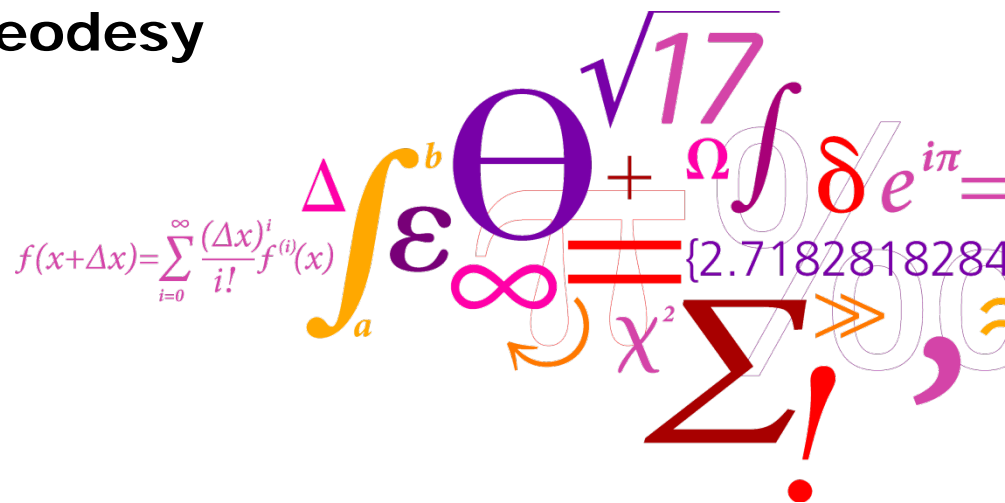


Spaceborne techniques for Gravity

Dr Ole B. Andersen,
DTU Space, Geodesy



A collage of colorful mathematical symbols and formulas. On the left, the Taylor series expansion is shown: $f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x)$. To the right, there is a large purple Greek letter Θ , a yellow integral $\int_a^b \varepsilon$, a red integral $\int \delta e^{i\pi}$, a purple square root $\sqrt{17}$, a red plus sign, a purple infinity symbol ∞ , a red equals sign, a purple curly brace containing the number $\{2.7182818284\}$, a purple Greek letter χ^2 , a red summation symbol Σ , a yellow greater-than sign $>$, a purple comma, and a red exclamation point.

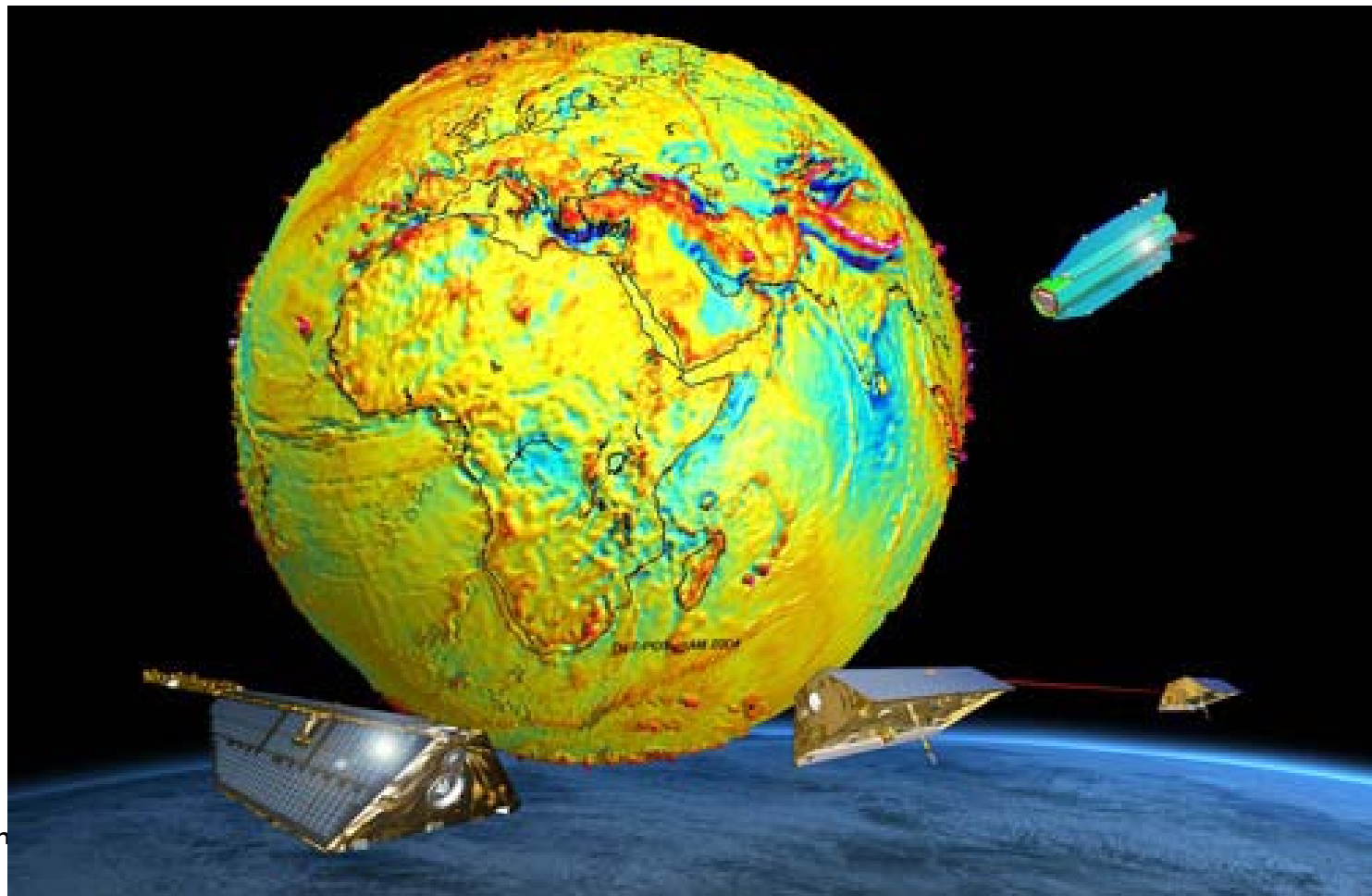
Overview

- Motivation
- The very basic.....

Gravity using satellite altimetry.

Gravity from GRACE (intersatellite ranging)

Gravity from GOCE (satellite gradiometry)

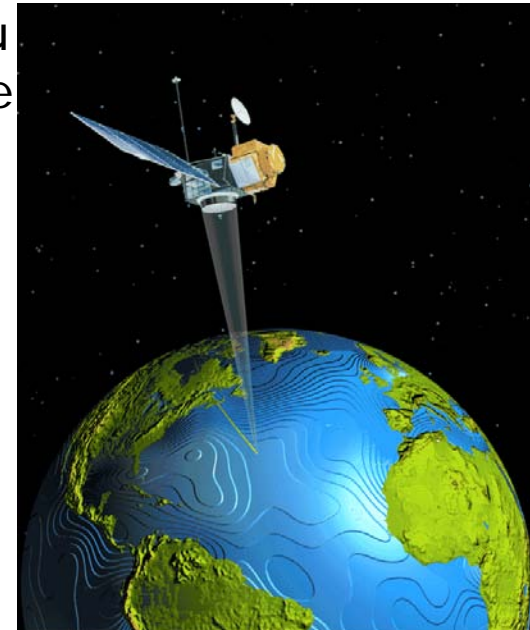


Motivation

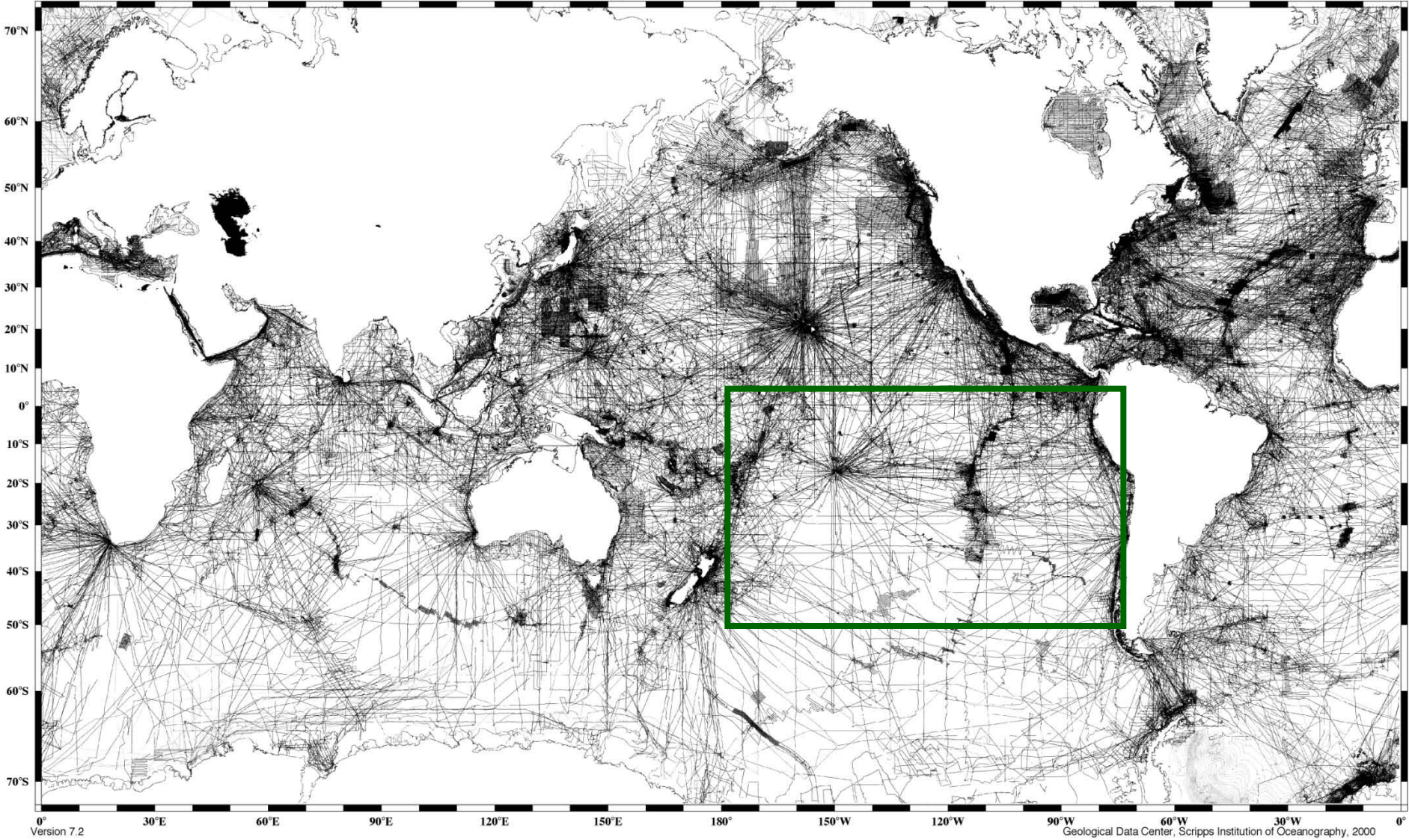
- Why:
 - Earth Resources
 - Earthquakes/tsunamies
 - Cheaper Sattellites
 - Better bathymetry
 - Climate research
 - Navigation /Safety

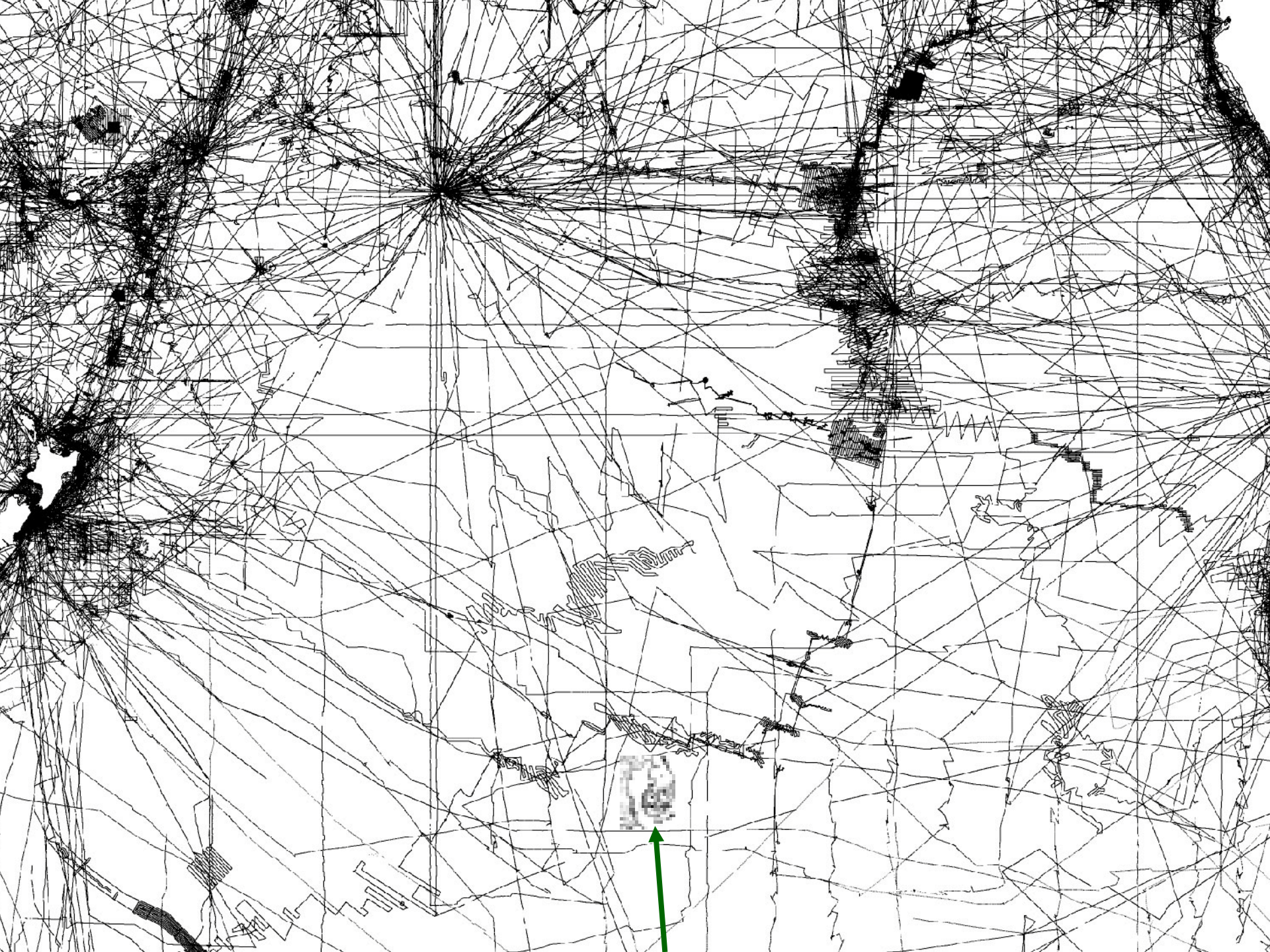


- In order to determine the gravity field of the Earth you need observations with uniform density throughout the world.
- The density of your observations determines which wavelength (and to which resolution) you can determine in the earth gravity field.
- Two thirds of the globe is covered with water, and many regions are NOT covered with marine gravity obs.



Ship Tracks





Space Measurement Techniques

- Satellite altimetry can provide information over virtually the entire Earth.

