SWARM AND GRAVITY:
POSSIBILITIES AND EXPECTATIONS FOR GRAVITY FIELD RECOVERY

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ABSTRACT

The CHAMP satellite, launched in 2000, has provided an enormous increase in accuracy of global satellite-only gravity field models. It is equipped with a GPS receiver for continuous 3-D positioning and an accelerometer to measure the non-gravitational forces. The three Swarm satellites are very similar to CHAMP. Therefore they are, in principle, also usable for gravity field recovery. Actually the expected accuracy cannot compete with the dedicated gravity field missions GRACE (launched in 2002) and GOCE (to be launched in 2007). Still it is worthwhile to investigate the potential of Swarm, especially considering the time line of the different missions. We will present general ideas on how to use Swarm for gravity field determination and results of error simulations based on a formal error propagation and a closed-loop simulation using the energy integral.

1. INTRODUCTION

The satellite mission CHAMP was the first satellite to carry both a GPS receiver for precise continuous 3-D positioning of the satellite and an accelerometer to allow the separation of gravitational and non-gravitational forces. This way CHAMP provided an enormous increase in the accuracy of global satellite-based gravity field models. The mission can thus be seen as a proof of concept for the following missions GRACE (in orbit since 2002) and GOCE (to be launched in 2007). All of these satellite missions fly in low altitude (around 450 km in case of CHAMP and GRACE and around 250 km in case of GOCE) in polar or near polar orbits. GRACE consists of two CHAMP-like satellites, which follow each other in the same orbit with a distance of about 200 km. In addition to GPS receivers and accelerometers the satellites are equipped with a microwave ranging system (K-Band ranging = KBR), which allows the measurement of the inter-satellite range with μm-accuracy. This way GRACE provides a much higher accuracy and spatial resolution of the derived gravity fields as CHAMP. The primary goal of GRACE is the detection of temporal variations of the long wavelength part, up to about degree $l \approx$ 20-30 (spatial resolution around 1000 up to 700 km). These variations are due to mass transport in the Earth system, e.g. changing water storage over the continents. The primary goal of GRACE is to monitor such variations. In contrast to this, GOCE is designed to determine a static field only, but with a much higher spatial resolution as GRACE, i.e. up to $l \approx 250$ corresponding to resolutions below 100 km. The GOCE satellite carries three pairs of very precise accelerometers. In differential mode they allow to derive gravity gradients in three orthogonal directions. GRACE and GOCE are complementary missions, improving both the temporal and the spatial resolution of the gravity field. For further details on these missions we refer to [1].

The Swarm mission consists of three CHAMP-like satellites. Each carries a GPS receiver and an accelerometer. Two of the satellites fly in low orbit (altitude of 450 km, similar to CHAMP and GRACE) next two each other (line of nodes of the respective orbital planes differ for about 1 degree). The third satellite orbits the Earth in 530 km altitude. In the beginning of the mission all three orbit panes will be more or less identical. Due to a slightly different inclination the orbit plane of the third satellite will drift away from the others, reaching a separation of around 130 degrees after 5 years mission duration (see [2]).

Using space-borne differential GPS one can determine the relative position vector between each of the three Swarm satellites. These GPS baselines can be used as observations for gravity field recovery. Just as pointed out in [1] this allows to derive a gravity field model tied to the Swarm mission allowing to derive optimal orbit positions and non-gravitational accelerations for atmospheric density studies. The objective of this paper is to give an estimate for the gravity field accuracy, which can be expected from Swarm GPS baselines. These relative positions can be determined with about one order of magnitude higher accuracy, than the absolute positions derived, e.g. for CHAMP (absolute position accuracy of some few centimeters). In [1] it is shown that GPS baselines between the two GRACE satellites (relative position) can be derived with mm-accuracy.

In the next section we will give some general ideas and considerations regarding the orbit constellation and observation geometry for Swarm. Based on this, the methodology for the simulation studies is presented in section 3. Finally the results are discussed in section 4.

