# Advantages of In Situ Data when Estimating the Global Neutral Atmospheric Density Using a Data Assimilation System

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# Abstract

Recently developed data assimilation techniques have improved time-dependent estimates of the neutral atmospheric density, making it possible to better estimate the drag perturbation force on low-Earth-orbiting satellites. These new systems primarily use surveillance of a satellite's orbital parameters to determine the neutral density along the satellite's flight path. A limitation to satellite surveillance comes from the need to observe each satellite over long periods (hours) to determine the satellite's average acceleration change. This limitation in temporal and spatial resolution translates into limited resolution in the data assimilation solution. In situ data, on the other hand, provide density measurements with a substantially increased temporal and spatial resolution. Evaluations show that, even though the global coverage from in situ sources is typically limited (usually 2 or fewer satellites) as compared with a constellation (50+) for satellite surveillance, the higher temporal and spatial resolution of in situ data minimizes errors and increases the resolution of the data assimilation output. Furthermore, the higher temporal and spatial resolutions of in situ data are shown to be especially important during geomagnetic storms when changes in the neutral density can occur on a minute-to-minute basis. Results of the data assimilation system are used to compare the use of satellite surveillance and in situ data with emphasis on CHAMP data.

# Background

- After the oblateness of the Earth, atmospheric drag is the next most significant natural perturbation force affecting satellite trajectories for low-Earth-orbiting (LEO) (<1000 km) satellites.
- Furthermore, this region, the neutral atmosphere, also called the thermosphere (95 to 500 km altitude), is constantly changing, and as a result, the estimation of the drag perturbation has a large uncertainty.

$$\boldsymbol{a}_{drag} = -\tfrac{1}{2} C_D A \rho \boldsymbol{v}_{rel} \left| \boldsymbol{v}_{rel} \right| / m$$

- Although difficulties can arise in estimating any of the terms in the drag force equation, the density term, *r*, consistently has among the largest uncertainties.
- Any improvement in estimating of *r* would significantly improve the drag estimate and consequently improve one’s estimate of the other orbital parameters.

# Limitations of Empirical Modelling

- Empirical models are a convenient ways to represent the neutral atmospheric density, but…
- Because the empirical model is statistical representation of the climate over a long period of time (years, months), they may not appropriately represent unusual, short-term features (hours, minutes) in the upper atmosphere.
- These hourly or minute-to-minute features in the thermosphere become even more pronounced during geomagnetic storms.
- Errors for MSIS-90 may reach 30 to 50%, even during low solar activity.
- NRLMSIS-00 can reach 30% at 200 km altitude and 70% at 600 km altitude.

# Improving Accuracy through Data Assimilation

- High Accuracy Satellite Drag Model (HASDM), which provides an improved specification of the neutral density with errors typically below 7 to 8% during geomagnetic quiet periods.
- Errors higher during storm periods, >20%.
- HASDM, for example, assembles data from a large (60+ satellites) constellation of LEO satellites using space surveillance.
- Density is obtained by observing perturbations in these LEO satellites' orbital parameters.
- If the ballistic coefficient is estimated along with the other orbital parameters, changes in the estimated ballistic coefficient can indicate changes in the neutral density.

# Limitations of Space Surveillance as an Observation Source

- However, the difficulty in using satellite surveillance arises from having to observe the satellite constellation over long periods, usually over many hours, to extract the changes in the ballistic coefficient estimate from the observation noise.
- In a validation study for the Dynamic Calibration Atmosphere (DCA) Phase I of HASDM (Casali and Barker, 2002), results indicate that only slight improvements could be obtained past a spherical harmonic resolution of degree 1 (2x2), with insignificant improvement beyond degree 2 (3x3) due to the long observation period requirement.
- To obtain this density resolution, the 0-degree correction to the exospheric temperature is solved every 3 hours. The higher-degree exospheric temperature coefficients are solved every 18 hours. Additionally, a 0-degree inflection temperature is solved for every 18 hours. A 3-hour segmentation was sufficient in conjunction with an *a priori* uncertainty of 3% (RMSE), as is the best level of drag modeling thought to be statistically obtainable by DCA.

# Questions to Be Answered

- This research seeks to answer the following two questions:
  - (1) How well can the higher temporal resolution of 2 in situ satellites resolve the hourly and minute-to-minute variability?
  - (2) Can the higher temporal resolution of only 2 in situ satellites provide a stable solution at a higher resolution as compared with the space surveillance?