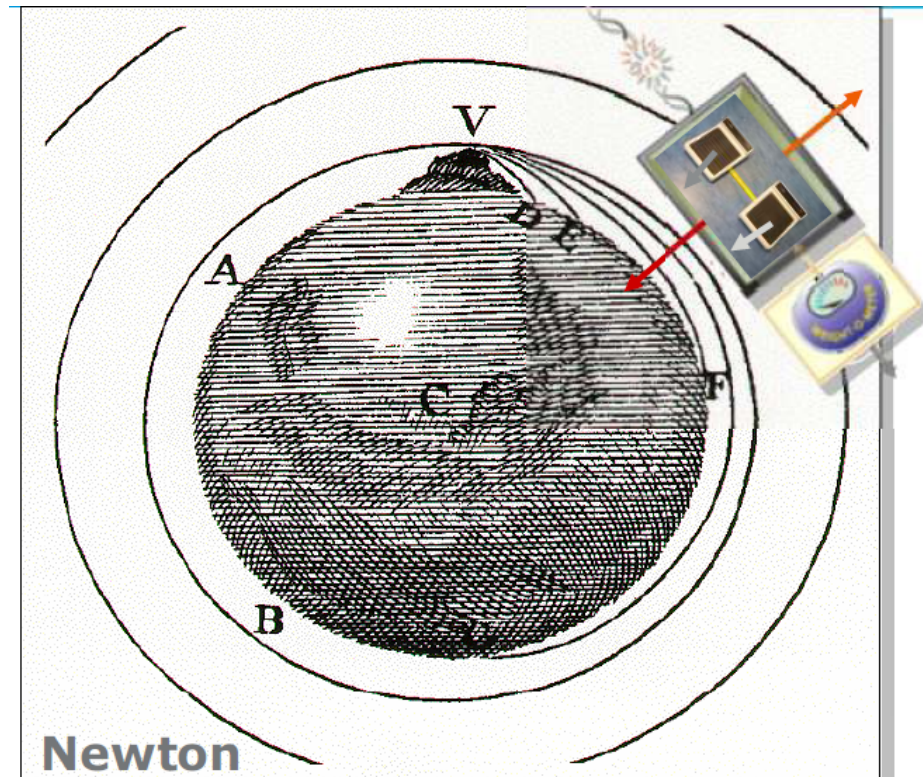
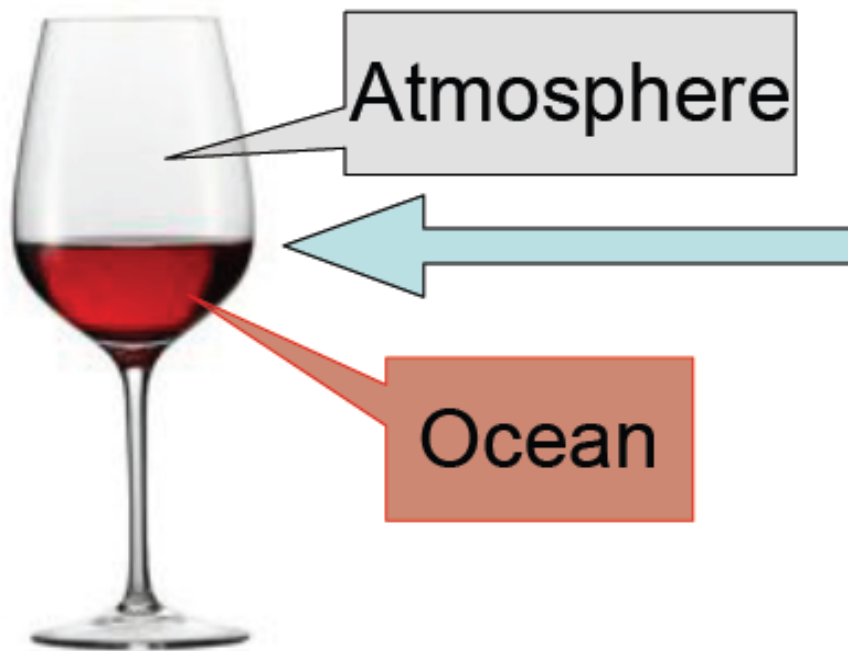
- Newton. $\vec{g} = -\frac{GM}{r^2} \vec{r}$ Notice r^2 dependence / un-ambiguity

$$\Delta g = -\frac{\partial V}{\partial r} - 2\frac{V}{r} \approx -\frac{1}{\gamma} \left(\frac{\partial N}{\partial r} + 2\frac{N}{r} \right)$$

- Two principles. Observe sea level or
- Satellite Altimetry

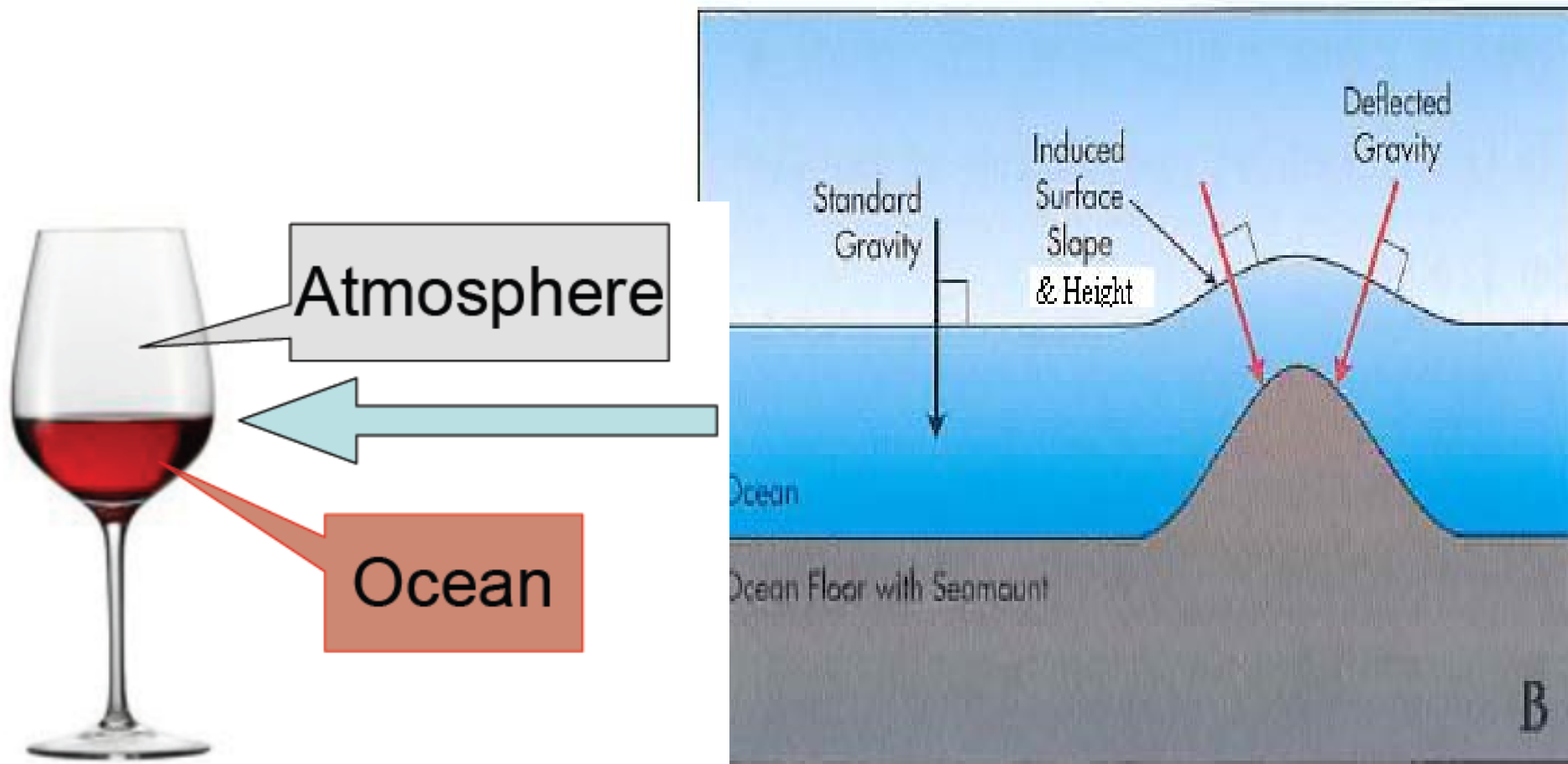
Put scales on satellites

Satellite gravimetry (GRACE/GOCE)



Satellite Altimetry

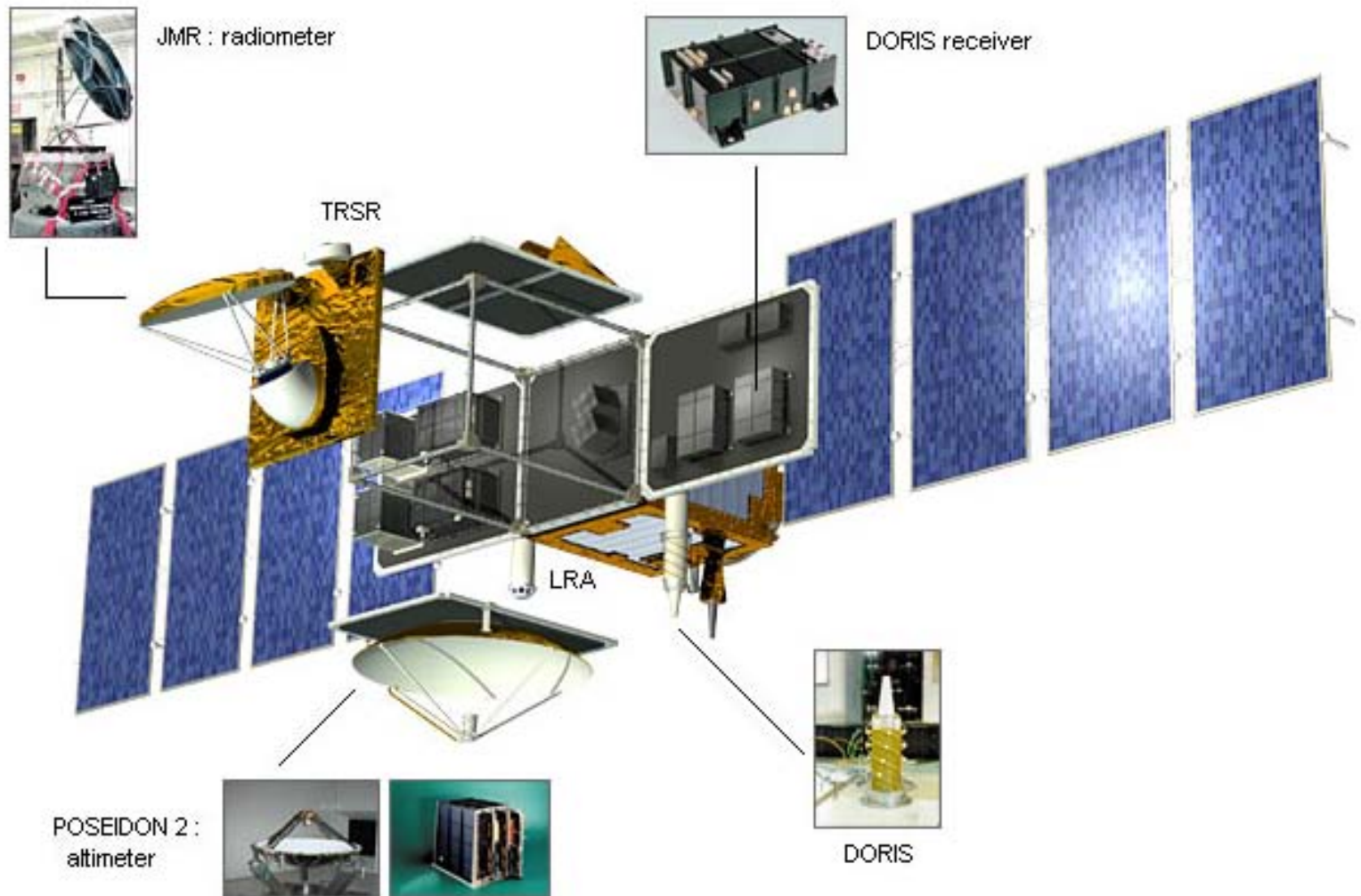
Measuring the sea surface height -> gravity.



Satellite Altimetry

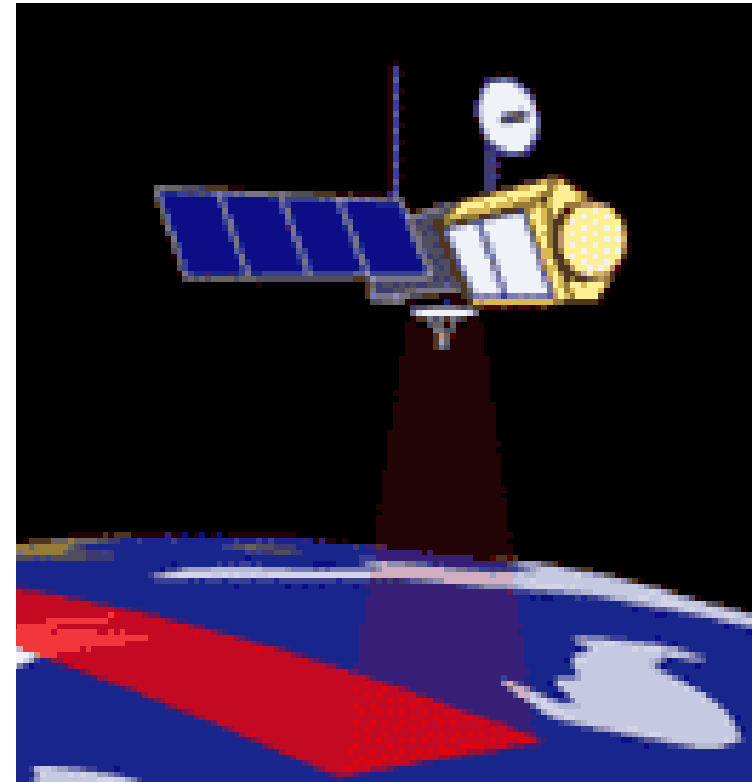


Altimetric Satellite (Jason, Envisat, ERS.....)



Measurement specification

Emitted Frequency (GHz)	Dual-frequency (K _u , C) - 13.575 and 5.3
Pulse Repetition Frequency (Hz)	2060 interlaced {3K _u -1C-3Ku}
Pulse duration (microseconds)	105
Bandwidth (MHz)	320 (K _u and C)
Antenna diameter (m)	1.2
Antenna beamwidth (degrees)	1.28 (K _u), 3.4 (C)
Power (W)	7
Redundancy	Yes
Specific features	Solid-State Power Amplifier. Dual-frequency for ionospheric correction, High resolution in C band (320 MHz)



Altimetric Observations

Accurate ranging to the sea surface is

Based on accurate time-determination $d = t * c / 2$

Where c must be adjusted slightly adjusted

For passage of ionosphere and troposphere.

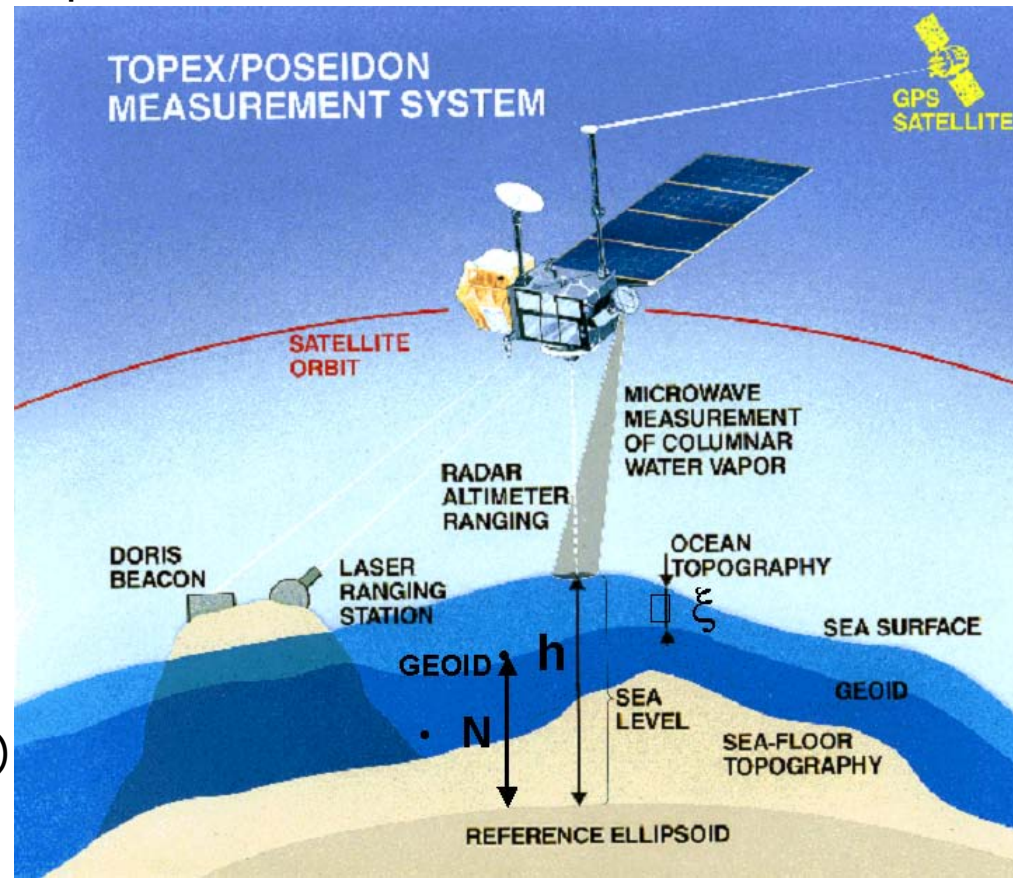
$$SSH = \text{Height}_{\text{sat}} - \text{Range}$$

