

## Satellite altimetry data processing

-Marine Gravity -

 $f(x + \Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)}{i!}$ 

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### **Motivation**

- Two thirds of the globe is covered with water
- Large regions are NOT covered with gravity/bathym
- If you were to cover the entire ocean with marine observations to 10 km res it would take 200 years
- Satellite altimetry can provide information of the Sea surface height over the oceans over nearly 60% of the Earth surface and its free.
- The height of the oceans closely assembles an equipotential surface of gravity.
- This way altimetry can be used to derive high resolution marine gravity field AT THE SURFACE.
- Satellite altimetry only provide the finer scale of the gravity /bathymetry.
- Individual satellite altimetry observations might not provide as accurate direct gravity field observations as marine gravity, but the ability to provide near global uniformly accurately gravity field makes satellite altimetry un-surpassed for determining the global marine gravity field of the earth.







### **Motivation**

- Due to satellite altimetry the gravity and bathymetry is now know FAR more completely over the oceans than before.
- This is important for many purposes (safety etc etc)
- Ship observations provide the long wavelength of the signal due to the shiptrack spacing
- Space missions (GRACE, GOCE) measuring AT 200 KM can only deliver the same SPATIAL RESOLUTION as marine gravity and NEVER the same resolution that satellite altimetry



### Content

Brief repetition (mearurement-technique)Radar altimetric observations

Isolating the Mean of the sea surface

Going from the Mean to the Geoid.
-> First Excercise
Going from the Geoid to Gravity

•Going from Gravity to Bathymetry.

Applications of Marine gravity



•Next generation development ←This is where we need students ☺ •-> Second Excercise.





### Altimetry observes the sea surface height (SSH)

The orbital height of the space craft (rel to the ref ellipsoid) minus the altimeter radar ranging to the sea surface corrected for path delays And environmental corrections Yields the sea surface height:

$$h(t) = SSH = MSS + \xi(t) + e$$

#### where

- *MSS* is the mean sea surface above the reference ellipsoid,
- $\xi$  is the ocean topography,
- e is the error



### The "mean" height mimicks the geoid.

## The reference Ellipsoid



Reference ellipsoid is the Mathematical shape of the Earth (using a, e etc). This enable establishing coordinates

Rather than working in Height of 6000 km +/- 100 meters

We isolate +/- 100 meters

One can cay that we REMOVE the ellipsoid



### Altimetry SSH observations (SSH)



$$h(t) = MSS + \xi(t) + e$$

Today: We work on data to get the MSS and the Geoid/Gravity/Bathymetry.

**Tomorrow:** 

We will work on data to get the time-varying signals  $\xi$  (t)

# Timevarying consist of tides, surges, currents and sea level change.







- Geoid is an equipotential surface of the Earth gravity potential N=W/γ.
- Change in gravity/potential is related to change within the earth.
- Moving horizontally does not require work by grav potential.
- So to determine bathymetry you FIRST need to determine gravity
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The MSS and the Geoid

MSS = N + MDT

If there was no currents then MSS = N and MDT = 0. N is +/- 100 meters



The Geoid of the Earth.

This "is" the surface that all Water in the oceans and would have. Also on land if you digged Channels.



The Geoid ontop of the Reference ellipsoid Greatly exaggerated.

Again its +/-100 meters.





The MSS and the Geoid

MSS = N + MDT

If there was no currents then MSS = N and MDT = 0. N is +/- 100 meters



However there are currents And the water does not Have the exact same density And temperature throughout The worlds ocean. MDT is +/- 1.5 meters.





### The Mean Dynamic Topography.



The MDT is an interesting quantity in itself as it contains info on all major Currents in the world. A change in the mean Current -> a change in the MDT IT IS WELL REPRESENTED BY A MODEL AND WE CAN TAKE IT OUT DTU Space

### National Space Institute Summing Altimetric observations



SEA SURFAC

GEOID



### The magnitudes of the contributors

RANGING ranges up to OCEAN ASER TOPOGRAPHY +/-100 meters The geoid  $N_{RFF}$ ANGING TATION Residual geoid  $\Delta N$ +/-2 meters +/-2 meters n Mean dynamic topography  $\xi_{MDT}$ GEOID 1 Time varying Dyn topography  $\xi(t)$ SEA +/-5 meters. (Tides LEVEL Error +/-10 cm



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REFERENCE ELLIPSOID

SEA-FLOOR

### Errors



$$e = e_{orbit} + e_{tides} + e_{range} + e_{retrack} + e_{noise}$$

- *e<sub>orbit</sub>* is the radial orbit error
- e<sub>tides</sub> is the errors due to remaining tidal errors
- $e_{range}$  is the error on the range corrections.
- *e*<sub>retrak</sub> is the errors due to retracking
- $e_{noise}$  is the measurement noise.