# Information as a Limit to Spatial Resolution

- The solution resolution is limited by the amount of information contained in the observations.
- As a result, there is a maximum stable resolution that can be obtained for a given observation source and data set.
- Therefore examine the potential of other observation sources, like in situ measurements, which have a higher temporal resolution and thus higher information content.

# Procedure

- This research seeks to answer these questions according to the following procedure:
  - A simulated thermosphere is created using a physical model, the Coupled Ionosphere-Thermosphere Model (CTIM), which is defined as the ‘truth’ thermosphere.
  - This ‘truth’ thermosphere simulates 24 hours of quiet, followed by 12 hours of geomagnetic storm conditions, followed by another 12 hours of quiet to respectively examine the quiet, storm, and storm-recovery capability of the system.
  - The storm conditions will reach an *ap* index of 300 during the 12-hour storm.
  - This ‘truth’ thermosphere will be sampled using a satellite simulation algorithm according to the orbital and instrument mechanics of two in situ observing systems, both of which are on two CHAMP-like satellites at 09:30 and 13:30 LT crossings.
  - These simulated observations (with 15% random errors to simulate noise) are used to calculate the thermospheric density via a least squares solution.
  - This solution is compared with the original ‘truth’ thermospheric density.
  - A root mean squared error (RMSE) and a pattern correlation coefficient are calculated to quantify the solution accuracy.
  - The maximum stable resolution is determined in the study by incrementally increasing the resolution until a limit on the maximum RMSE and minimum pattern correlation coefficient is found.

# Solution Method

## Solve for spherical harmonics…

$$T(\psi,\lambda) = a_0 + \sum_{i=1}^{\infty} a_i \left[P_i(\sin\psi)\right] + \sum_{i=1}^{m}\sum_{j=1}^{i}\left[P_{i,j}(\sin\psi)\right]\left\{c_{i,j}\cos(j\lambda) + s_{i,j}\sin(j\lambda)\right\}$$

$$P_i(\sin\psi) = \frac{1}{2^n n!}\frac{d^i}{d(\sin\psi)^i}\left[(\sin\psi)^i - 1\right]^i \qquad P_{i,j}(\sin\psi) = \left[1 - (\sin\psi)^2\right]^{j/2}\frac{d^j P_i}{d(\sin\psi)^j}$$

## Using batch least squares…

$$\hat{\boldsymbol{X}}_k = \left(H_k^T R_k^{-1} H_k + \overline{P}_k^{-1}\right)^{-1}\left(H_k^T R_k^{-1}\left[\boldsymbol{y}_k - \boldsymbol{y}_k^*\right] + \overline{P}_k^{-1}\left[\overline{\boldsymbol{X}}_k - \boldsymbol{X}_k^*\right]\right) + \boldsymbol{X}_k^*$$

$$P_k = \left(H_k^T R_k^{-1} H_k + \overline{P}_k^{-1}\right)^{-1}$$

$$H_k^T R_k^{-1} H_k = \sum_{l=1}^{m} \tilde{H}_l^T R_l^{-1} \tilde{H}_l \qquad H_k^T R_k^{-1}\left[\boldsymbol{y}_k^{obs} - \boldsymbol{y}_k^*\right] = \sum_{l=1}^{m} \tilde{H}_l^T R_l^{-1}\left[\boldsymbol{y}_l^{obs} - \boldsymbol{y}_l^*\right]$$

# System Evaluation

- Root mean square error (RMSE)

$$\mathbf{RMSE} = \sqrt{\sum_{n=1}^{N} \left(\hat{\rho}_n - \rho_n^{CTIM}\right)^2 \Big/ N}$$

- Pattern correlation coefficient, $\rho$

$$\gamma = \mathrm{cov}(\hat{\boldsymbol{\rho}}, \boldsymbol{\rho}^{CTIM}) \Big/ \sqrt{\mathrm{var}(\hat{\boldsymbol{\rho}}) \cdot \mathrm{var}(\boldsymbol{\rho}^{CTIM})}$$