2. GENERAL ASPECTS ON GRAVITY FIELD RECOVERY FROM SWARM

As stated in the introduction, gravity field recovery from Swarm is based on the GPS baselines between the three satellites. Fig. 1 shows the Swarm constellation with satellites A and B in low orbit next two each other and satellite C about 80 km above these two. The red lines indicate the GPS baselines. It is obvious from Fig. 1, that the baseline A-B gives cross-track information, while A-C and B-C give radial information. In general observations in different directions are sensitive for different parts of the spherical harmonic spectra. This will be shown in more detail in section 4. In contrast to the cross-track and radial geometry of Swarm, the GRACE-mission gives along-track information (the two satellites follow each other in the same orbit, one behind the other).

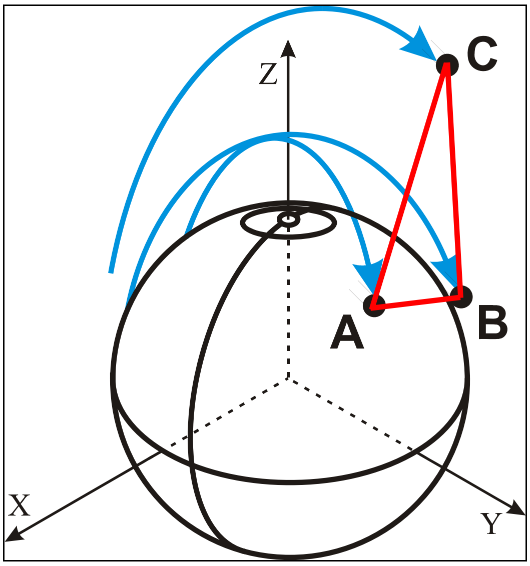


Figure 1. GPS baselines between the 3 Swarm satellites.

The satellite constellation of Swarm will not stay fixed during the mission life time. This is due to (1) the different orbital altitudes and (2) slightly different inclinations of the orbit planes of satellites A and B as compared to C. Issue number (1) results in different orbital velocities of satellites A/B and C. Therefore the baselines A-B and B-C will not always be in radial direction. This is indicated in Fig. 2. The baseline will in general be a combination of radial and along-track components (in extreme cases pure radial or pure along-track). At certain epochs it will even not be possible to determine the GPS baseline at all, since about 5-6 GPS satellites need to be in common view. One might be able to derive the A-B and A-C baselines only during ¼ of the mission. At least this holds for the beginning of the mission, when all three orbit planes are close. Since the plane of satellite C will drift away from the others during the mission life time, the visibility will be further reduced. In addition the A-C and B-C baselines will then also contain a cross-track component. In contrast the A-B baseline will not change during the mission and there should always be enough GPS satellites in common view. In this respect the Swarm A-B situation is very similar to the GRACE constellation with two close satellites.

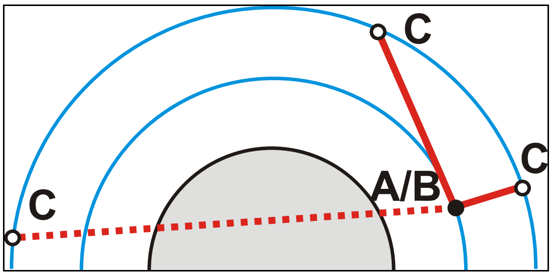


Figure 2. Time varying observation geometry.

As indicated in section 1 the GRACE baseline can be determined with an accuracy of about 1 mm, as compared to the very precise KBR measurement. Therefore one can assume a similar accuracy for the Swarm baseline A-B. Until now there are no results for such high-low GPS-baselines as Swarm A-C and B-C. Therefore, we use the same mm-accuracy as for the cross-track baseline. Pre-Swarm studies for such a constellation could be done using e.g. CHAMP and GRACE data or future GRACE and GOCE data.