$\text{Height}_{\text{sat}}$ Is determined using

GPS or DORIS/Laser ranging

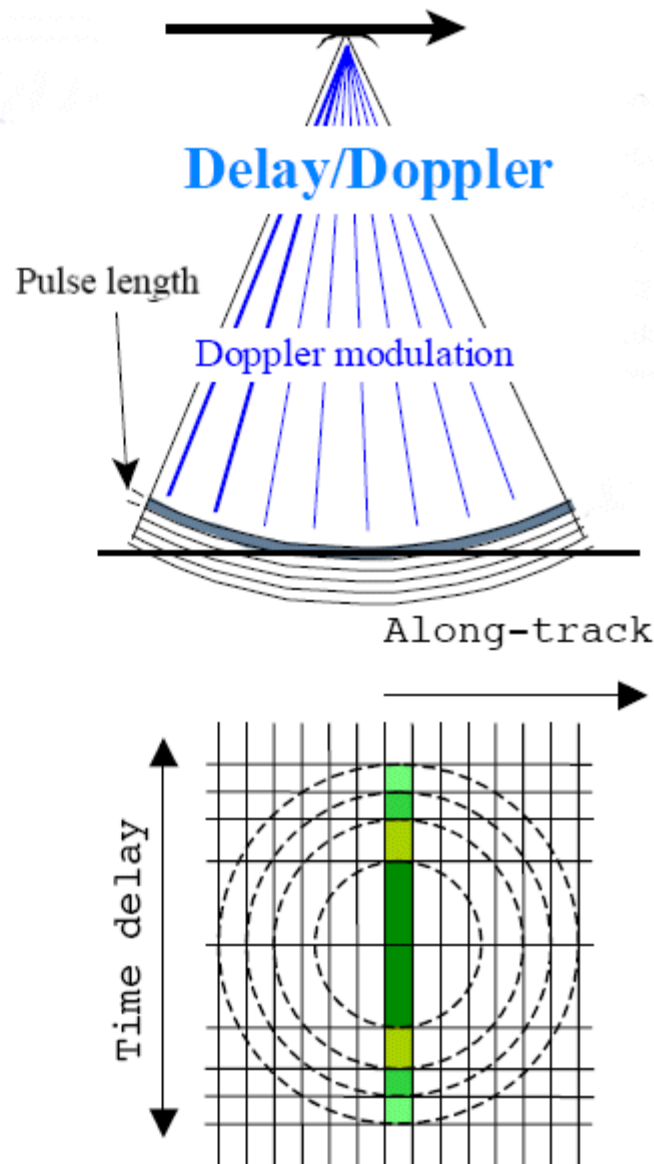
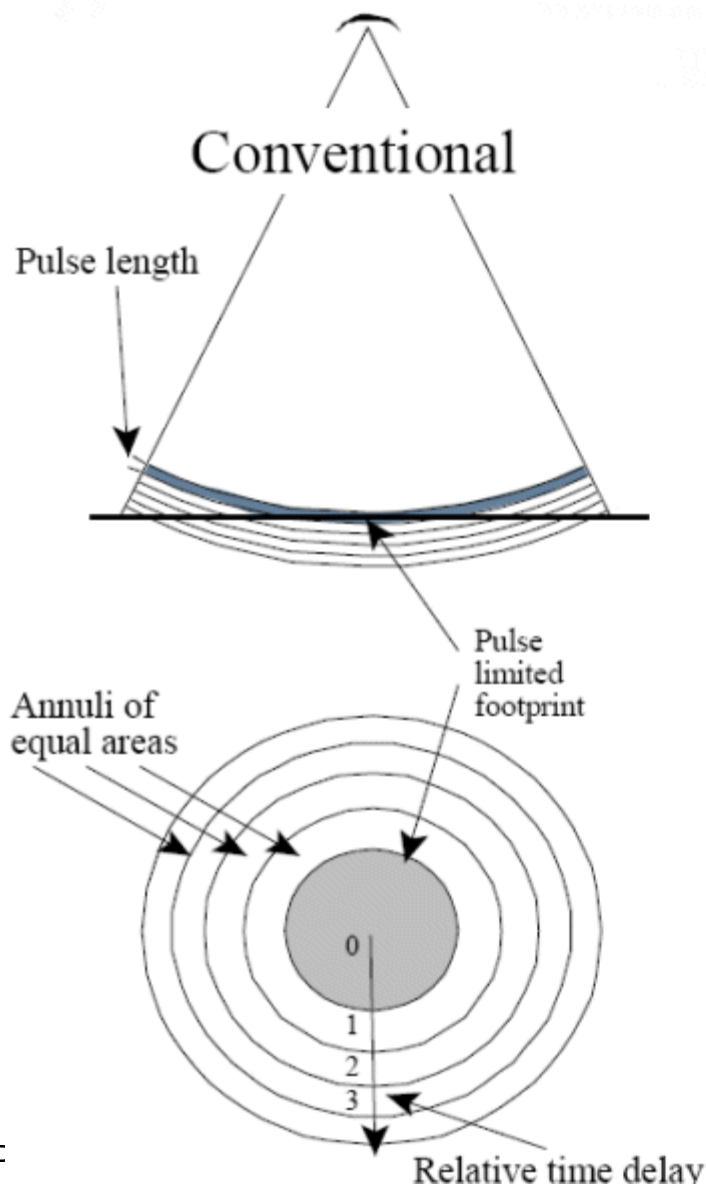
Relative to the reference ellipsoid

Ellipsoid is "best" mathematical model of the Earth Shape (WGS84)

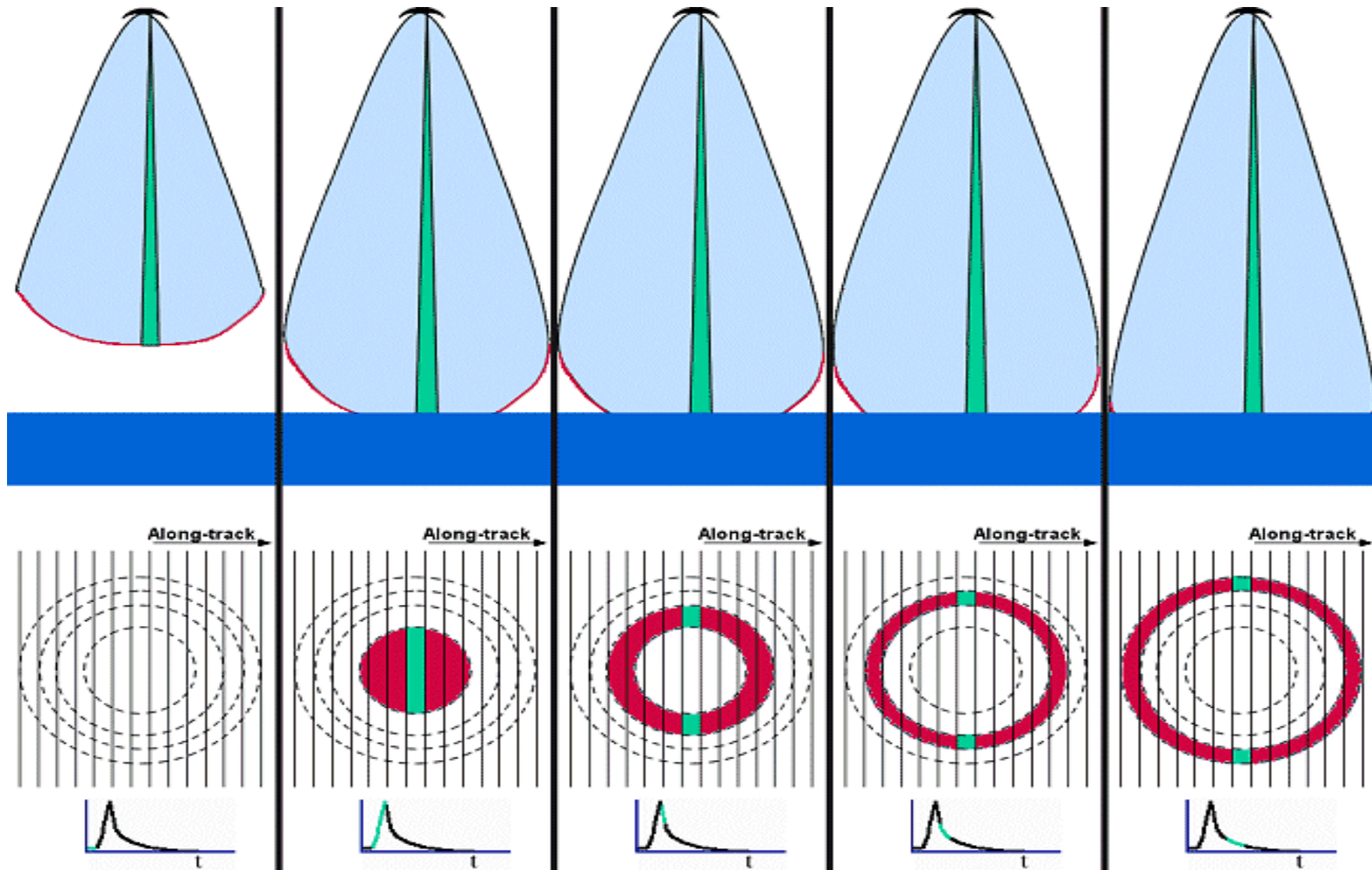


Jason/ENVISAT

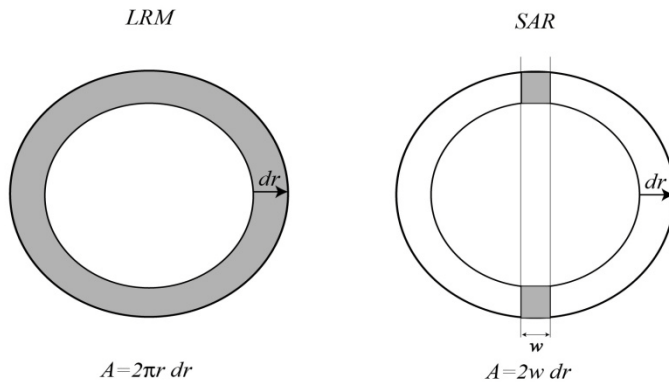
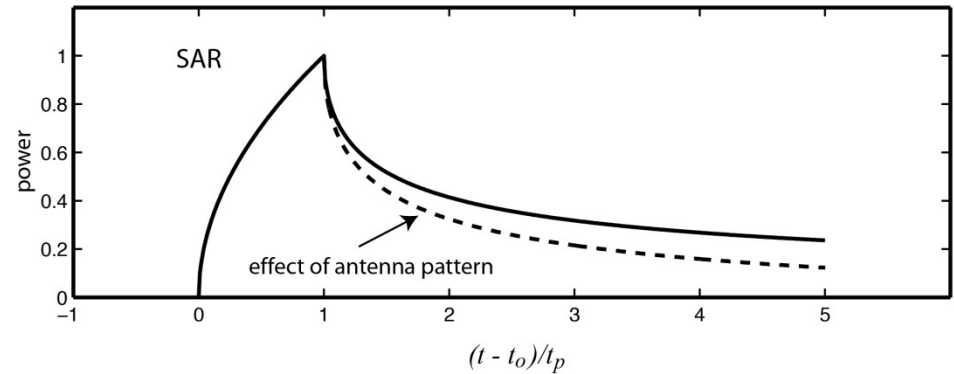
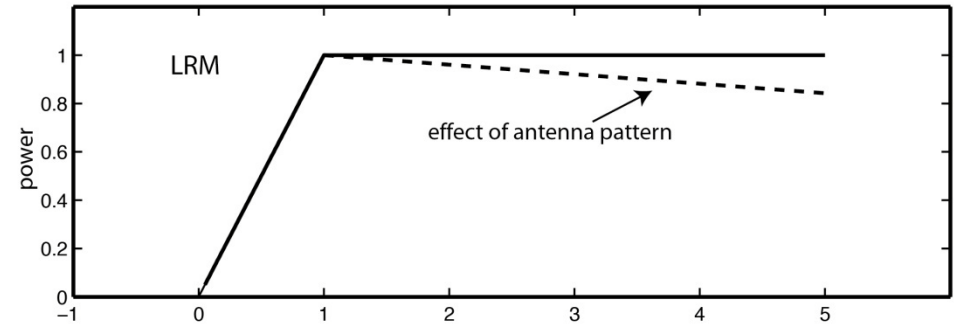
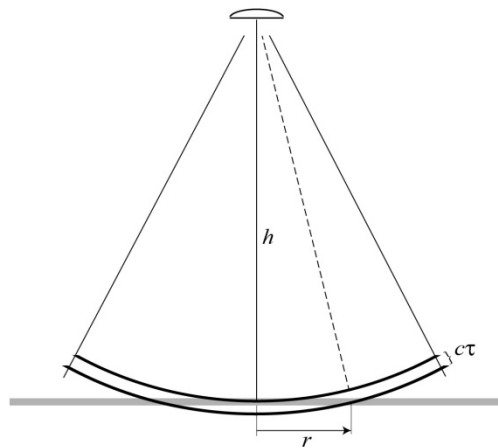
Cryosat-2, Sentinel-3



Cryosat-2, Sentinel-3. Doppler processing



flat surface response – LRM vs. SAR

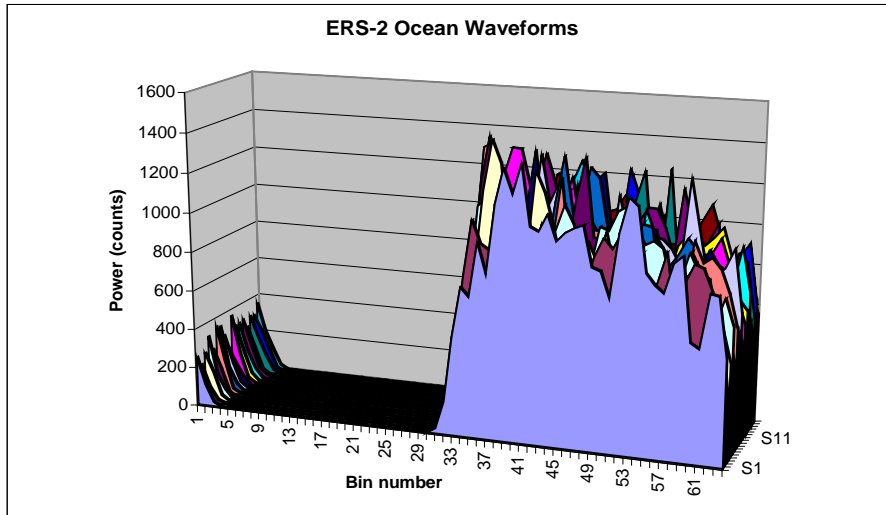


$$M(A, t_o, \sigma) = \frac{A}{2} \{1 + \text{erf}(\eta)\};$$

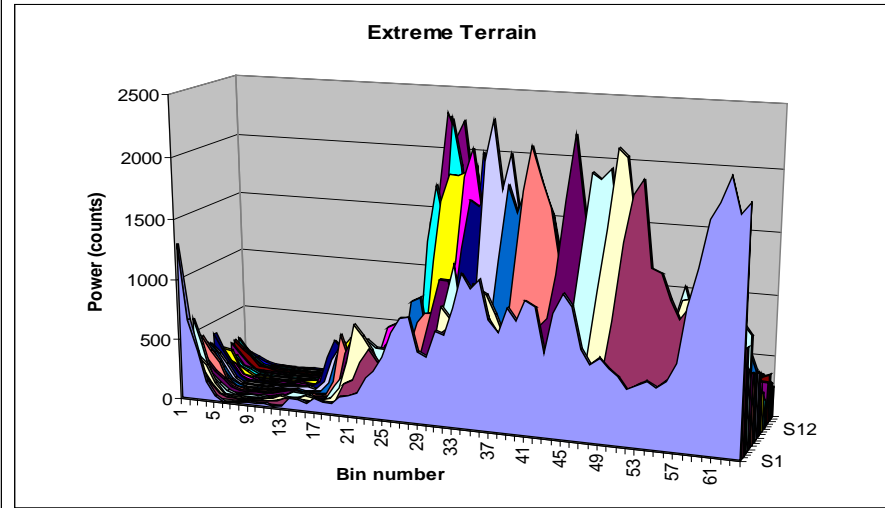
$$\eta = \frac{t_o}{\sqrt{2}\sigma}$$

Typical Echoes

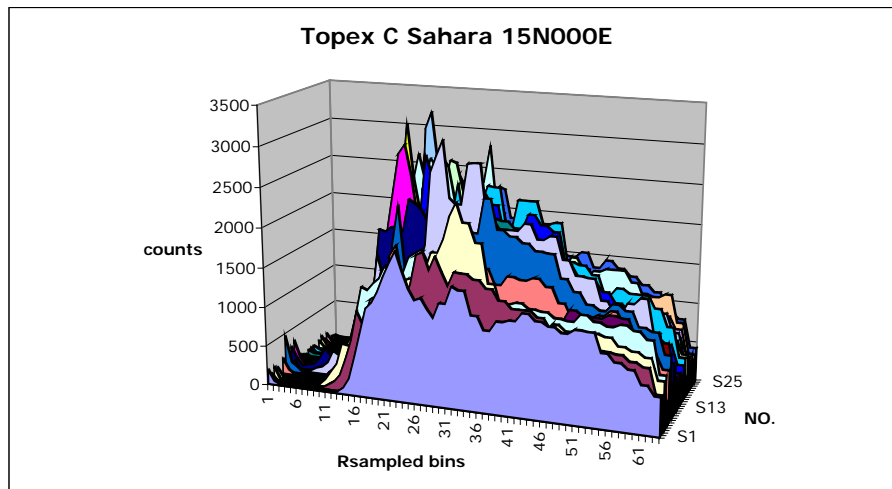
Ocean Echoes



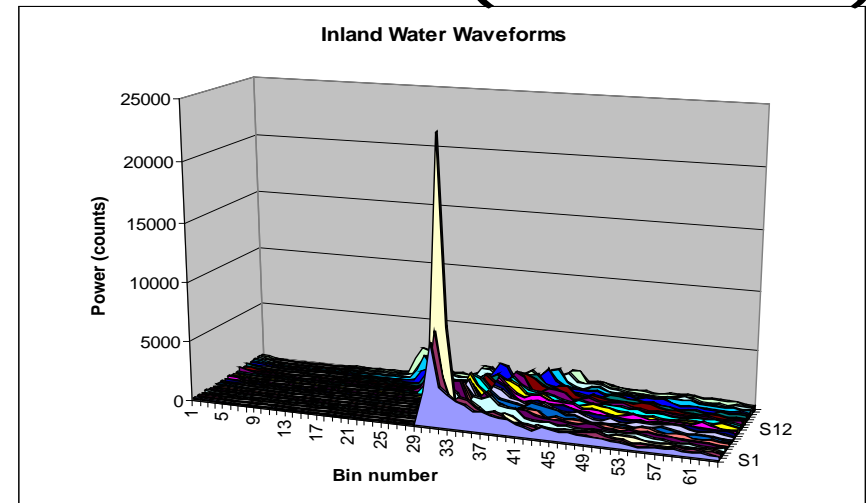
Extreme Terrain



Desert – Australia

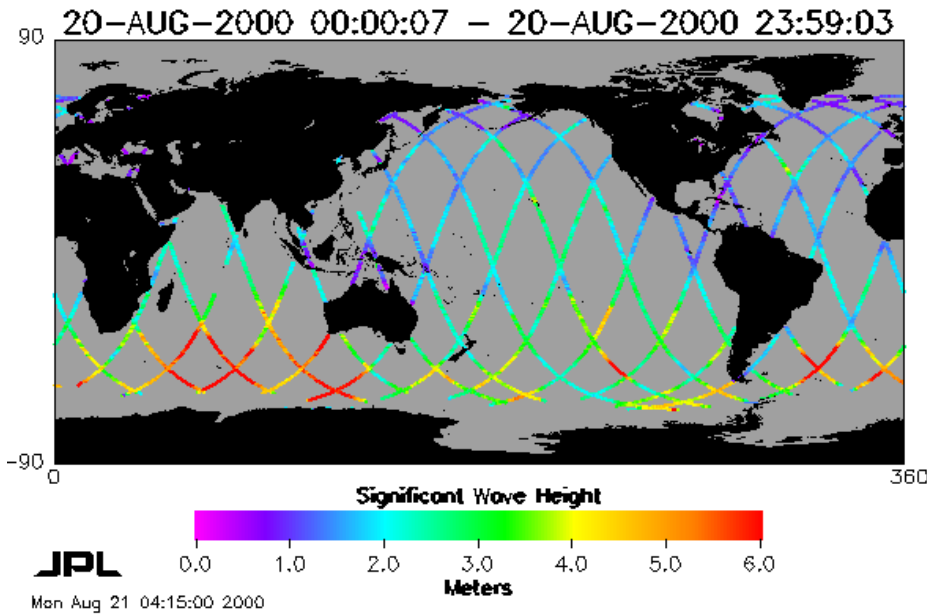


Inland Water (River – lake)