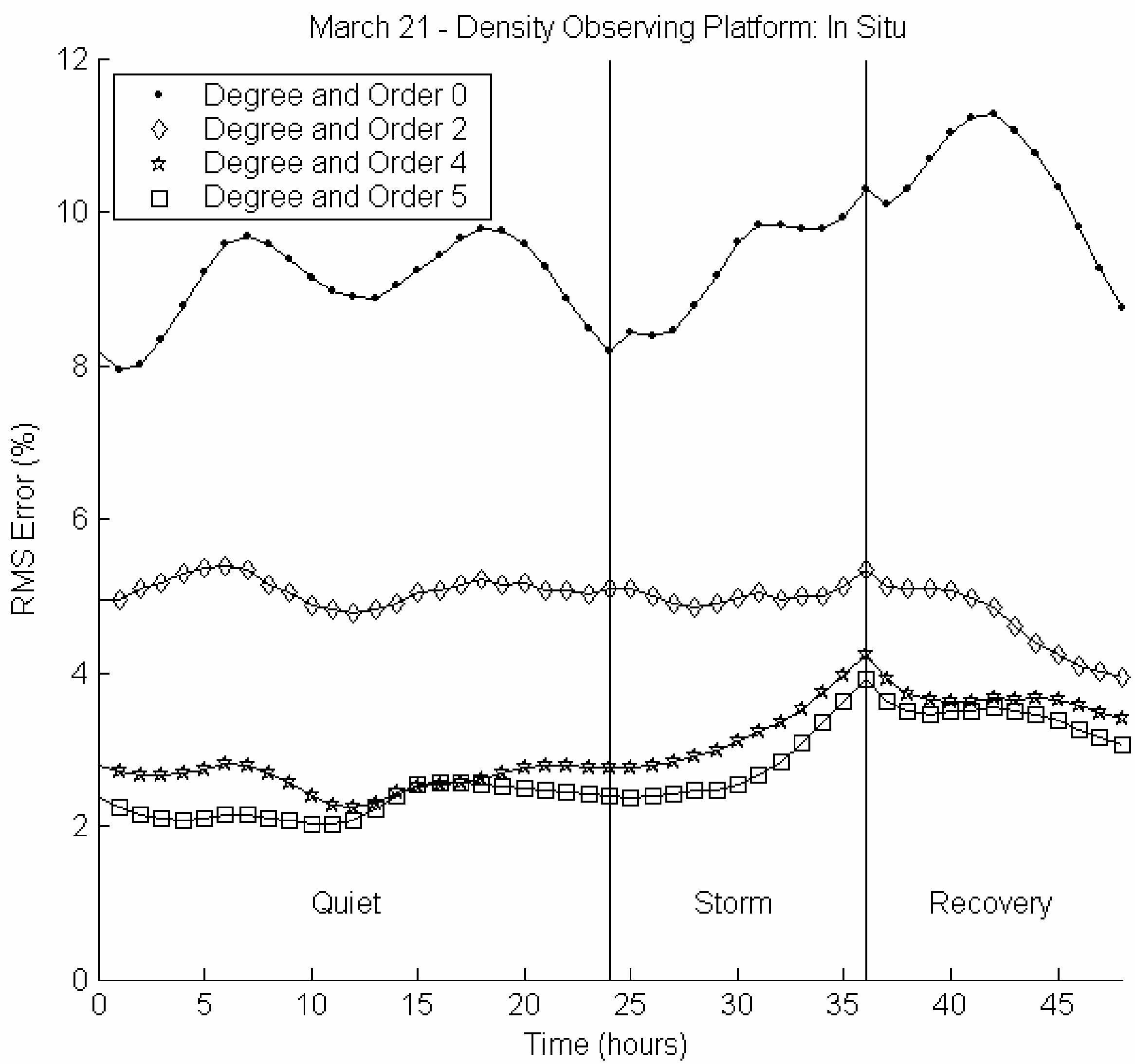
$$\mathrm{cov}(\hat{\boldsymbol{\rho}}, \boldsymbol{\rho}^{CTIM}) = \sum_{n=1}^{N} \left[\left(\hat{\rho}_n - \hat{\mu}\right)\left(\rho_n^{CTIM} - \hat{\mu}^{CTIM}\right)\right] \Big/ N$$

$$\mathrm{var}(\hat{\boldsymbol{\rho}}) = \sum_{n=1}^{N} \left(\hat{\rho}_n - \hat{\mu}\right)^2 \Big/ N$$

$$\mathrm{var}(\boldsymbol{\rho}^{CTIM}) = \sum_{n=1}^{N} \left(\rho_n^{CTIM} - \hat{\mu}^{CTIM}\right)^2 \Big/ N$$