3. METHODOLOGY OF THE ERROR STUDY

In order to give an estimate on the accuracy level, that can be expected from Swarm, two different strategies were used:

(1) a pure error propagation
(2) a full closed-loop simulation.

Both strategies should give comparable results, where (1) gives formal errors, i.e. error variances per spherical harmonic coefficient and (2) gives empirical errors, i.e. differences of potential coefficients between input model and the retrieved field. Both strategies are explained in more detail in the following subsections. The results are shown in section 4.

3.1. Error propagation

The gravitational potential can be expressed as a series of spherical harmonics. In orbit coordinates this reads

$$V(r,u,\Lambda) = \frac{GM}{R} \sum_{l} \left(\frac{R}{r}\right)^{l+1} \sum_{m} \sum_{k} \bar{K}_{lm} \bar{F}_{lmk}(I)\, e^{i(ku+m\Lambda)} \qquad (1)$$

with (r,u,Λ) being the orbit coordinates geocentric radius, argument of latitude and Earth fixed longitude of the ascending node, respectively; $\overline{K}_{lm}$ are the spherical harmonic coefficients (the unknowns in our case); $\overline{F}_{lmk}(I)$ is the so called inclination function (due to a rotation of the set of spherical harmonics to the orbit frame); R is the Earth's radius and (l,m) denote the spherical harmonic degree and order. Following [3] we can write equation (1) in form of a 2D Fourier series

$$V(u,\Lambda)=\sum_m \sum_k A_{mk}^V e^{i(ku+m\Lambda)}, \qquad (2)$$

where A_{mk}^V are linear combinations of potential coefficients of the same order m, the so called lumped coefficients

$$A_{mk}^V=\sum_l H_{lmk}^V \overline{K}_{lm} \qquad (3)$$

and H_{lmk}^V are the transfer or sensitivity coefficients of a specific gravity field functional, in this case the gravitational potential V, for which they read

$$H_{lmk}^V=\frac{GM}{R}\left(\frac{R}{r}\right)^{l+1}\overline{F}_{lmk}(I)\,. \qquad (4)$$

Observing, e.g. the potential V one gets the lumped coefficients by way of a Fourier analysis (equation 2). Equation number (3) is then the linear model for the determination of the unknown potential coefficients $\overline{K}_{lm}$. The design matrix contains the transfer coefficients. If a noise model is available for the observations one can perform a pure error propagation. This can be done not only for the potential V, but for arbitrary gravity field functionals, say orbit perturbations, potential gradients, gravity gradients or ranges between satellites. The latter two cases can be used to simulate the expected accuracy for the GOCE and GRACE mission, respectively. Further details on the error propagation can be found in [4].

In the sequel we will make use of potential gradients to perform the error propagation for Swarm, i.e. we use the components of the gravitational acceleration vector in different observation directions. This is done, because we will use the so called energy integral method (see section 3.2.) in the closed loop simulation. This method gives potential differences ΔV from the inter-satellite GPS baselines. For short baselines, as it is the case for GRACE and at least also for Swarm A-B, ΔV is proportional to potential gradients, e.g. for the along-track component it holds

$$\Delta V/\Delta x\approx V_x\,, \qquad (5)$$

Where Δx is the length of the along-track baseline. In case of GRACE, e.g., one can derive the along-track potential gradient V_x, from Swarm A-B one gets the cross-track gradient V_y. Swarm A-C and B-C can be approximated by the radial component V_z. Especially the latter approximation can only give qualitative information, since the A-C and B-C baselines might be considerably long and are in general combinations of along-track, cross-track and radial directions. Still the error propagation for potential gradients reveals interesting aspects of the error characteristics (see section 4).