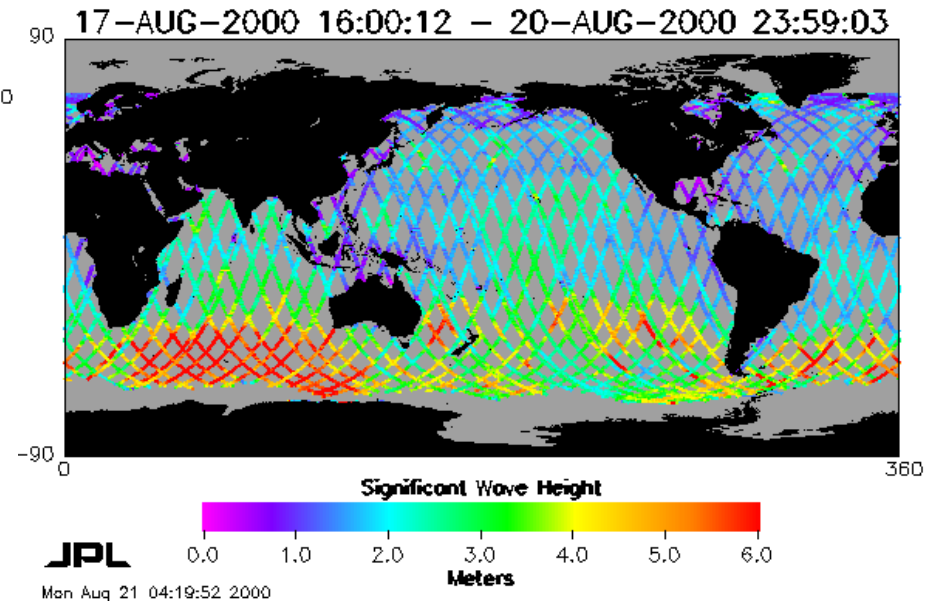
When launching a satellite.

You Choose an orbit



1 Day

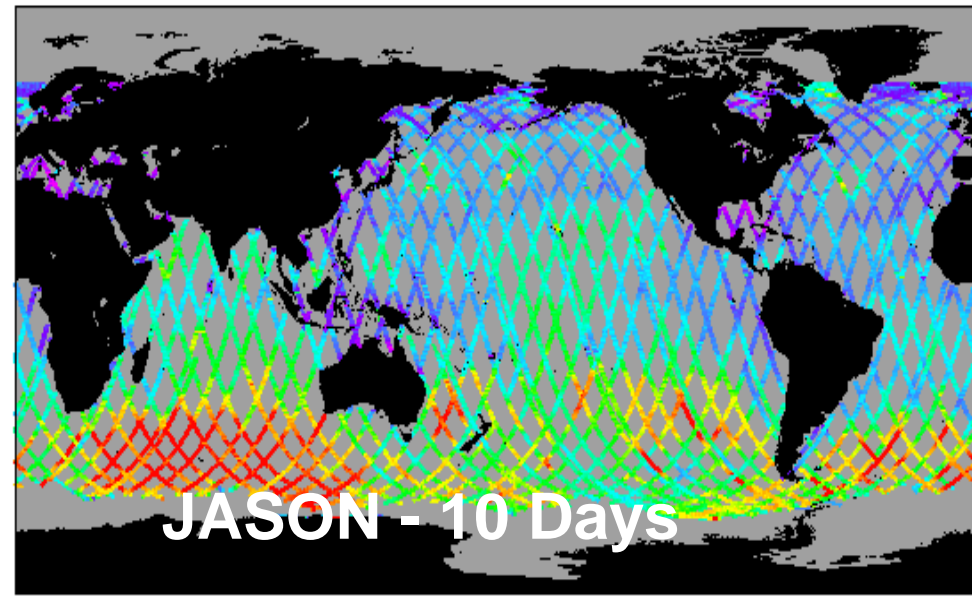
3 Days



Crossing of tracks essential for accuracy

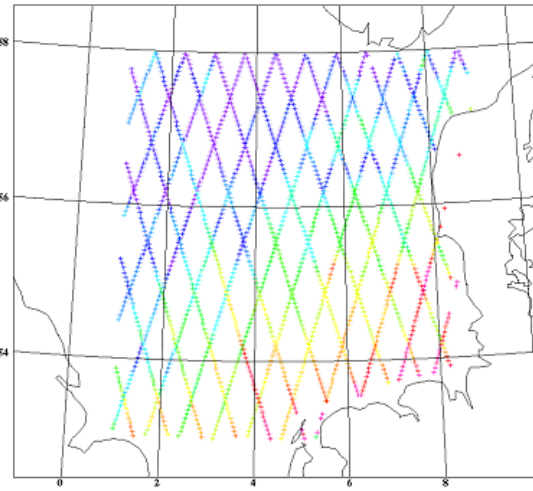
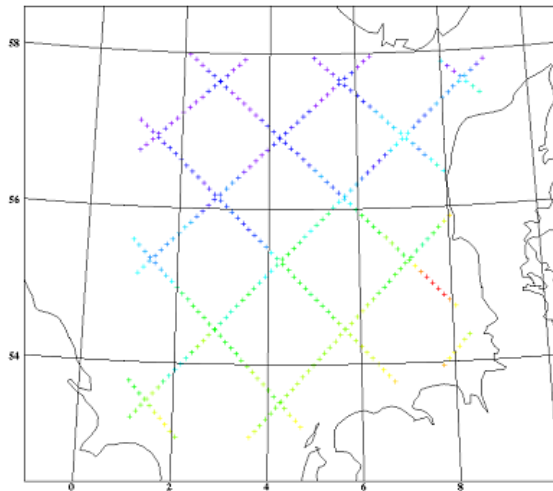
Orbit Parameters

The coverage of the sea surface depends on the orbit parameters (inclination of the orbit plane and repeat period).

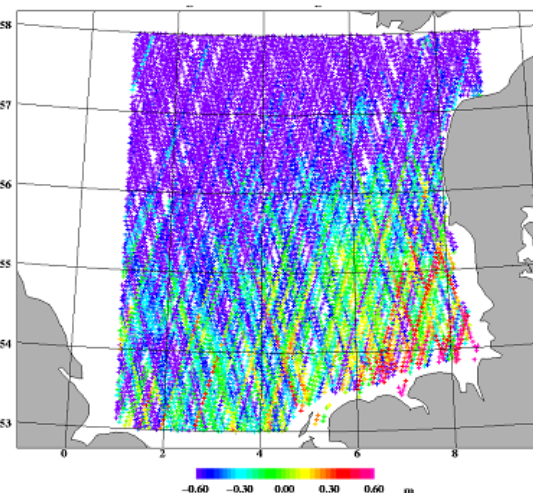
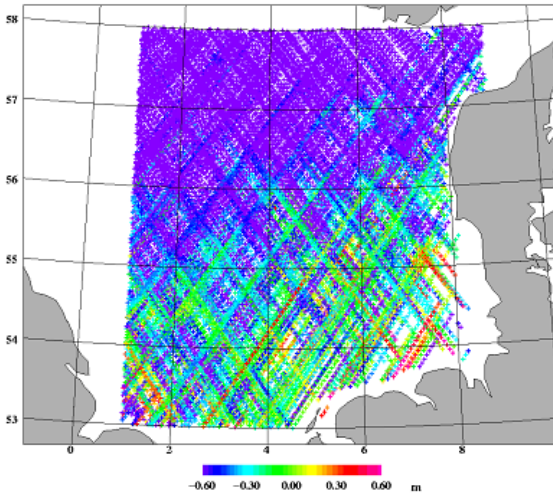


	Satellite	Repeat Period	Track spacing	Inclination Coverage
Repeating (ERM)	ENVISAT/Sentinel-3	30 days	95 km	98° (+/- 82°)
	JASON 1-2-3	10 days	315 km	66.5°
Geodetic	Cryosat-2	369 days	7 km	88°

ERM – GM data.



ERM Data
 TOPEX/JASON –
 (280 km)
 ERS/ENVISAT
 (80 km)



Geodetic Missions
 ERS/GEOSAT or Cryosat
 (5 km track spacing)

GEOSAT+ERS GM data is ESSENTIAL for high resolution Gravity Field mapping.

Getting Gravity from Sea surface height (SSH)

Gravity does not change with time.....

So SSH must agree when tracks crosses.....
Crossover adjustment

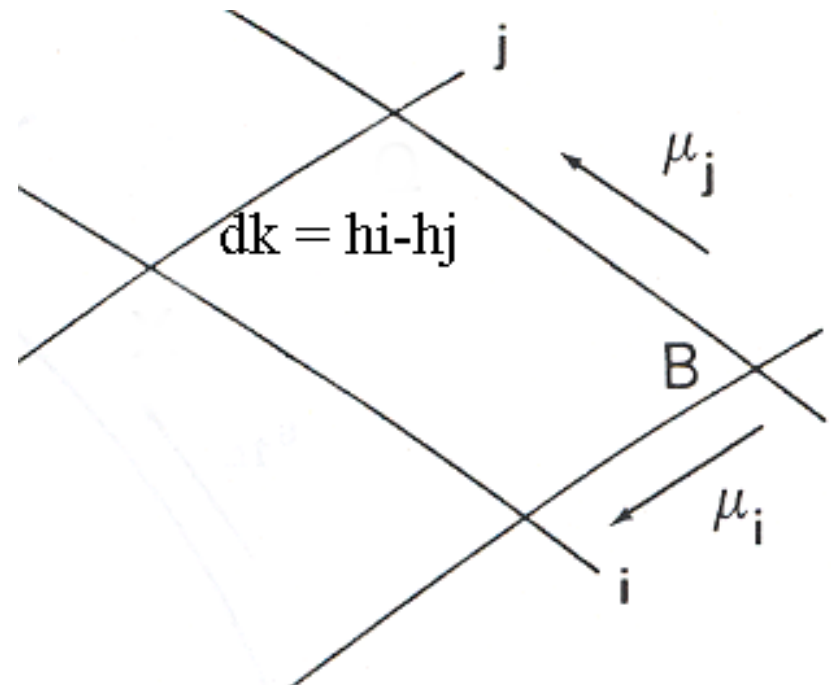
- $\underline{d}_k = h_i - h_j$
- $\underline{d} = A\underline{x} + \underline{v}$
- where \underline{x} is vector containing the unknown parameters for the track-related errors.

\underline{v} is residuals that we wish to minimize

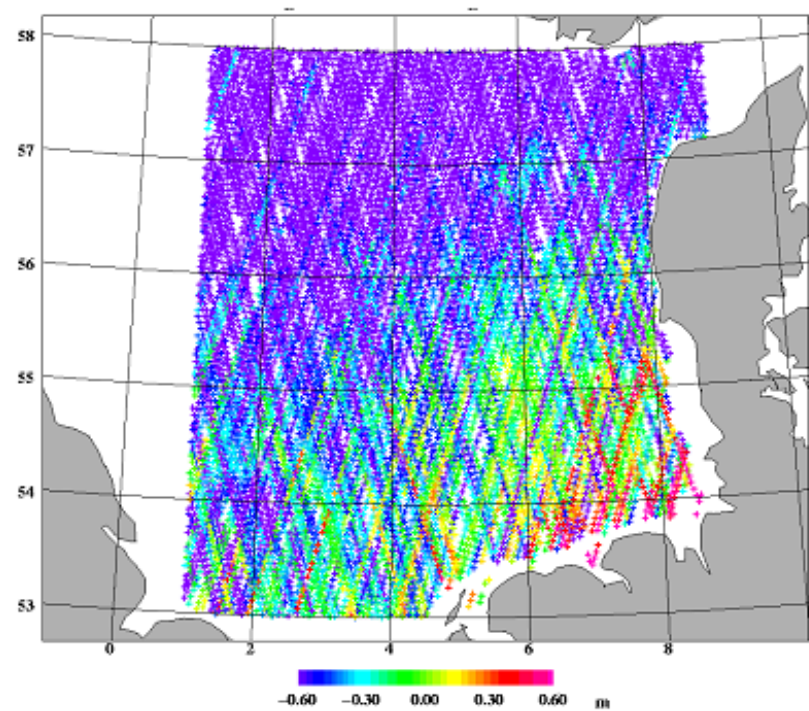
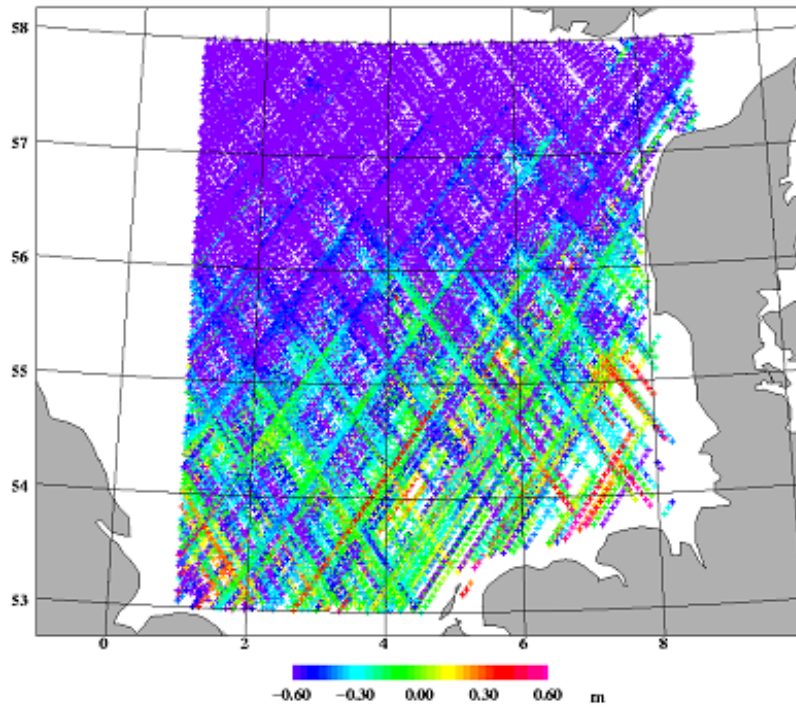
- Least Squares Solution to this is

$$\underline{x} = (A^T C_d^{-1} A + c c^T)^{-1} A^T C_c^{-1} \underline{d}$$

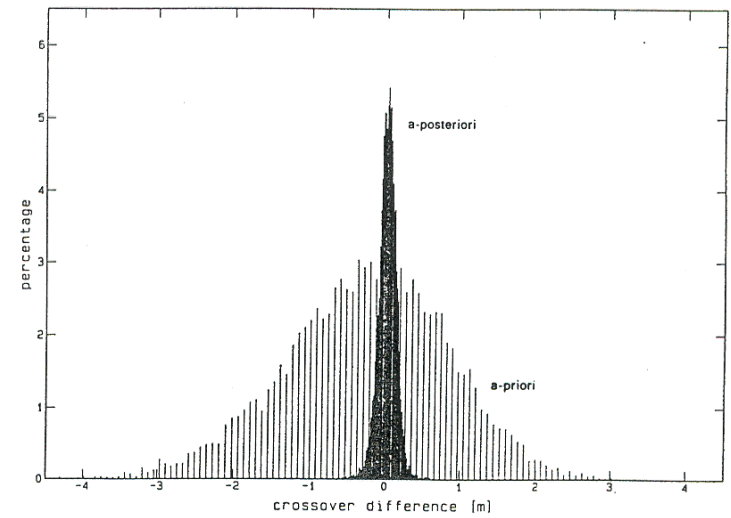
- Constraint is needed $c^T \underline{x} = 0$
- Case of bias – mean bias is zero



