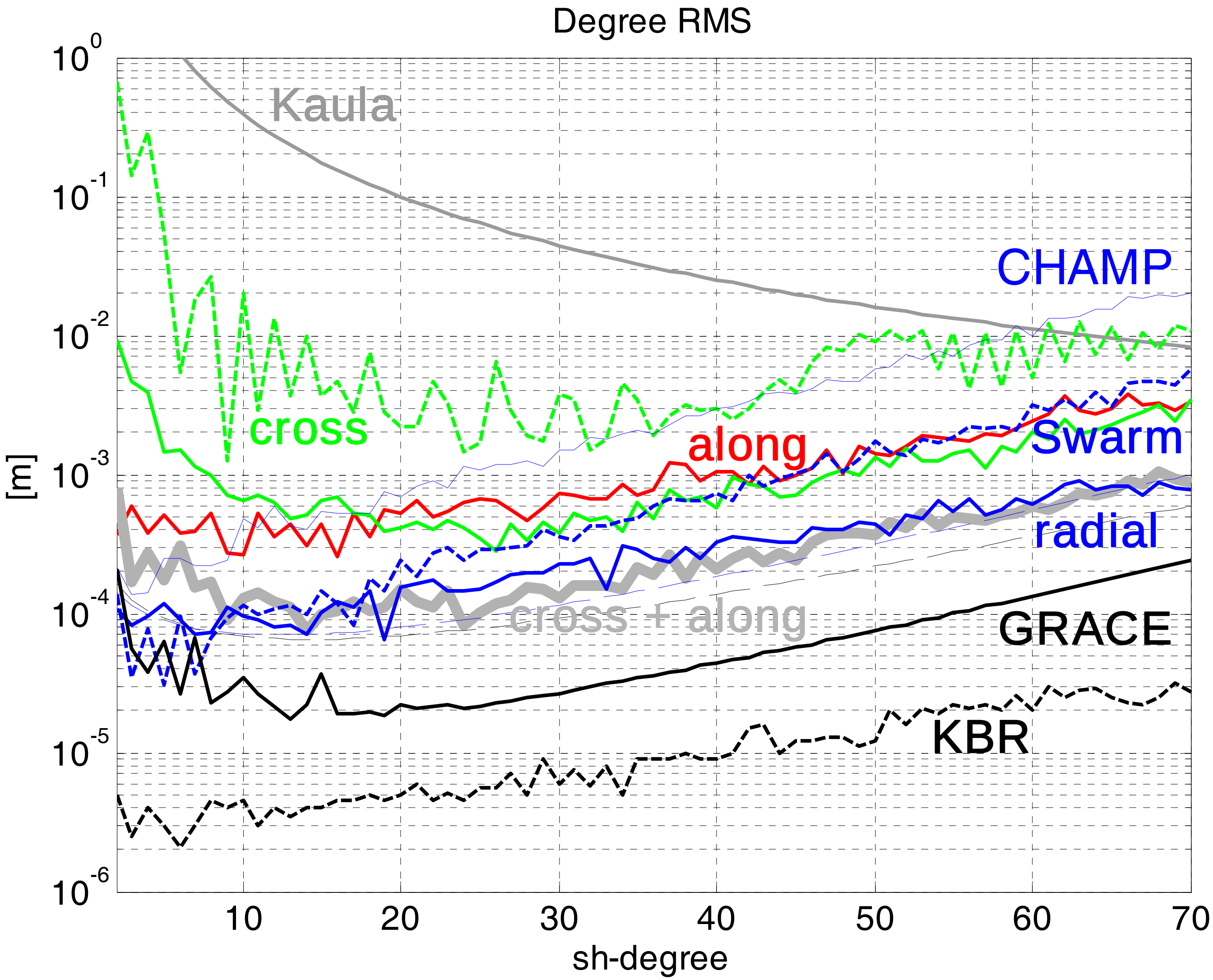
$$\sigma_{\text{cross}} = 2.0 \text{ mm}$$