3.2. Closed-loop simulation

The closed-loop simulation is based on a "true-world" model of the global gravity field. This is used to integrate orbits of the three Swarm satellites. The resulting positions of the satellites A, B and C at identical epochs are then used to compute the GPS-baselines, which in reality are the basic observations. Using these baselines, one can try to recover the "true-world" model. In our computations the gravity field recovery is based on the energy integral method, already successfully applied to real CHAMP (see [5], [6]) and GRACE data (see [7]). The method (as applied to GRACE or Swarm) gives potential differences ΔV between two satellites from the relative velocity vector, i.e. from the first temporal derivative of the GPS-baseline (=relative position) vector. The energy integral is described in more detail below. In order to study the error behavior of Swarm, the GPS-baseline is contaminated with error time series, both for the baseline itself as well as for the relative velocity vector. In both cases white noise is used. Of course this scenario is not necessarily realistic, especially for the velocity, but we use white noise here as a first guess to estimate the order of magnitude one can expect. In future studies a more realistic error spectra will be used.

Energy integral method

The energy integral method is based on the energy conservation law, which states, that the sum of potential and kinetic energy is constant. The kinetic energy is proportional to the square of the satellite's velocity, while the potential energy contains the desired gravitational potential V. In an Earth fixed reference frame the energy integral of a single satellite reads

$$V=\frac{1}{2}\dot{\boldsymbol{x}}^2-Z+\int_x \boldsymbol{a}\cdot \mathrm{d}\boldsymbol{x}+C\,, \qquad (6)$$

where Z is the centrifugal potential, C is an unknown constant and the line integral in equation (6) sums up the effect of all non-conservative forces like air drag (measured by the onboard accelerometer) or tidal effects of Sun and Moon. All these forces are not subject to the current study. Therefore neither air drag nor tidal forces are included in the present simulations. However, in reality they can simply be included using equation (6). For reasons of simplification (neglecting the constant C for a moment) we may only consider the velocity vector and state, that the potential is a function of the absolute velocity $\dot{\boldsymbol{x}}$ (and centrifugal potential), i.e.

$$V = \frac{1}{2}\dot{\boldsymbol{x}}^2 - Z \,. \tag{7}$$

In a similar sense we can give the following equation for the potential difference (e.g. between satellites A and B) as function of the relative velocity vector $\Delta\dot{\boldsymbol{x}}_{A,B}$

$$\Delta V_{A,B} = \frac{1}{2}\Delta\dot{\boldsymbol{x}}_{A,B}^2 + \dot{\boldsymbol{x}}_A \cdot \Delta\dot{\boldsymbol{x}}_{A,B} - \Delta Z_{A,B} \,. \tag{8}$$

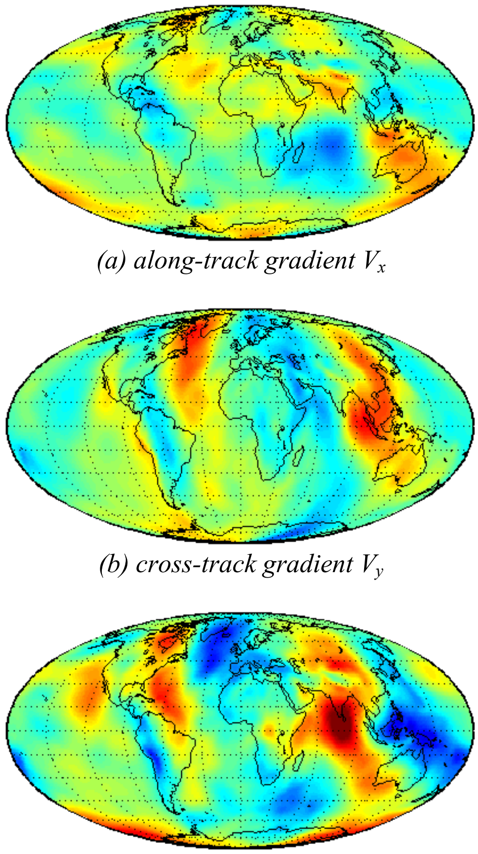
The potential differences between several points are used in an inversion step as pseudo observations to recover the potential coefficients of the gravitational field. The difference between the true-world model and the recovered field is the empirical error of the analyzed baseline. The results for several constellations are discussed in the next section.