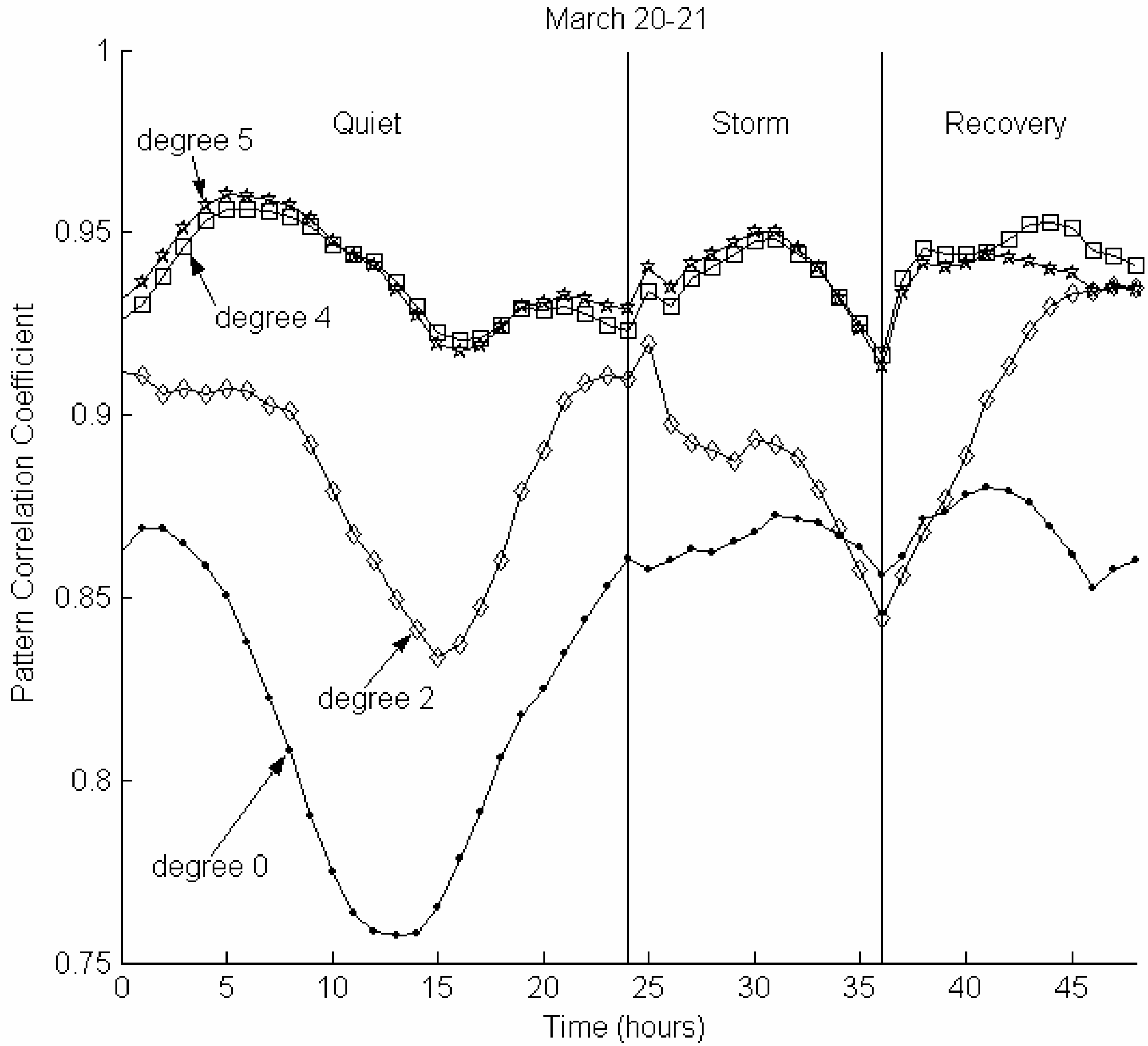
# Results (RMSE)

- RMSE indicates a maximum resolution of degree 4 (5x5) for two in situ satellites.