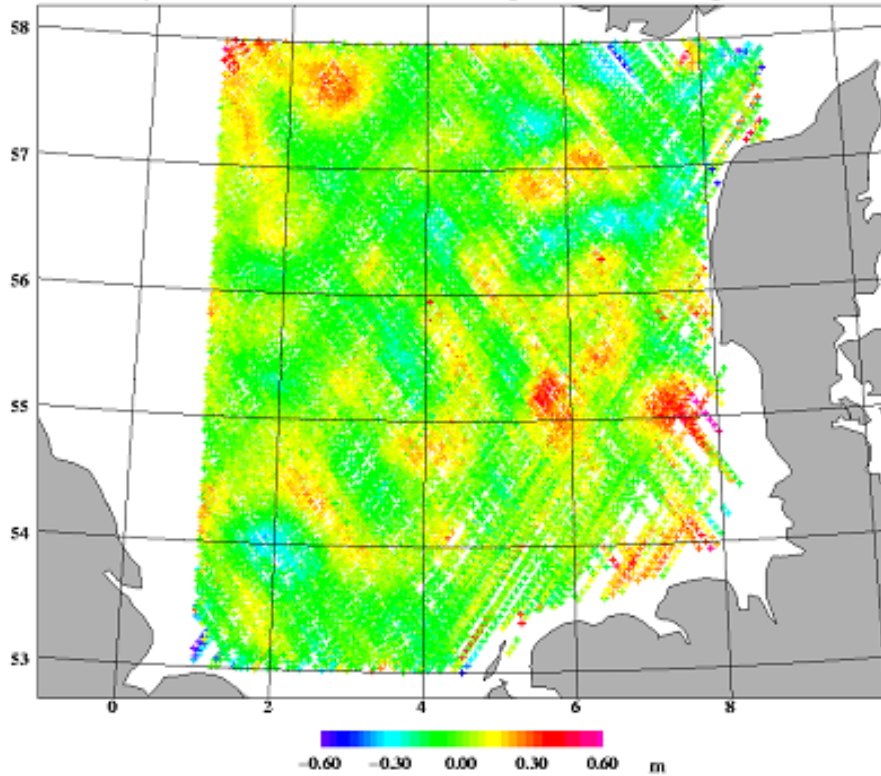
Before



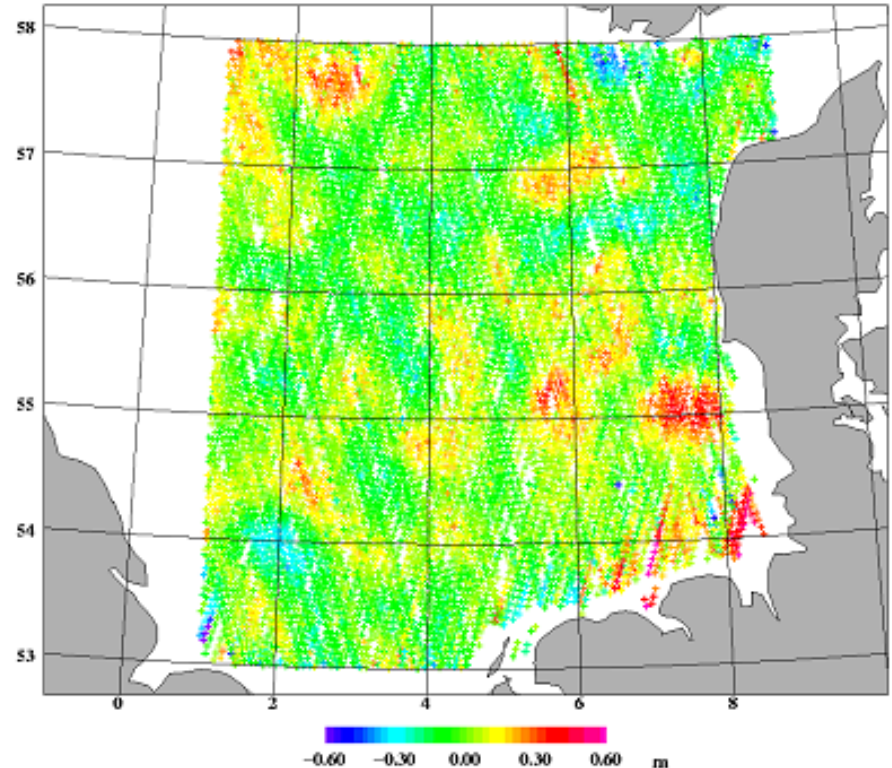
After Crossover



X-over adjusted GEOSAT-GM altimetric heights minus EGM96 geoid

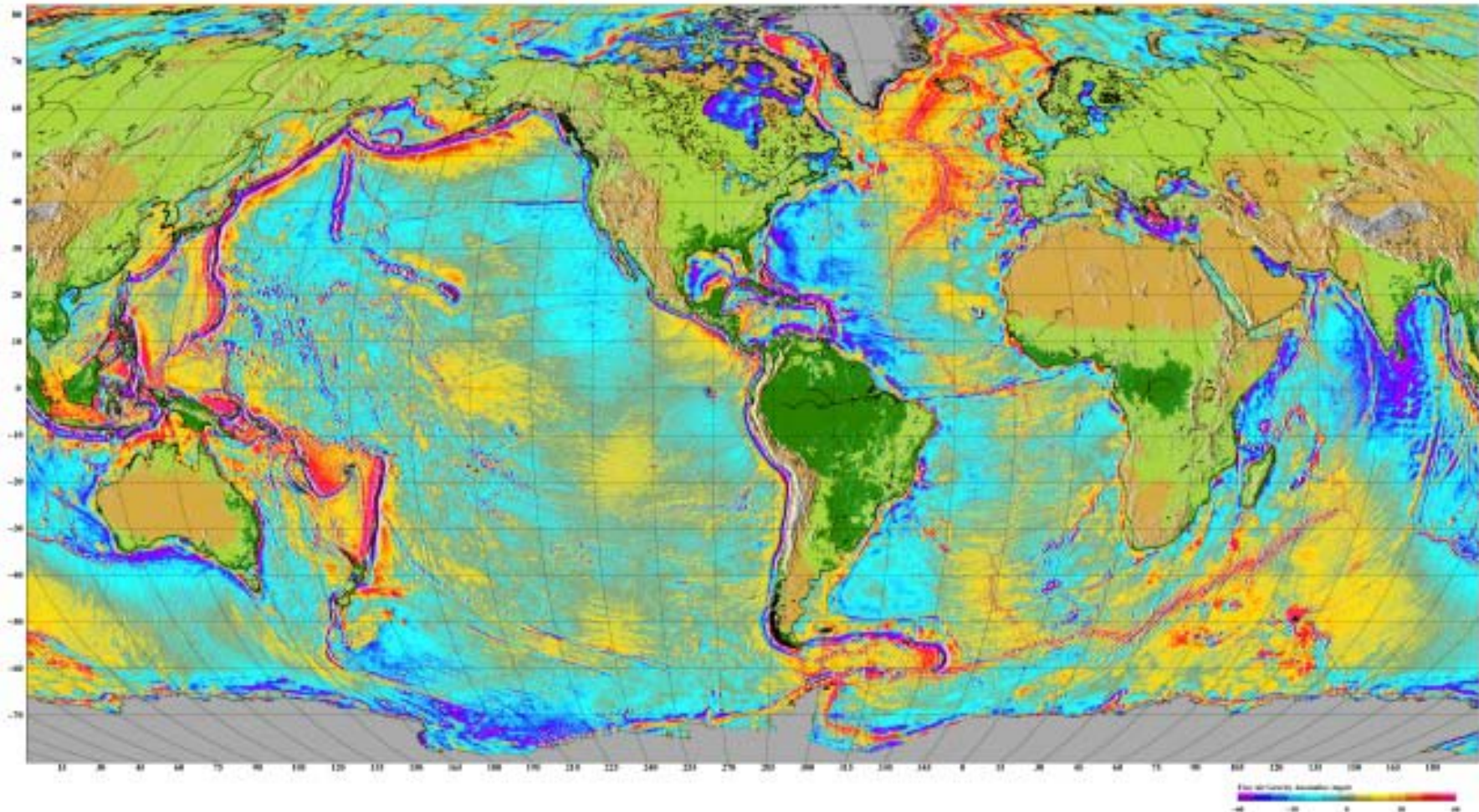


X-over adjusted ERS-1-GM altimetric heights minus EGM96 geoid

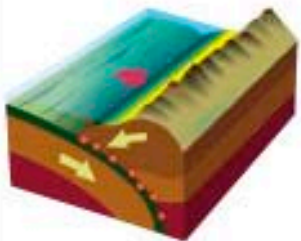


DTU10 1 min global marine gravity grid.

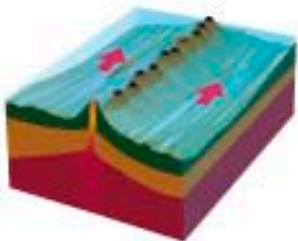
Free Air Gravity Anomalies from Satellite Altimetry



Subduktionszoner



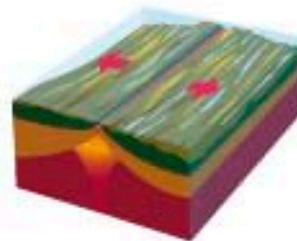
Hot spot



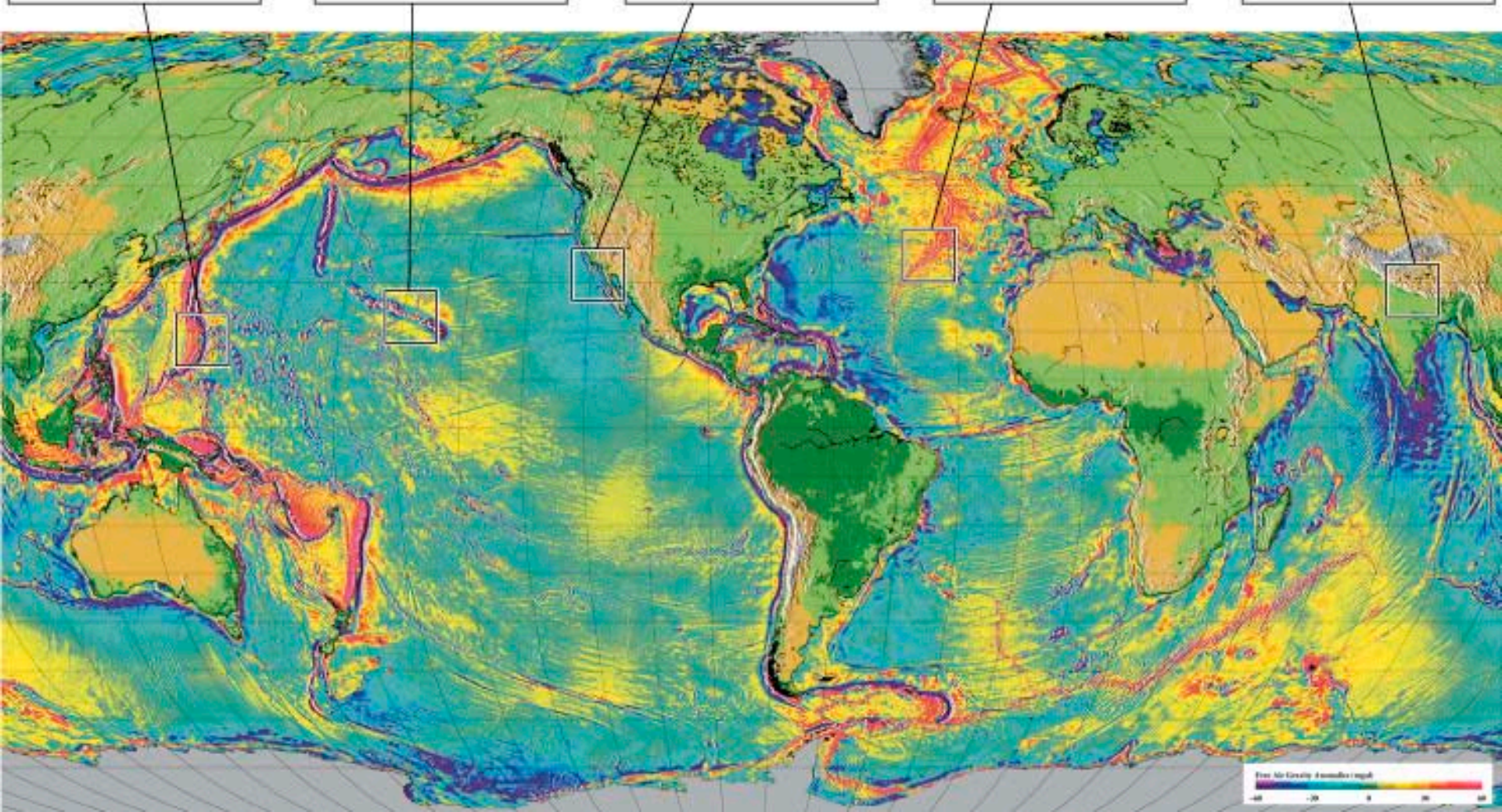
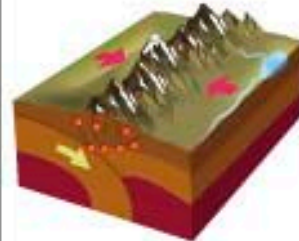
Bevarede



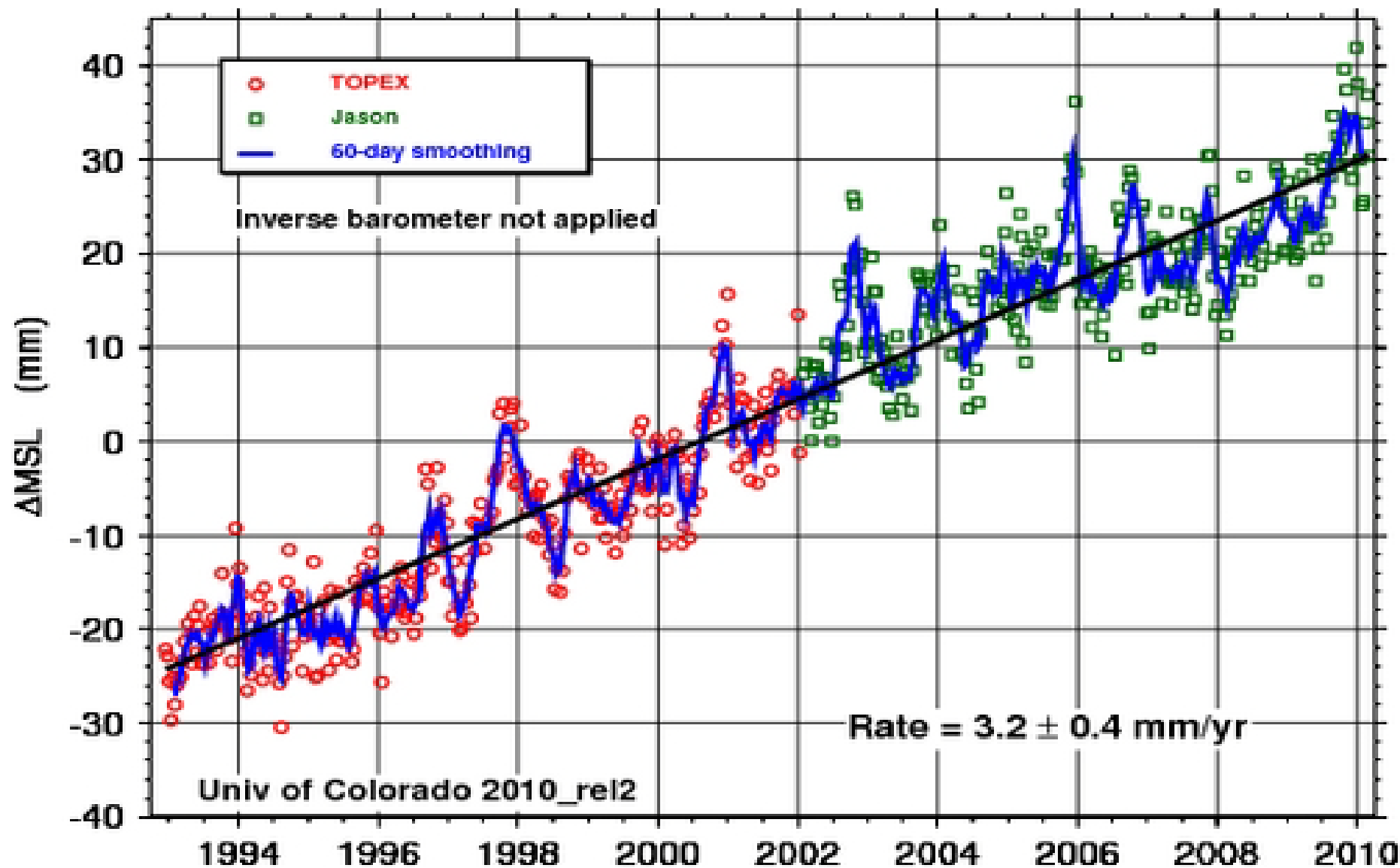
Spredningszoner



Bjergkædedannelse



But you can use altimetry for many purposes



Satellite Altimetry.