4. SIMULATION RESULTS

4.1. Potential gradients

Before we show the characteristics of the error spectra, let's first take a look at the observations themselves. Fig. 3 (a-c) shows the along-track, cross-track and radial component of the potential gradient, respectively. It is obvious, that they reveal different parts of the gravity field. While the radial component seems to be more or less isotropic (no clear preference for different direction – only the high frequencies are amplified) the along-track and cross-track components show different sensitivity for different directions. The along-track component reveals East-West structures of the field (like the Himalaya massive), while the cross-track contains mostly North-South structures (like the Andes – the Himalaya is hardly visible). For an intuitive explanation let's consider an ellipsoidal gravity field (e.g. one like the normal reference field GRS80) with variations in latitude direction only: two satellites in cross-track mode (like Swarm A-B) would measure the same signal and therefore the gradient between them vanishes. Such a constellation is not sensitive to the zonal part of the spectra (coefficients with order *m=0*). In contrast a GRACE-like along-track constellation is not very sensitive to the sectorial structures of the field.

As already discussed in section 3.1, the potential gradients shown in Fig. 3 are only approximations for the observed potential differences along GPS baseline vectors between different satellites. Still, Fig. 3 gives a qualitative description of what can be expected from different constellations. The actual error characteristics as derived from the pure error propagation as well as the closed-loop simulation are presented in the next section.



(a) along-track gradient V_x

(b) cross-track gradient V_y

(c) radial gradient V_z

Figure 3. Different components of the potential gradient.

4.2. Qualitative description of error characteristics from potential gradients and potential differences

To derive a qualitative description of the spherical-harmonic error characteristics both, a pure error-propagation (based on transfer coefficients of potential gradients) as well as a closed-loop simulation (based on the relative energy integral between two satellites) were performed. Fig. 4 shows the results in a matrix of triangle plots of the complete spherical harmonic error spectra. According to [3] only the radial component of the gradient is isotropic (only dependent on the spherical harmonic degree *l*; no preference for different directions), while the cross-track and along-track components are direction dependent. This is clearly visible from the triangles in the top line of Fig. 4. As already indicated in section 4.1, the along-track component is not very sensitive to the sectorial structures (coefficients at the left and right edges of the triangles), while the cross-track component is not sensitive to the zonal part of the spectra (vertical line in the center of the triangles). The mid row of the triangle matrix shows the results of the closed-loop simulation. Even though the picture is not that clear as in the top row, the error characteristics are in principle the same. For comparison the bottom row shows the estimated errors of the EIGEN-GRACE02s model [8], derived by GeoForschungsZentrum Potsdam (GFZ) from real GRACE data, i.e. from an along-track constellation. It shows the same error structure, with less sensitivity for the higher sectorials. Apart from the different sensitivity of different constellations for different parts of the spherical harmonic spectra it is also worthwhile to notice, that the along-track and cross-track components are complementary and their combination reveals the same signal content in all parts of the spectra as the radial component alone.

Comparing the different rows of the triangle matrix, one must take into account, that the spectra is shown with different resolution: while the error propagation (top row) was performed up to degree $l=200$, the close-loop simulation was performed only up to $l=70$; the real model EIGEN-GRACE02s is shown complete up to degree $l=150$.