### Accuracy versus precision.

Accuracy is the relationship between the mean of measurement distribution and its "true" value, whereas *precision*, also called reproducibility or repeatability

Different applications may have different requirements in terms of accuracy and/or precision. For instance, the estimation of the rate of global sea level rise from altimetry requires accuracy, but not necessarily precision given the huge numbers of measurements available to compute the mean rate.

Ocean studies like of El Niño require *both* accuracy (to discriminate the anomalous raised or lowered SSH value with respect to the mean)

#### Gravity and Bathymetry only requires precision.

### PRECISION IS 2-4 CM WITH MODERN ALTIMETERS

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### Lets isolate N

Look at wavelength content.

Altimetry is most accurate for wavelength between 10 and 150km

$$h_{RED1} = N + \xi(t) + e$$





Two mearusement "modes" Repeating low resolution (ERM) vs Non repeating (geodetic mission) mode.





GEOSAT+ERS GM data is ESSENTIAL for high resolution Gravity Field mapping. 17 DTU Space, Technical University of Denmark Databehandling 30210, 2013

### The $\xi(t)$ time varying signals.

- ERM data. Most time+error averaged out
- Geodetic mission data  $\xi(t)$  is not reduced
- Must limit errors to avoid "orange skin effect"
- 95% OF ξ(*t*) IS LONG WL >150KM
- PERFORM X-OVER ADJUSTMENT







### **Crossover Adjustment**

Assumption: the geoid does not change and is identical where tracks meets. Timevarying sea level changes like long wavelength from track to track. "THIS IS NOT PART OF PENSUM"

- <u>d<sub>k</sub>=h<sub>i</sub>-h</u>;
- <u>d</u>=A<u>x</u>+v
- where <u>x</u> is vector containing the unknown parameters for the track-related errors.
- v is residuals that we wish to minimize
- Least Squares Solution to this is

 $\underline{x} = (A^T C_d^{-1} A + cc^T)^{-1} A^T C_c^{-1} \underline{d}$ 

- Constraint is needed c<sup>T</sup>x=0
- Problem of Null space Rank
- Bias (rank=1) mean bias is zero
- Bias+Tilt (Rank = 4)
- Constrain to zero (geoid)





### Effect of Crossover adjustment







**Isolating The geoid: N** 

The timevarying signal is "gone"

# Altimetry is most accurate for wavelength between 10 and 150km



Reference geoid models are used to model long wavelength (where altimetry is not accurate) BTU Space, Technical University of Denmark







### Global Earth Geopotential Models (EGM's)

Example EGM96 or EGM08 also EIGEN 6S or EIGEN 6C Exist as Satelllite only (S) = GRACE/GOCE/LAGEOS ETC - low resolution Or as Combination models (C) = Sat+Land/ship/altimetery





### Global Earth Geopotential Models (EGM's)

Rather than giving the Geoid or Gravity (vector) you provide the

Gravity potential as Geoid and Gravity can be derived from this (will return to that). However the potential is not easily derived. It requires global integration...

$$V = G \iiint_{Jorden} \frac{dm}{l}$$

$$= G \iiint_{Jorden} \frac{\rho}{l} dv, \quad \rho = massetætheden$$

Global convolution is nearly impossible so you turn to Spherical Harmonics.

Like Fourier Transformation on a sphere.