Summary - Limitations / Possibilities

- Altimetry observes the surface of the ocean (geodesy and oceanography)
- Satellites altimetry observes repeatedly under all weather conditions
- Satellite altimetry measures the "gravity related quantity at the sea surface"
- This is very close to the sources – so you get very high spatial resolution....
- Altimetry maps wavelength between 20 and 100 km of the gravity field.
- -> How do we get the longer wavelength parts of the gravity field....
- Satellite altimetry observes along predefined ground tracks at predefined intervals. Orbital issues are important to consider. No observations "at will".
- There will be polar Gap around the North Pole with no coverage

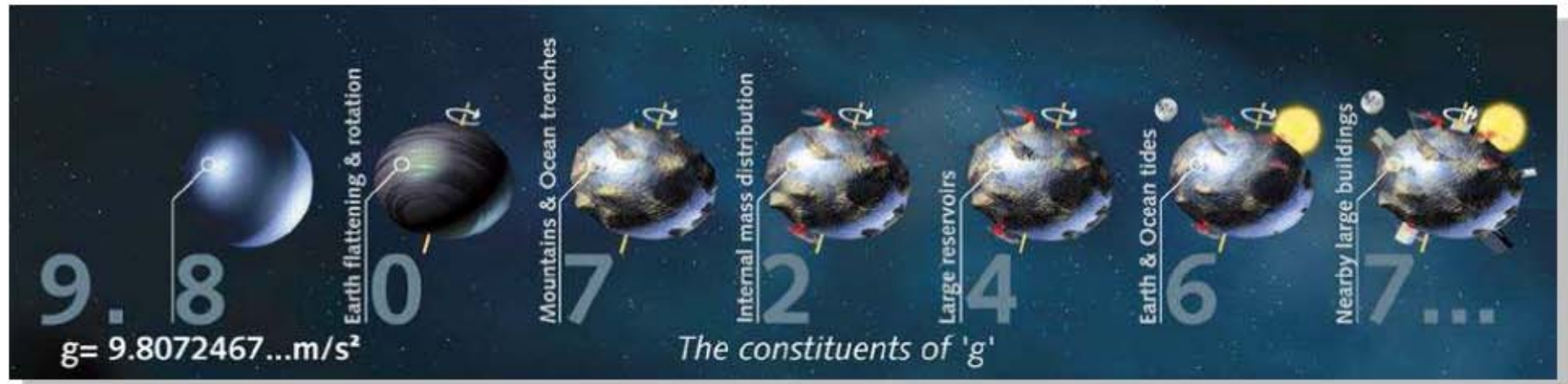
Lets go for the longer wavelength >200 km.

- Now its adequate to measure in space, but height of satellite
- Determine which wavelength can be measured.

- Rule of thumb. Resolution equals height

Satellite Gravimetry

GRACE AND GOCE



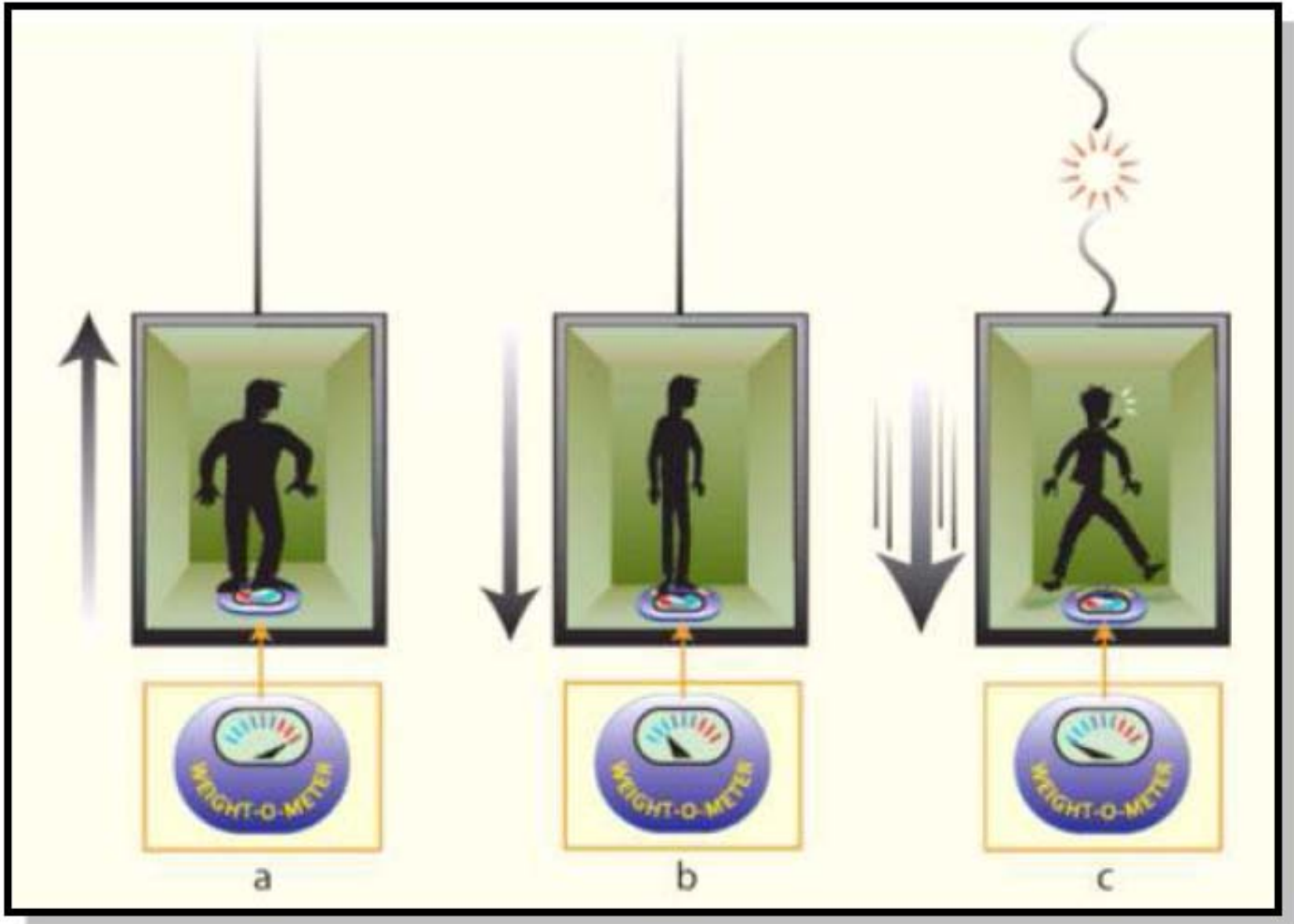
ales at fixed dista

9.81135829**5133** m/s² gradiometry

difference

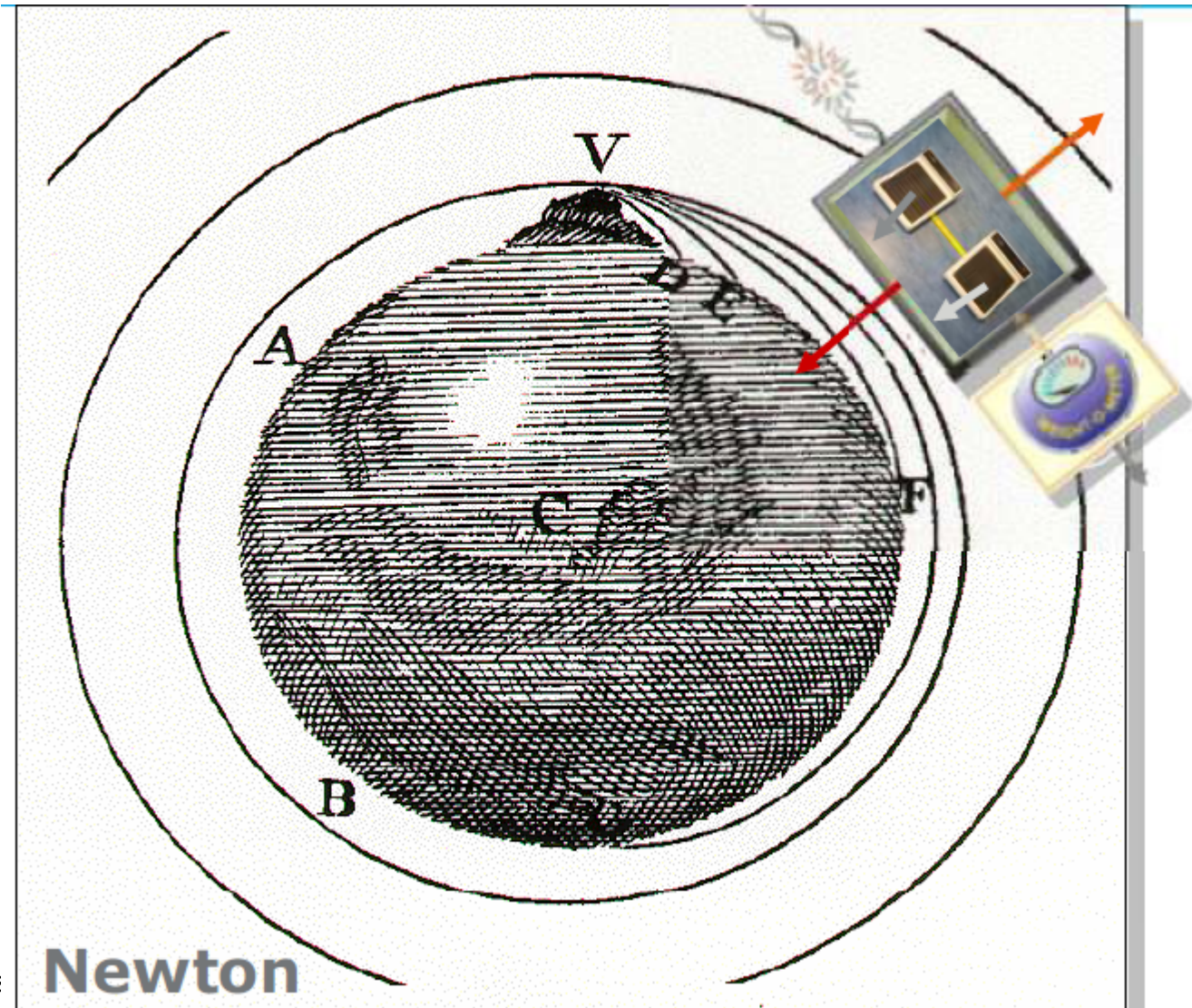
9.81135829**4298** m/s²

Scales in motion.....

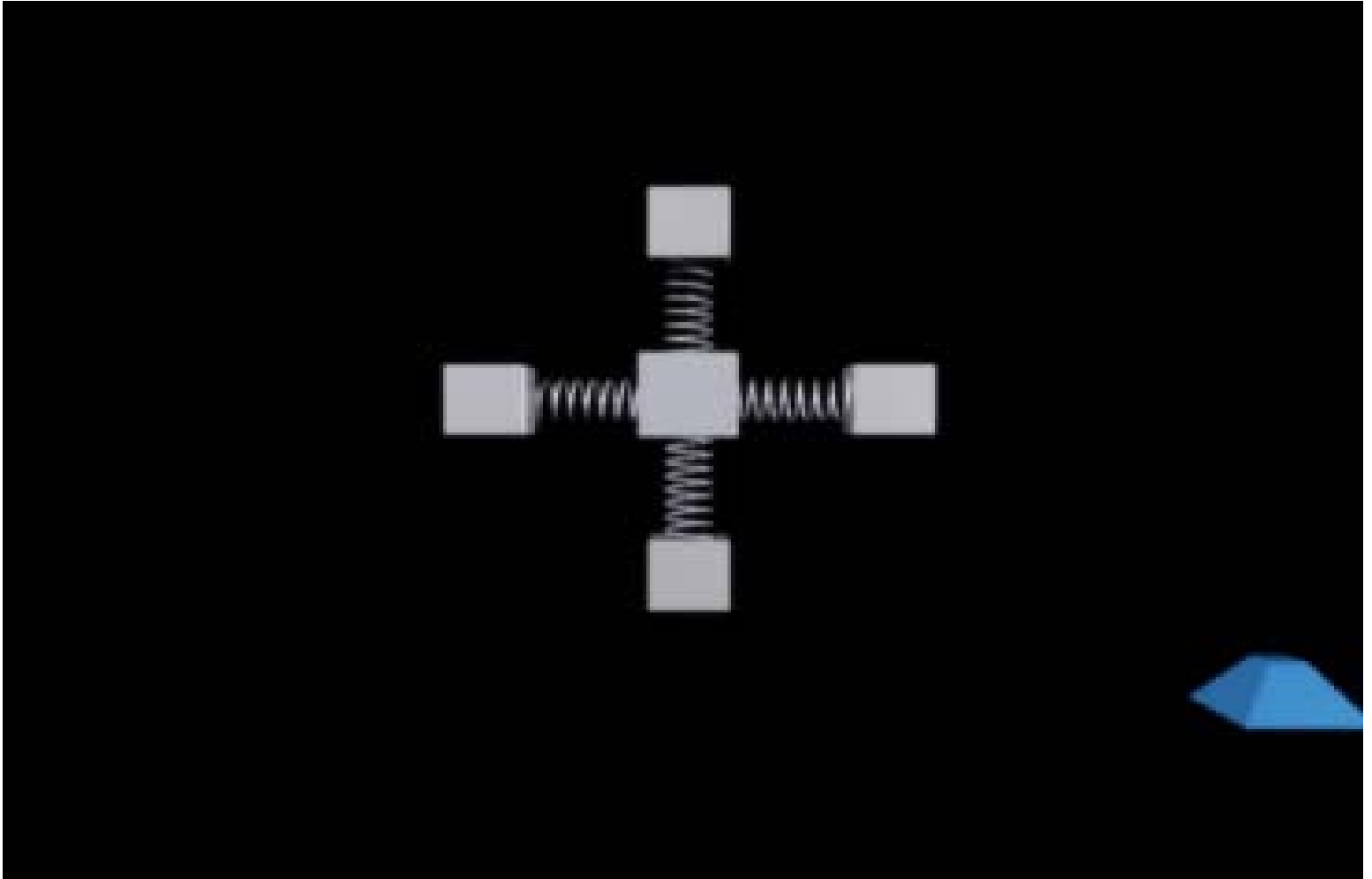


Gravity and satellite orbits

GOCE is launched in 250 km orbit.....



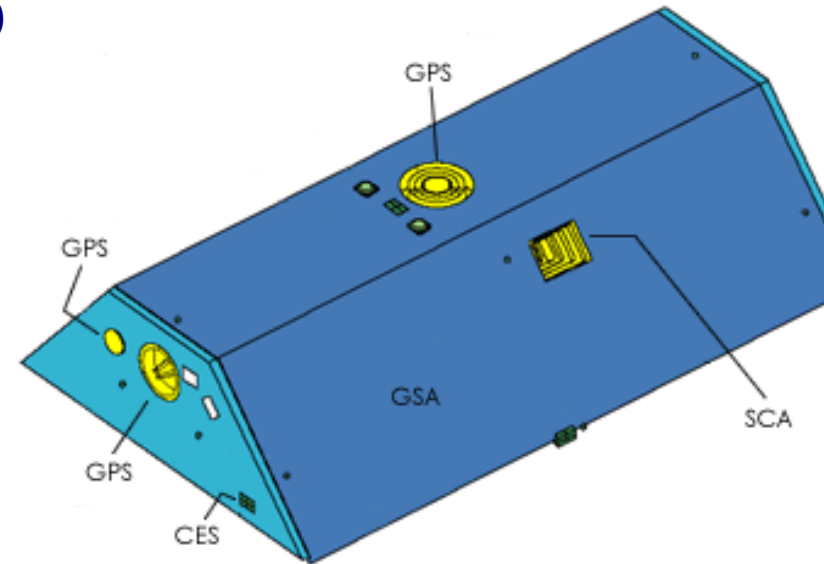
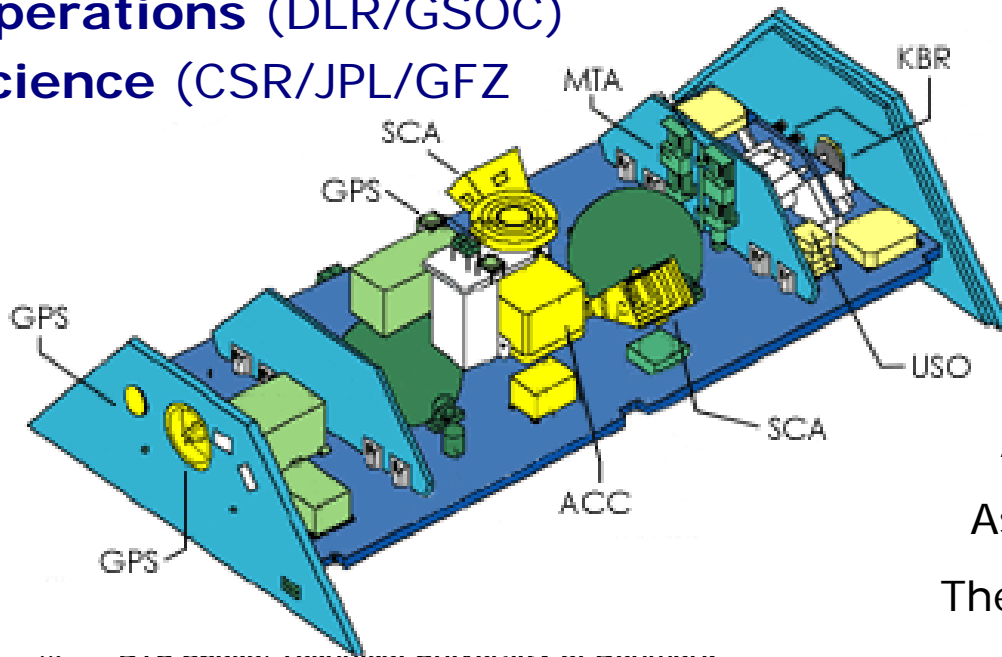
Gradiometry



GRACE Formation Flying Satellites

- KBR+USO Ranging (JPL/SSL/APL)
- ACC SuperSTAR Accelerometer (ONERA)
- SCA Star Cameras (DTU)
- GPS Receiver (JPL)

- **Satellite** (JPL/Astrium)
- **Launcher** (DLR/Eurockot)
- **Operations** (DLR/GSOC)
- **Science** (CSR/JPL/GFZ)



In essence.