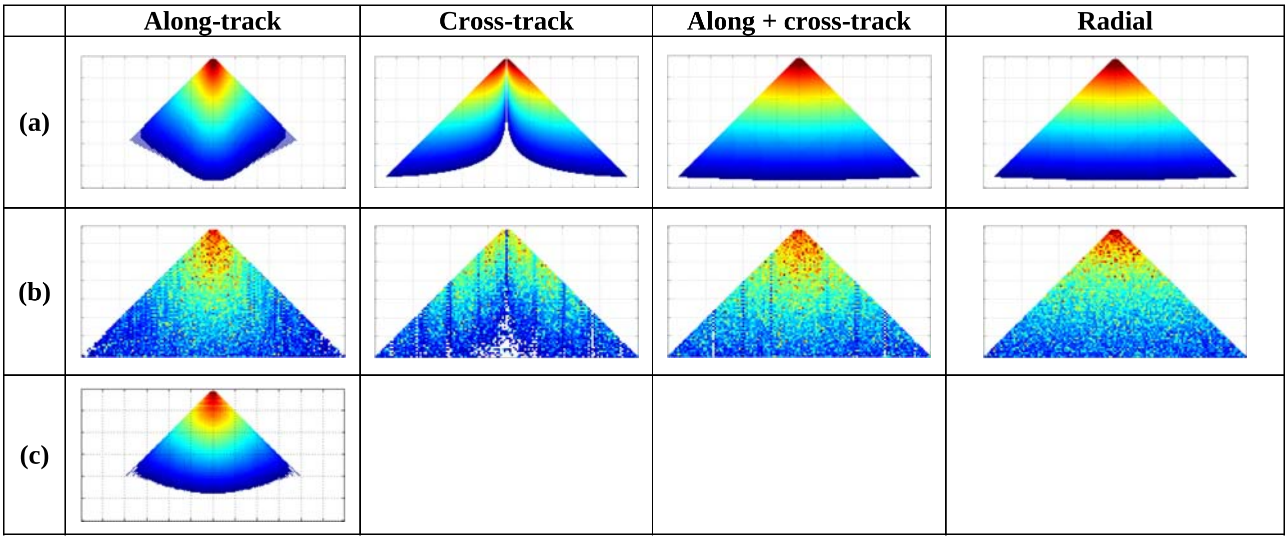


Figure 4. Qualitative comparison of sh-error characteristics for (a) error propagation, (b) closed-loop simulation and (c) real error estimates of GFZ's EIGEN-GRACE02s model. The triangles show the number of significant digits, with red indicating high numbers (accurate result) and blue indicating only a low number of significant digits (recovered coefficients close to signal-to-noise ratio, SNR=1:1). Estimates which are not significant (SNR<1) are not plotted at all.

4.3. Quantitative analysis of GPS-baselines for gravity field recovery

Based on error estimates for the GPS baseline vector a quantitative description of the expected accuracy of the recovered gravity field was performed. The error estimates for the relative position vector is 1 mm/s, which corresponds to the accuracy obtained by different groups (see [1] and [9]) for the GRACE GPS-baseline. The accuracy in case of GRACE is verified against the microwave link between the satellites, which is about a factor 1000 more accurate that the GPS derived range. In case of CHAMP, the numerical differentiation of positions with cm-accuracy led to absolute velocities of about 0.1 mm/s accuracy. The CHAMP data sampling is 30 seconds, so the accuracy holds for smoothed values rather than for point values. Now for GRACE the relative position accuracy is about one order of magnitude higher than the absolute CHAMP positions. Therefore we assume in the simulation also an increase of one order of magnitude between the absolute and relative velocities, i.e. we assume the baseline velocity to have errors in the order of 0.01 mm/s. This is about 2 orders of magnitude worse, that the range-rate derived from GRACE KBR.