#### Here Convolution becomes Multiplication (will come back to this in plane)

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Strykowski G. Lecture 02:

National Space Institute The Spherical Harmonics and the Gravity Field of the Earth

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Expansion of the reciprocal distance into zonal harmonics and decomposition formula [see HWM2006, sec 1.11]:

Given: 
$$Pig(r, artheta, \lambdaig)$$
 and  $P'ig(r', artheta', \lambda'ig)$ 

$$l^2 = r^2 + r'^2 - 2rr'\cos\psi$$

trigonometrical relations for the spherical triangles yield

$$\cos \psi = \cos \vartheta \cos \vartheta' + \sin \vartheta \sin \vartheta' \cos (\lambda' - \lambda)$$

Assume that: r' < r (or change the notation)

Important and remarkable result

$$\frac{1}{l} = \sum_{n=0}^{\infty} \frac{r'^2}{r^2} P_n\left(\cos\psi\right)$$



Fig. 1.7. Spherical distance  $\psi$ 



and, in fully normalized solid spherical harmonics:

Fig. 1.8. The spatial distance l

$$\frac{1}{l} = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{1}{2n+1} \left[ \frac{\overline{R}_{nm}(\vartheta,\lambda)}{r^{n+1}} r'^{n} \overline{R}_{nm}(\vartheta',\lambda') + \frac{\overline{S}_{nm}(\vartheta,\lambda)}{r^{n+1}} r'^{n} \overline{S}_{nm}(\vartheta',\lambda') \right]$$







### **Global Geoid Models**

Global geopotential models are given as spherical harmonics.

These can be expanded in to regular grid

(i.e. Matlab geoid Heigh routine)

The Degree gives the "resolution"

Of the model

Resolution = 40000km /(degree\*2)

All geoid models can be downloaded From.

http://icgem.gfz-potsdam.de/ICGEM

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Model	Year	Degree	Data
GO_CONS_GCF_2_DIR	2010	240	S(GOCE)
GO_CONS_GCF_2_TIM	2010	224	S(GOCE)
GO_CONS_GCF_2_SPW	2010	210	S(GOCE)
GOC001S	2010	224	S(GOCE,Grace)
EIGEN-51C	2010	359	S(Grace,Champ),G,A
AIUB-CHAMP03S	2010	100	S(Champ)
EIGEN-CHAMP05S	2010	150	S(Champ)
ITG-Grace2010s	2010	180	S(Grace)
AIUB-GRACE02S	2009	150	S(Grace)
GGM03C	2009	360	S(Grace),G,A
GGM03S	2008	180	S(Grace)
AIUB-GRACE01S	2008	120	S(Grace)
EIGEN-5S	2008	150	S(Grace,Lageos)
EIGEN-5C	2008	360	S(Grace,Lageos),G,A
EGM2008	2008	2190	S(Grace),G,A
ITG-Grace03	2007	180	S(Grace)
AIUB-CHAMP01S	2007	90	S(Champ)
ITG-Grace02s Corel	2006	170	S(Grace)
FICEN CLAASA	2006	150	S(Grace Lageon)



### **Global Geoid Models**

By using close formulas the potential can be turned into geoid heighe or the associated gravity field (se Strykowskis lectures)





### **Remove - Restore.**



$$h_{RED3} \approx h_{RED2} - N_{REF} = \Delta N + e'$$
  
*Take These Out"*

$$h_{REDUCED} \approx \Delta N + e'$$

- Remove-restore technique changes signal to noise unify signal spectrum.
- "Remove known signals and restore their effect subsequently"
  - **Remove** a global spherical harmonic geoid model (EGM2008)
  - Compute Gravity
  - **Restore** EGM2008 global gravity field (Pavlis)



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gnal?

The importance of removing long wavelength signal? (signals longer than around 100 km)

 $h_{RED3} \approx \Delta N + e''$ 

- 1) We only need to consider a local computition of V and hence gravity.
- 2) Therefore No global convolution involved any more......
- 3) We can approximate the sphere by a plane (works well within 300 km)
- 4) We dont need to use spherical harmonic coordinates. Cartesian will do.
- 5) Formulas for computation become linear and much simpler.
- 6) We can compute small regions in parallel.
- 7) Deep sources will not contribute. Shallow sources will dominate.
- REMEMBER WE ONLY USE ALTIMETRY FOR THE 10-100 KM SCALES

## TODAY THIS IS THE **ONLY** FEASABLE WAY TO COMPUTE GLOBAL MARINE GRAVITY FROM SATELLITE ALTIMETRY



### FIRST DATA EXCERCISE

• Use altimetry in the Northsea to determine the MSS and the Geoid.



### LEARNING FROM THE FIRST DATA EXCERCISE

• Use altimetry in the Northsea to determine the MSS and the Geoid.