The two satellites become the scale
And you use orbit perturbation
As measurements.

There aint no moving parts in satellite

Launch GRACE (look for DTU Starcameras)

- QT - LAUNCH

Orbit

Launched: March 17, 2002

Initial Altitude: 500 km

Current Altitude: 456 km (30 m/d)

Inclination: 89 deg

Eccentricity: ~0.001

Distance: ~220 km

Nominal Mission : 5 years

Non-Repeat Ground Track,
Earth Pointed, 3-Axis Stable

2005 GRACE 1-2 Orbit Switch

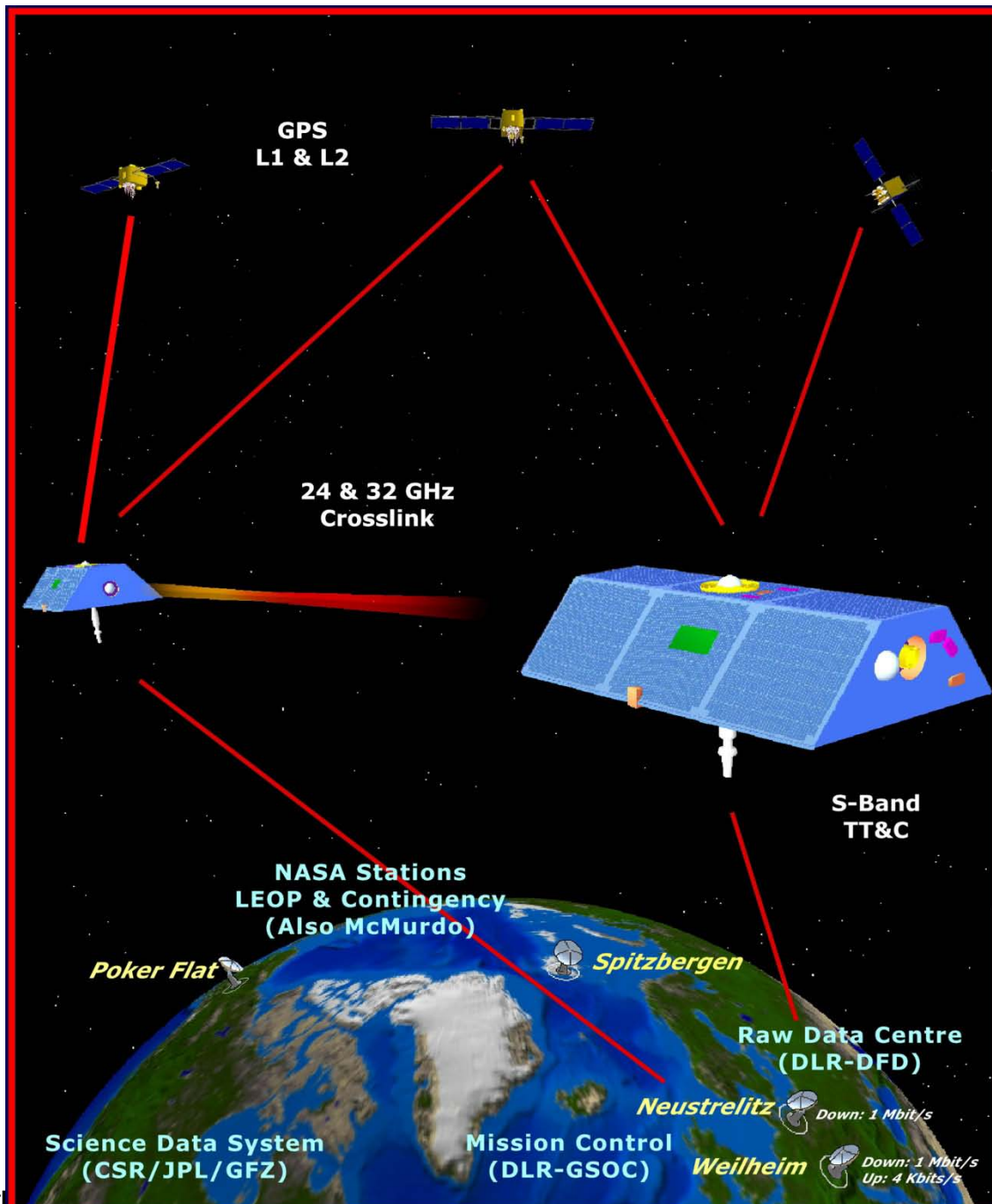
Mission Lifetime

5 Years

Extended to 9 Years

Future

GRACE Follow On



GRACE over LAND (QTIME)

- QT - Land

GRACE measurement technique

The two satellites act like a very big scale because of ORBIT perturbation.....

Orbit perturbation/ranging is measured by KBR ranging

Accuracy currently 2 Micrometer

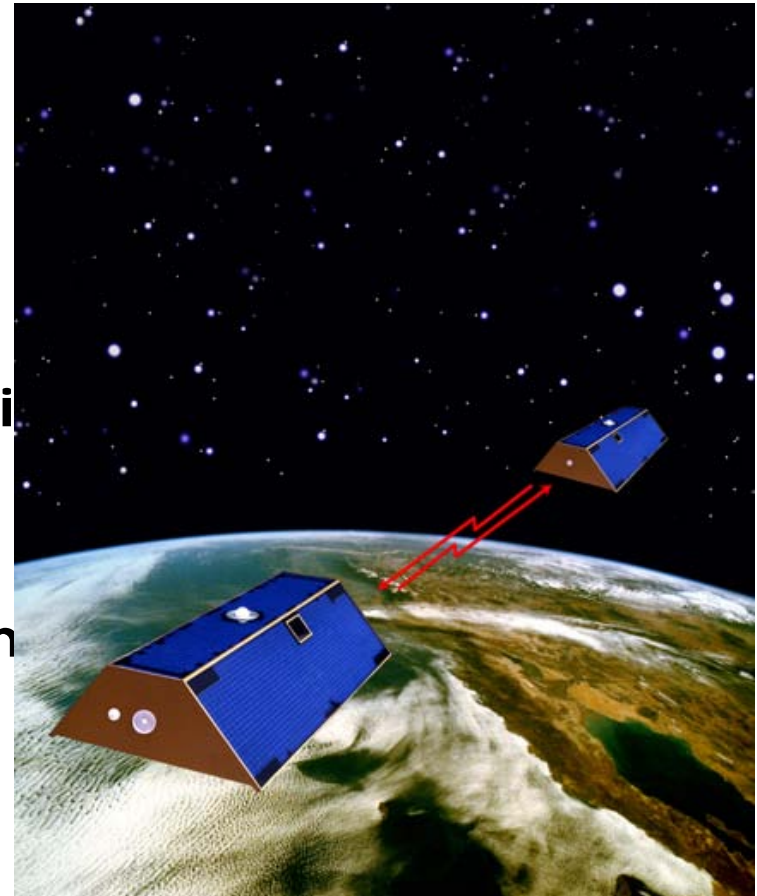
1/100 thickness of a hair

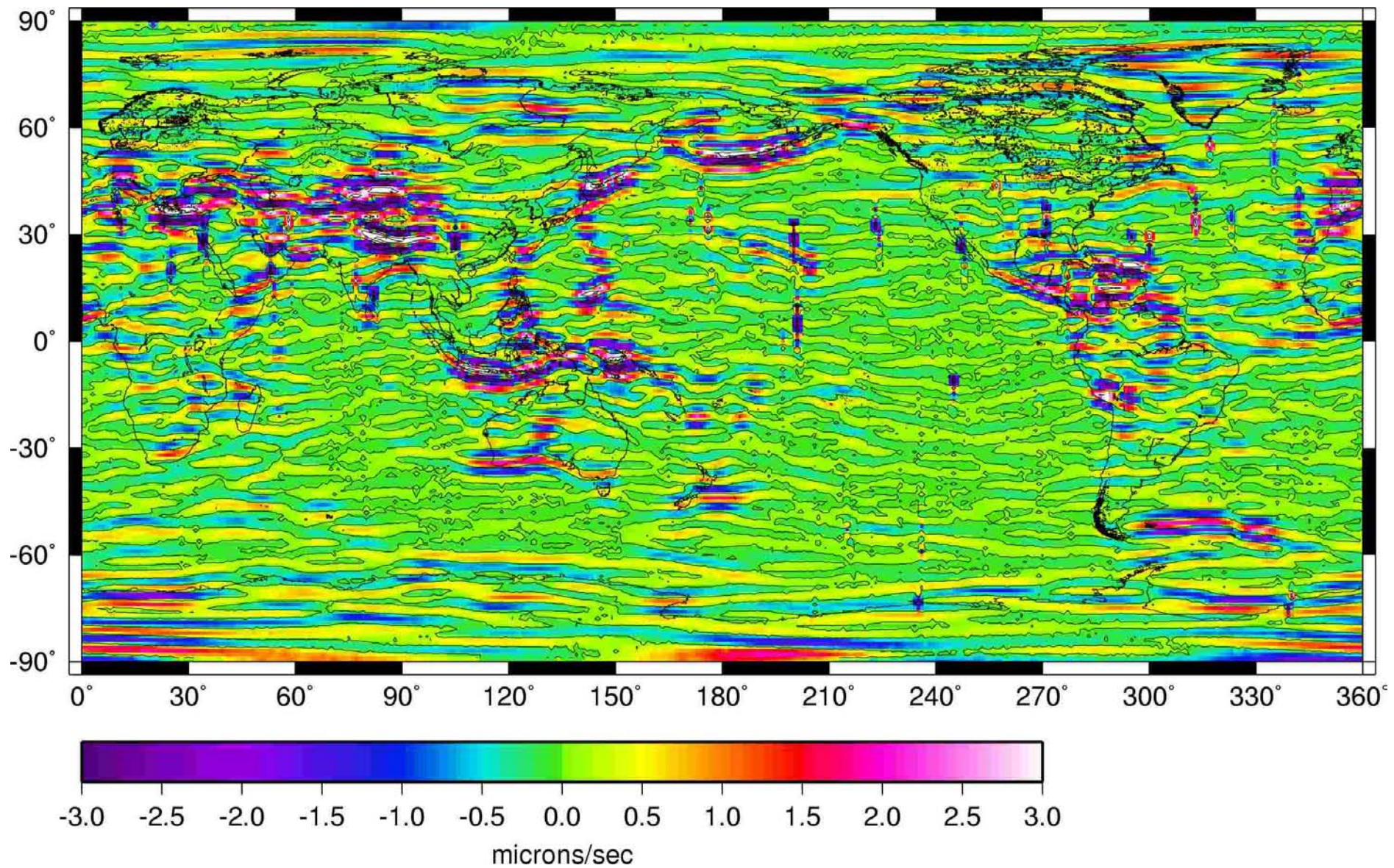
Better than diameter of blood cell

Delivers 10-Day / Monthly gravity field
From 2002 Onwards

Study gravity field changes with time

Limit in resolution 400 km

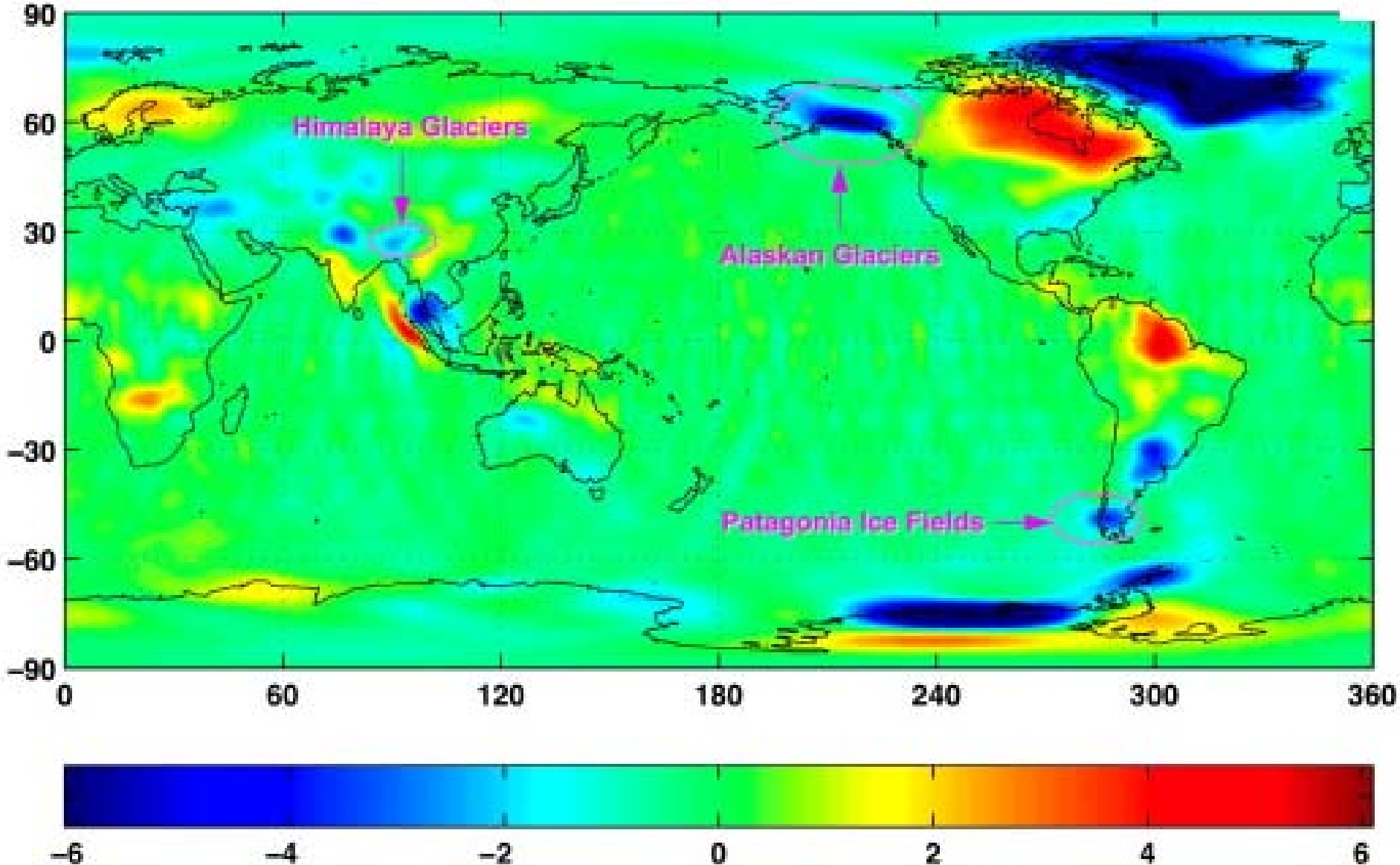


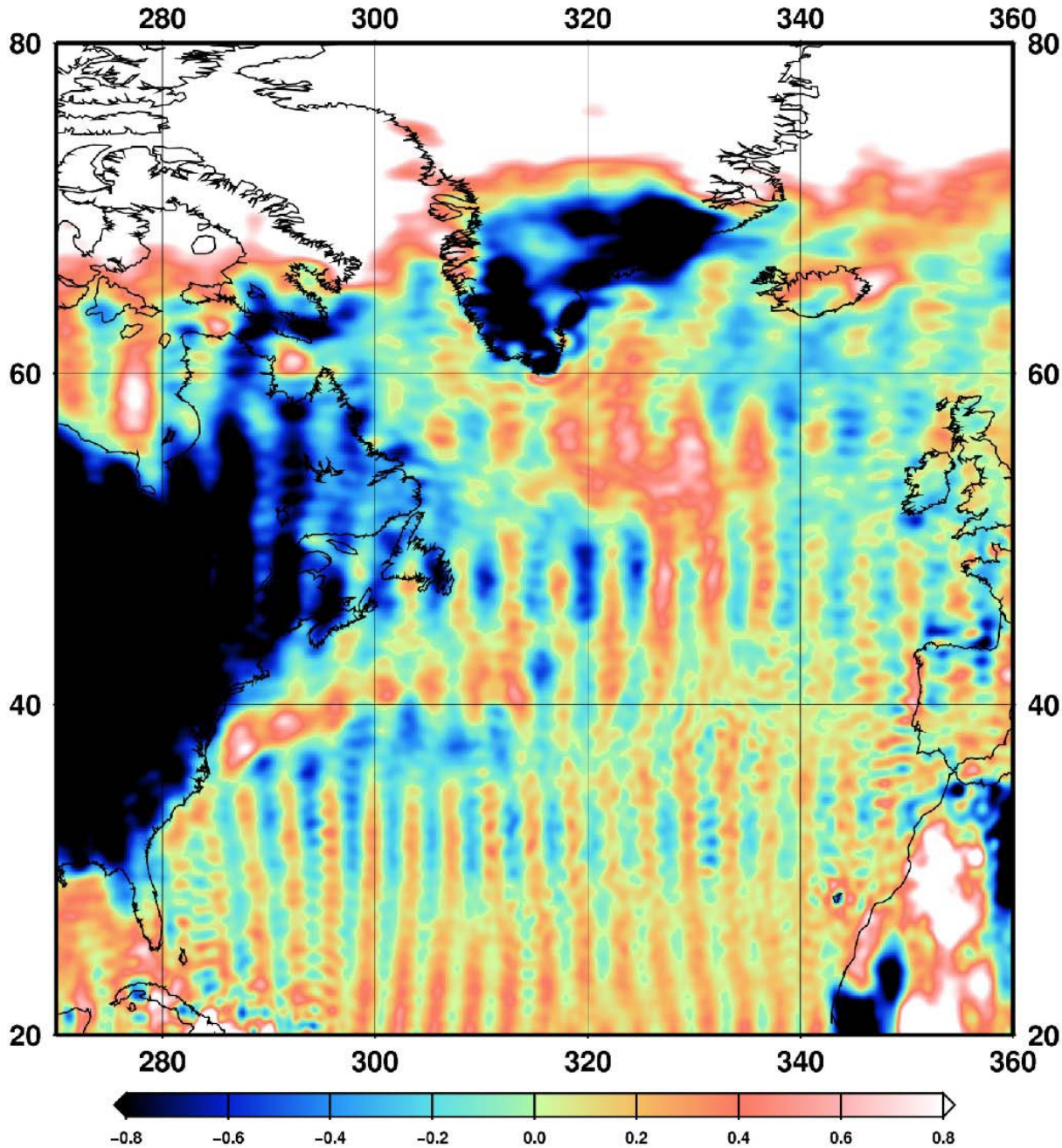


Grace can derive a gravity field every month..... > Does gravity change ?????