The results of the full closed-loop simulation are shown in Fig. 5 by means of degree RMS values. The figure contains the Kaula-curve, indicating the gravity signal, along with several error curves for different observation scenarios. The 'CHAMP'-curve is derived from a simulation with one satellite in an altitude of 450 km and an absolute velocity accuracy of 0.1 mm/s. The 'along'- and 'cross'-curves show the results derived from two-satellite constellations also in 450 km altitude but with relative velocity accuracy of 0.01 mm/s. the ‘along’-constellation corresponds to GRACE (without KBR) and the ‘cross’-constellation corresponds to Swarm A-B. The combination of ‘cross’ and ‘along’ leads to the same accuracy as a pure radial constellation. This was already shown in section 4.3. Of course the radial component is sensitive to the orbit altitude: lower constellations will lead to a higher resolution of the gravity field than higher ones. The curve denoted ‘radial’ corresponds to a constellation with one satellite in 450 km and another in 300 km altitude, which might fit to the Swarm A-C or B-C baselines at the end of the mission life time. At the beginning of the mission, the lower pair is in an altitude of 450 km and the third satellite is in an altitude of 530 km. This constellation leads to the curve denoted ‘Swarm’. Obviously the swarm constellation is more sensitive to the higher degrees of the gravity field at the end of the mission life time as compared to the beginning. Unfortunately the orbit planes of the lower pair and the upper satellite will drift apart, leading to a decreased observability of the A-C and B-C GPS-baselines. Therefore there will be only less epochs available for gravity field recovery. In Fig. 5 all curves are scaled to one month of data. For comparison there are also two GRACE-KBR curves included. The ‘KBR’ curve corresponds to a full simulation using KBR range-rates and indicates the GRACE baseline errors. This baseline is not yet reached with real data. The ‘GRACE’ curve is computed from the standard deviations given for GFZ’s model EIGEN-GRACE02s.

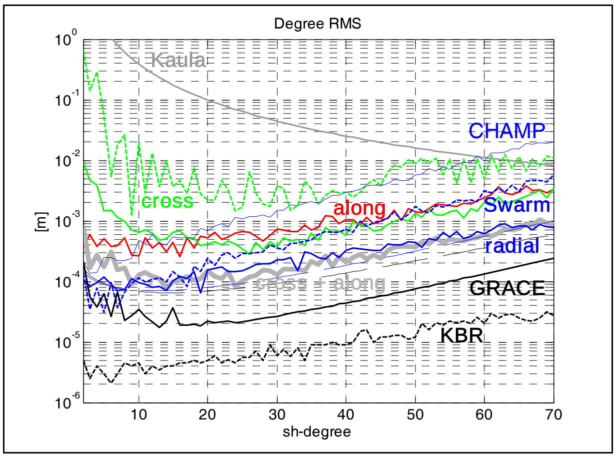


Figure 5. Degree errors for different constellations

5. CONCLUDING REMARKS

Each of the Swarm satellites is very similar to CHAMP. Therefore they can, in principle, be used for gravity field recovery. In contrast to CHAMP, where absolute GPS positions are the basic observations, Swarm allows to observe GPS-baselines between the three satellites, thus increasing the accuracy of the derived gravity field. The quality of the gravity field solution depends on the constellation (which baselines are used) and the altitude of the satellites. Close to the end of the mission life-time, the quality will profit from the low altitude, but suffer from bad observation geometry. The simulation contains this bad geometry only roughly: constraints were put on the spherical distance between satellites and the baselines were only used when this distance was not larger than a given limit. In order to make the whole simulation more realistic, a full GPS-simulation should be included.

Of course Swarm cannot compete with the KBR-accuracy of GRACE, neither with the projected baseline accuracy nor with the currently achieved results (which will probably converge towards the baseline in the future). But Swarm might become interesting considering the life-time of the different missions. GRACE might continue till 2010 or 2011, while CHAMP will probably last only till 2008. The GOCE mission is planned for the period 2007-2009. This means, that the launch of Swarm (planned for 2010) is close to the end of the decade of dedicated gravity field missions. Without any direct follow-on missions, Swarm could continue this decade – of course with reduced accuracy. Further investigations need to be carried out to see, if Swarm could contribute in some way to temporal variations in the gravity field, such as semi-annual, annual or secular changes in the low spherical harmonics.

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