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- Applications of Marine gravity
- Next generation development





## Potential and "Anomalous" Potential.

The anomalous potential *T* is the difference between the actual gravity potential *W* and the normal potential *U* from the ellipsoid (that we ample removed using EGM2008)

 $V(\varphi, \lambda, z) = W(\varphi, \lambda, r) - U(\varphi, \lambda, r)$ 

What is important is that V is a harmonic function outside the masses of the Earth.

There fore V is satisfying  

$$\frac{\partial^2 V}{\partial r^2} + \frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial \lambda^2} = 0$$
(\nabla 2 T = 0) Laplace (outside the masses) (\nabla r^2) + (\nabla 2 \nabla r)^2 + ((\nabla 2 \nabla r)^2 + (

 $(\nabla^2 T = -4\pi\gamma\rho)$  Poisson (inside the masses ( $\rho$  is density))

But let us work in Cartesian coordinates:





### **Geoid to Gravity**





Gravity and T is approximated through

$$\Delta g(x, y, z) = \Delta g(\bar{x}, z) = -\frac{\partial V}{\partial z} = \frac{1}{\gamma} \left( \frac{\partial N(x, y)}{\partial z} \right)$$

By deflection of the Deflection of the vertical or the GEOID SLOPE

$$\xi(\bar{x}) = -\frac{1}{\gamma} \frac{\partial V}{\partial y} = -\frac{\partial N}{\partial y}$$
$$\eta(\bar{x}) = -\frac{1}{\gamma} \frac{\partial V}{\partial x} = -\frac{\partial N}{\partial x}$$



### Laplace becomes:



This means that the VERTICAL derivative of the gravity field is related to The horizontal derivatives of the deflections of the vertical and hence the Geoid (slope)



### Use FFT to get Gravity from altimetry.

Fast Fourier Techniques.

Requires gridded data. SO THIS MUST BE DONE FIRST

- Very simple to use 2D version with Flat Earth approximation
- FFT is a fast version of the Discrete Fourier Transform requiring 2<sup>n</sup> observations
- Typical input is then the grid of the  $\Delta N(x,y)$  values

$$F(f) = F(k_x, k_y) = \iint f(x, y) e^{-i(k_x x + k_y y)} dx dy$$
  
$$F^{-1}(F) = f(x, y) = \iint F(k_x, k_y) e^{i(k_x x + k_y y)} dk_x dk_y$$

Where  $k_x$  and  $k_y$  are wavenumbers= 1/wavelength,  $k_x$  = 1/ $\lambda_x$ 

### The importance of FFT



• Using the FFT called F The geodetic relations between geoid N and g becomes.

$$F(\Delta g) = \frac{k}{\gamma} F(N), k = \sqrt{k_x^2 + k_y^2}$$

What is important is that the Fourier transform of N is multiplied by the Wavenumber. So higher wavenumbers or shorter wavelength will be amplified.

### **Upward continuation.**



One other interesting property of the FFT is the fact that the gravity field at some height (using the Laplace equation to compute this) is related to the gravity field at sea surface (z = 0)

$$F(\Delta g(k,z)) = F(\Delta g(k,0))e^{-2\pi k z}, k = \sqrt{k_x^2 + k_y^2}$$

So the gravity at height z will have large k or short wavelength suppressed Vice versa the gravity at depth –z will have short wavelength increased. (however if you enter the sources Laplace is no longer valid......)

Upward continuation (suppressing short wavelength) Downward continuation (enhancing short wavelength)



### From height to gravity using 2D FFT

$$F(\Delta g) = \frac{k}{\gamma} F(N), k = \sqrt{k_x^2 + k_y^2}$$

The conversion enhaced showr wavelength.

Optimal filter was designed to handle white noise + power spectral decay obtained using Frequency domain LSC with a Wiener Filter (Forsberg and Solheim, 1997)

$$F(\Delta G) = \frac{\Phi_{N\Delta g}}{\Phi_{NN} + \Phi_{ee}} F(N)$$

Power spectral decay follows Kaulas rule (k<sup>-4</sup>)

$$F(\Delta G) = \frac{k}{1 + ck^4} F(N) = k\beta(k)F(N)$$

Resolution is where wavenumber k yields  $\beta(k) = 0.5$ 

Getting the global field computed.