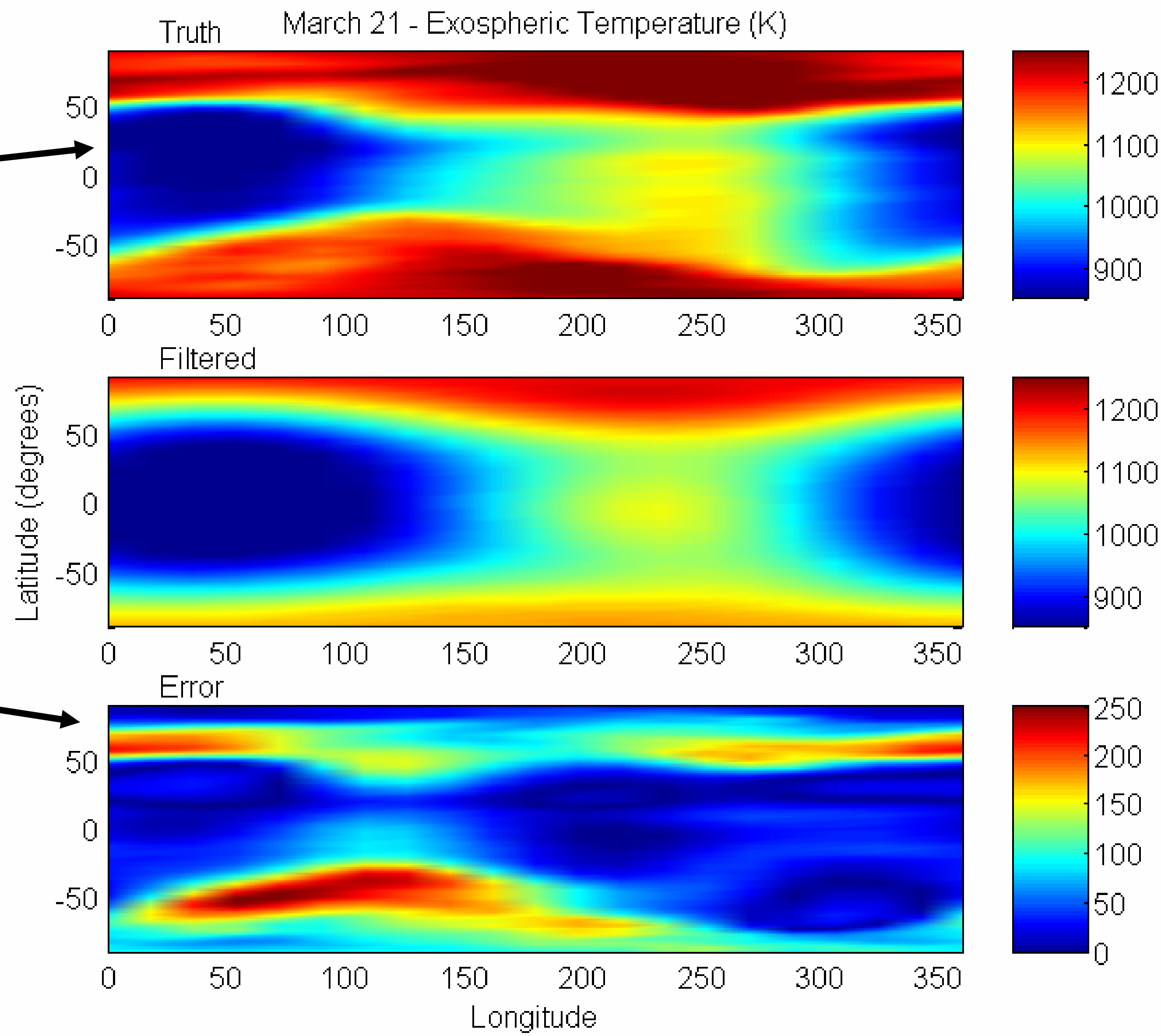
# Results (pattern correlation)

- The pattern correlation coefficient also indicates a maximum resolution of degree 4 (5x5) for two in situ satellites.



# Results (at storm peak, 36 hours)

- Simulated 'truth' thermosphere
- Maximum resolution for 2 in situ satellites.
- Absolute difference between the 'truth' and the data assimilation result.



# Conclusions

- Results presented herein provide a preliminary measure for the potential of in situ measurements for its use in discerning the small-scale, short-term neutral density structure.
- Results show that in situ measurements may be better adept at capturing the small-scale, short-term density structure due to its high temporal resolution as compared with space surveillance.
- This research comes to this conclusion based on the ability of in situ measurements to obtain a higher stable solution through the data assimilation system as compared with space surveillance.
- Using 2 in situ satellites demonstrates that the neutral density can be specified with a stable solution up to and including a spherical harmonic resolution of degree 4 (5x5) every 10 minutes.
- This result indicates that in situ measurements can provide a higher degree of resolution as compared with space surveillance systems, which have a lower degree 2 (3x3) maximum resolution every 18 hours with a degree-0 (1x1) correction every 3 hours due to the longer observation period required.
- Although coverage from 2 in situ satellites is limited, the coverage is sufficient to provide a pattern correlation coefficient consistently higher than 0.92.