$$\sigma_{\text{radial}} = 2.7 \text{ mm}$$

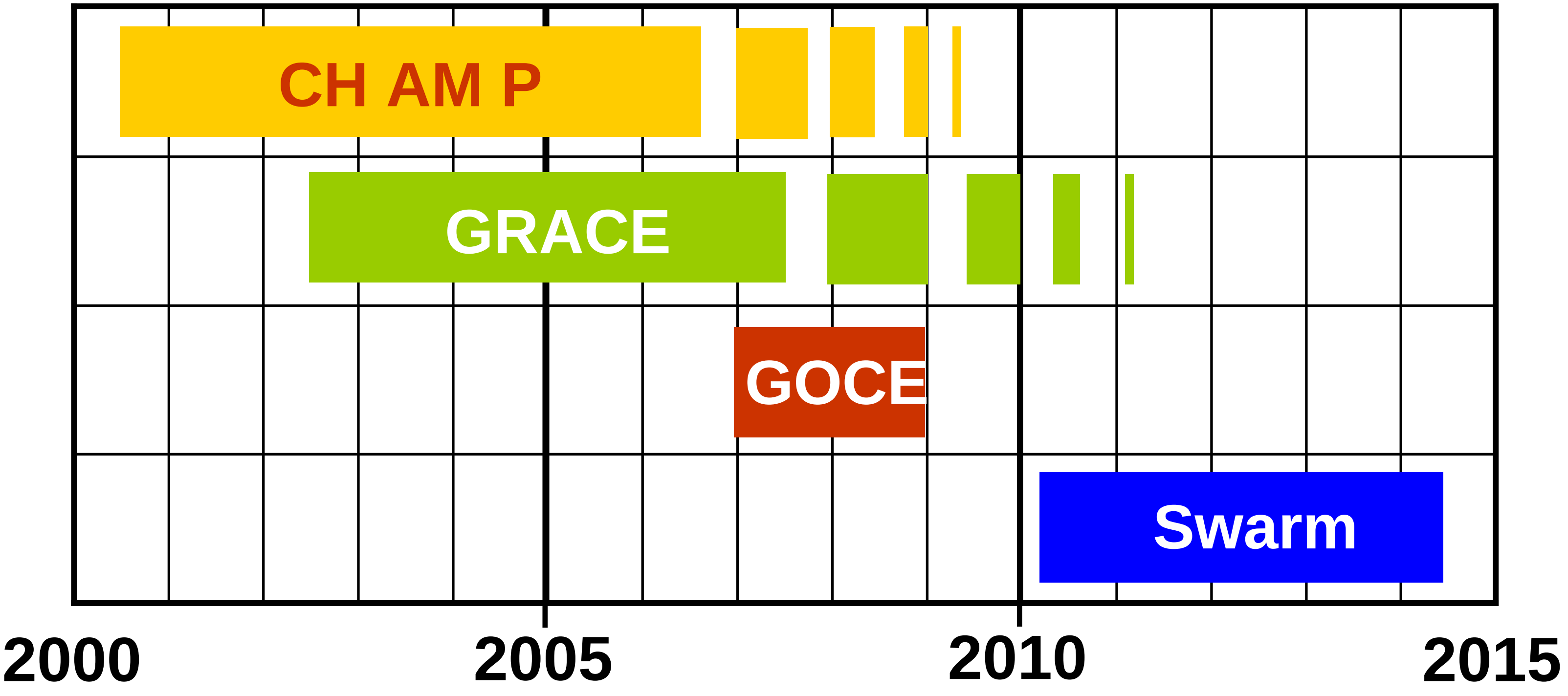
⇨ Expected range-rate accuracy:

$$\sigma \approx 10^{-2} \text{ mm/s}$$

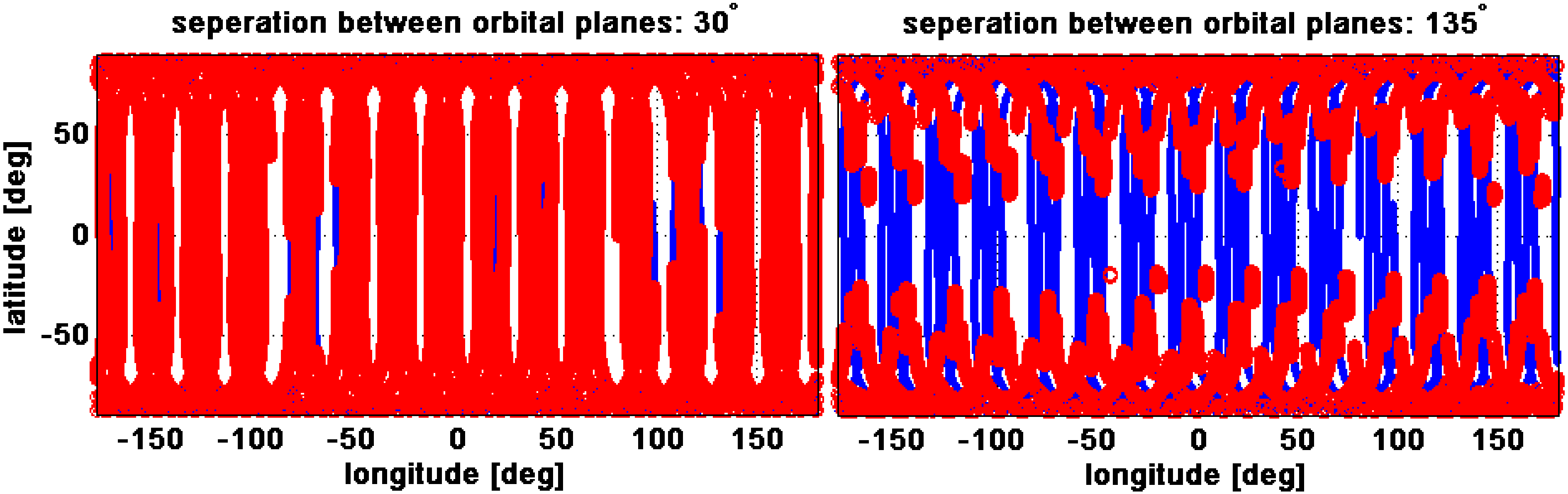
Simulation results



Time line of geopotential missions



Data distribution during the mission life time



Concluding Remarks

- Gravity field recovery with Swarm is possible
- Continuation of decade of gravity field mapping possible (but, with reduced accuracy)
- Radial component most sensitive at end of mission (but, bad visibility)
- Simulation must include full GPS-simulation (visibility)
- Realistic error estimates for GPS?
- Different analysis methods possible (not necessarily energy integral)