The gravity can be computed in parallel in the small planar cells covering the Earth.

Typically 2 by 5 degree cells are used this gives a total of say 6400 cells to compute.

In each cell several 100.000 reduced altimetric height observations are used.

These are subsequently merged to derive global gravity field A

# FINALLY THE gravity contribution of the EGM model must be

RESTORED.....



### Data and models

- Satellite Altimetry (Major points).
  - Altimetry Pathfinder: http://topex-www.jpl.nasa.gov/
  - RADS/NOAA (Remko): http://rads.tudelft.nl/rads/rads.shtml
  - NASA, ESA (Raw data).
- DNSC08/DTU10 suite of Global Fields
  - (<u>http://space.dtu.dk</u>. ftp.space.dtu.dk/pub/DTU10
  - Marine Gravity (1 min res).
  - Mean Sea Surface (1 min res)
  - Bathymetry (1 min res)
  - Mean Dynamic Topography (1 min res)
  - Interpolation Error file

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### Earthquakes





### **Bathymetry Prediction.**



#### **Bouguer constant – Upward Continuation**

Example: Using sea water and rock (1 and 2.6 g/cm3)

Bouguer constant = 75 mGal per km of topography.

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### Gravity and bathymetry are highly correlated



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Bathymetry prediction example. The Mid Atlantic Spreading Ridge.





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### **DTU 10 Bathymetry**





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Ex:Applications of Marine gravity





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**Seamount maping** 

The number of known seamounts in the Pacific have increased 6 fold from satellite altimetry

### **GEODESY -> GEOPHYSICS**







### History of Improvemet

321.400 obs	Mean	Std Dev.	Max Dev	Note
KMS99	0.60	5.69	73.74	
KMS02	0.44	5.15	49.38	
DNSC05	0.48	4.79	46.88	Retrack ERS-1, PGM04
DNSC08 DTU10	0.39	3.91 3.82	36.91	Double Retrack ERS1+ PGM+ Retracked GSAT
SS V12.1	0.62	5.79	82.20	
GSFC 00.1	0.68	6.14	89.91	
NTU01	0.79	6.10	92.10	
SS V18.1	0.39	3.98	38.29	Retracked ERS1+GSA

### **Next Generation**



Satellite altimetry have revolutionized marine gravity field mapping During last 15 years retracking have improved gravity significantly.

### New problems are emerging.

New sensors ICESAT Cryosat

However these requires retracking. But we can now go into getting gravity in the Arctic Ocean. Around Antarctica In larger lakes In coastal region.

We can also get heights of rivers etc.

This way we can also enhance topography maps of the world using altimetry.

### <sup>DTU Space</sup> Nati**The Arctic Ocean – "Problems"**



### ICESAT mapping the Arctic with Laser

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## Difference - Radar and Laser Altimetry

GLAS has much smaller footprint than radar altimeter instruments such as ERS and ENVISAT's RA-2 (3-10 km)

Small footprint enables GLAS to measure smallscale features on the ice sheet, previously unresolved in radar altimetry (65-70 meters)

Icesat will give unprecedented elevation information containing exquisite detail across ice sheet features such as: Ice shelf rifs/edges etc (examples).







### Cryosat



### Sea Ice and Gravity from ESA's mission CRYOSAT

### ESA's ice mission CryoSat-2

The question of whether global climate change is causing the polar ice caps to shrink is one of the most hotly debated environmental issues we currently face. CryoSat-2 aims to answer this question.

CryoSat-2's radar altimeter operate in SAR and Interferometric modes called SIRAL (SAR Interferometric Radar Altimeter). CryoSat-2 will reaching latitudes of 88° North and South.



### **Delay Doppler vs Altim**



#### Advantages: PRECISION IS ENHANCED BY A FACTOR OF TWO

Much less sensitive to sea state (random errors) Coastal regions / Narrow Footprint -> Closer to the coast

Retracking





## EAST GREENLAND

#### WITHOUT RETRACKING

#### WITH RETRACKING



### Land Hydrology – The Amazon



Satellite altimetry In rivers Retracking is Essential<sub>5's</sub> (P. Berry–De Montford)

SeaIce-Like

Ocean





### Afternoon excercises.

- Look at the global MDT.
- Derive gravity using 2D FFT of the grid.
- Trying to upward continuate the gravity field to GOCE altitude.