Whats fantastic about GRACE is that it can measure how gravity is changing because our planet is changing.

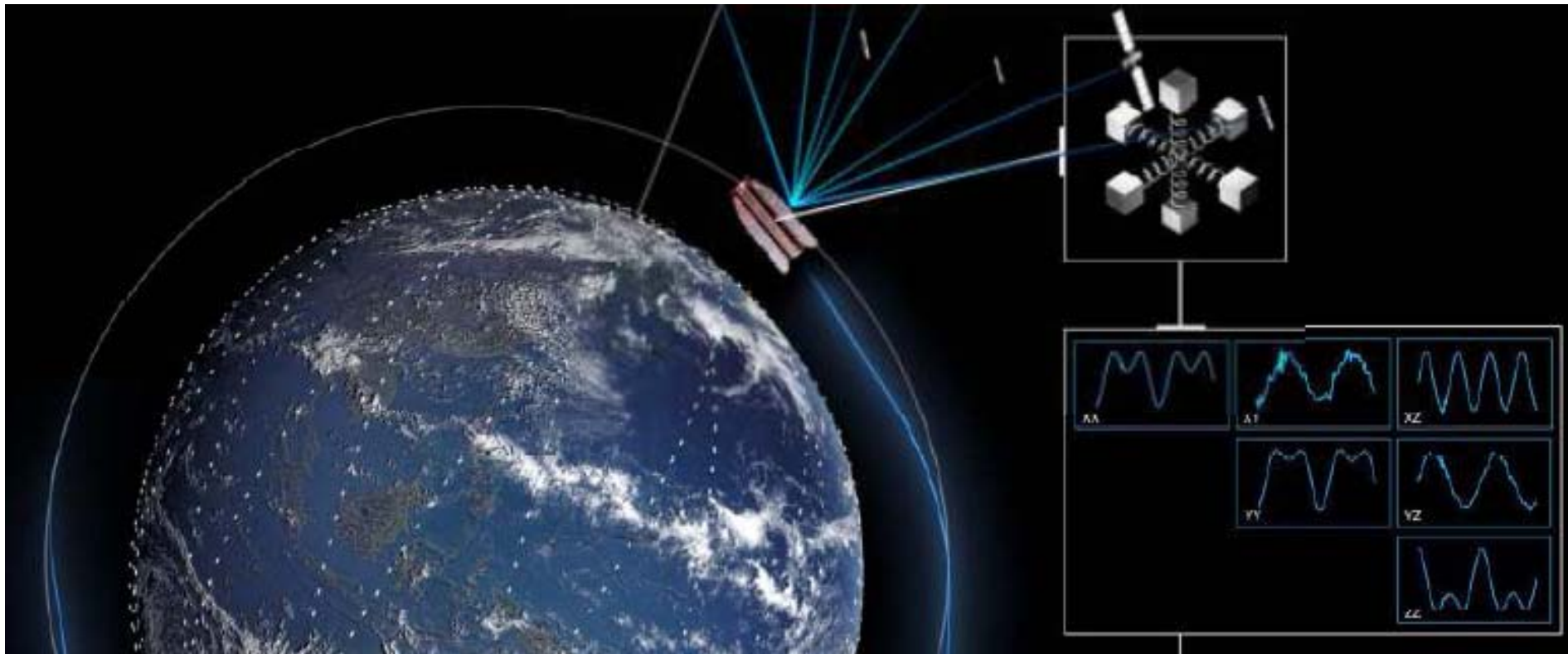
GRACE Mass Rate (cm/yr), CSR RL04, 2002.09 – 2009.08





However GRACE
 Has one big problem.
 Because GRACE
 Flies north-south
 It doesnt measure
 East west accurately –
 What do we do.....

ESA Launch GOCE in 2010



Measure gravity/accelerations -> no moving parts


Extreme stability->fabulous engineering achievement

Atomic diameter is ➔ 1 Ångström = $1\text{E}-10$ m

A picometre = $1\text{E}-12$ m ➔ 1% of an atom !!!



Get a feeling for what GOCE can do.....

0.2 gram 

Super-tanker acceleration
due to attracting snowflake:

0.0000000000002 m/s²

smallest acceleration measurable
in space by GOCE



1 000 000 metric tonnes

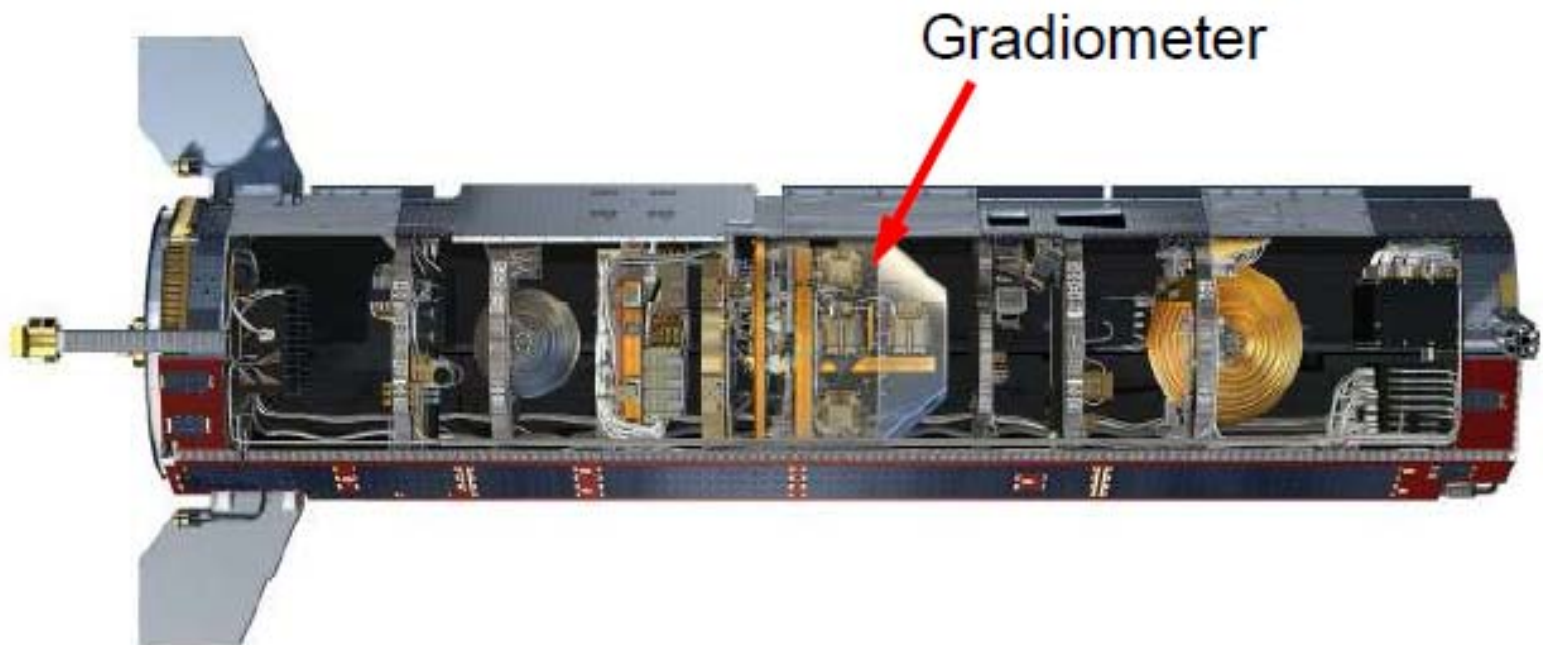
GPS positioning of GOCE in orbit



Sun-synchronous, 61 day repeat orbit at 256 km altitude

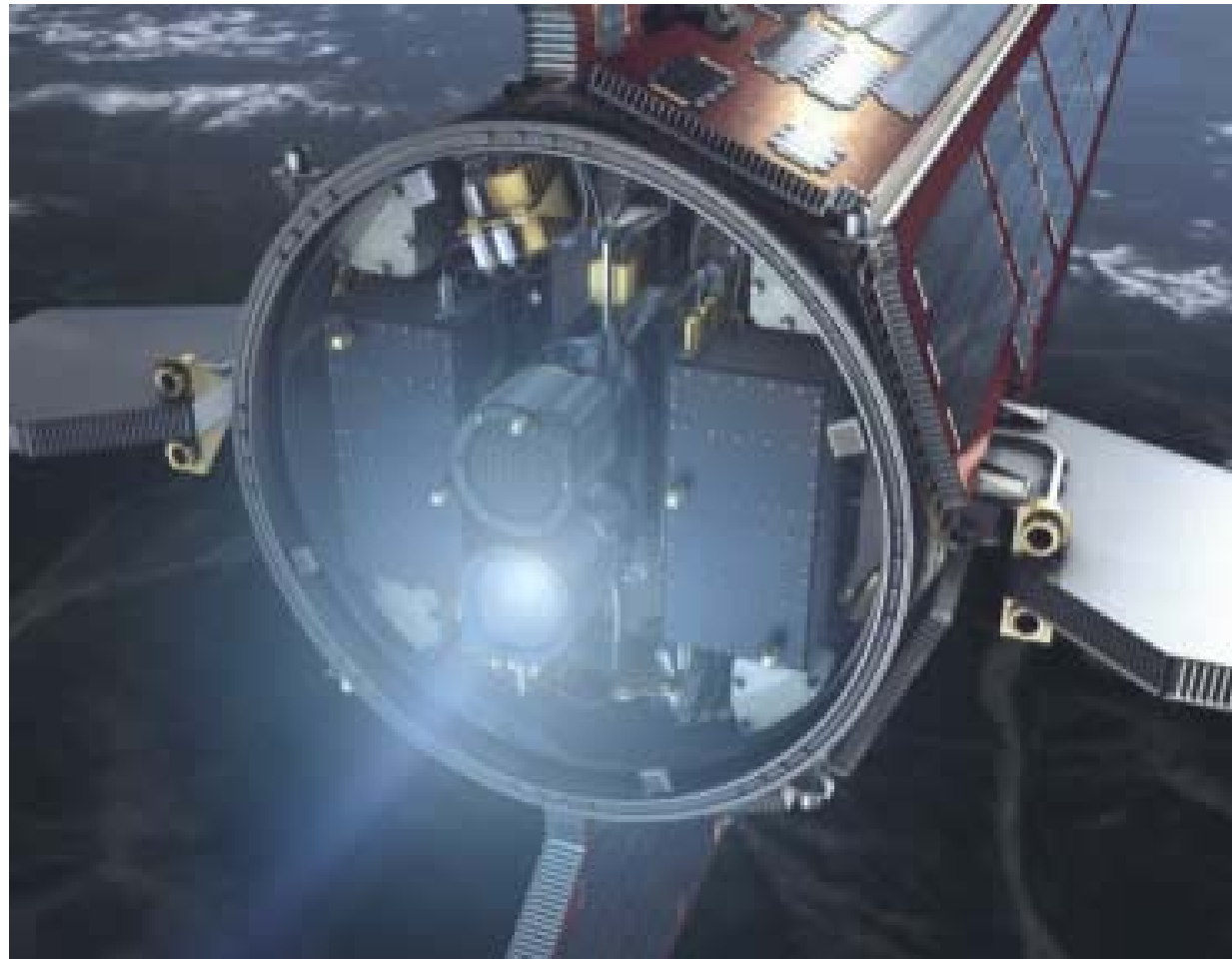
- Orbit as low as possible to maximize signal strength
- Significant aerodynamic forces and torques act on satellite

Gradiometer needs 'quiet environment'



Flying low \rightarrow Atmosphere disturbs the Free Fall

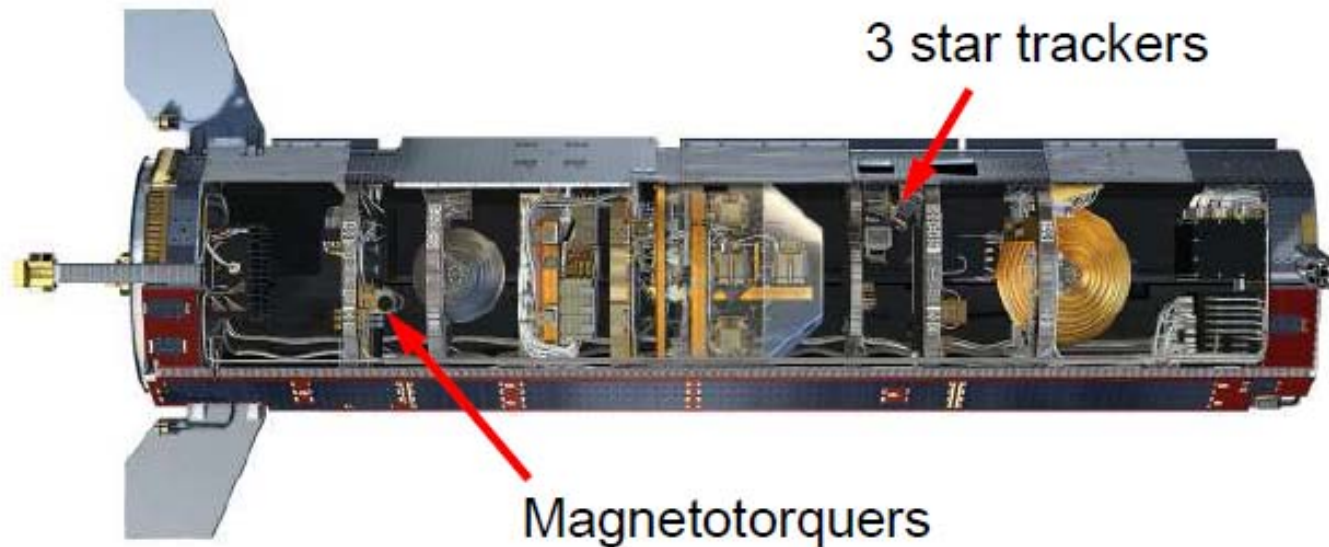
Counteracting the atmospheric drag.....



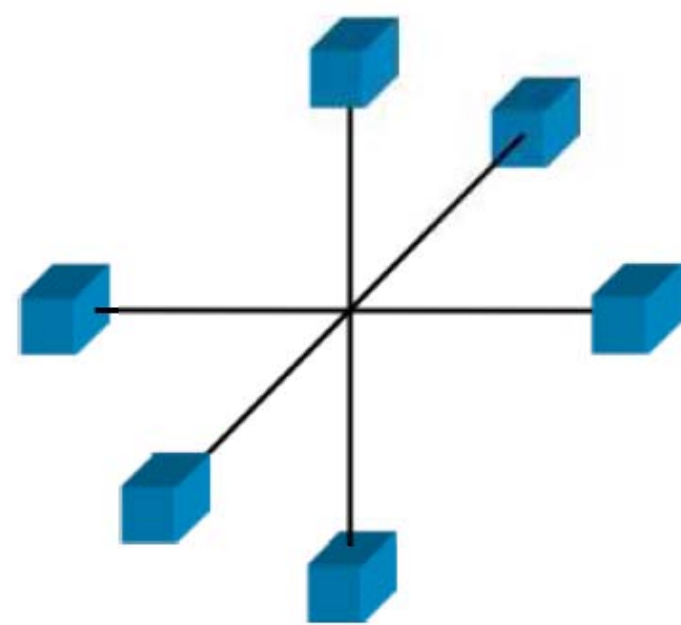
Navigating GOCE - DTU startrack/camera

Attitude

- Attitude control by magnetotorquers
 - Sun- and nadir-pointing
 - No control about yaw axis at magnetic poles and roll axis at magnetic equator
- Attitude measured by three star trackers
 - Capture also angular rates and angular accelerations

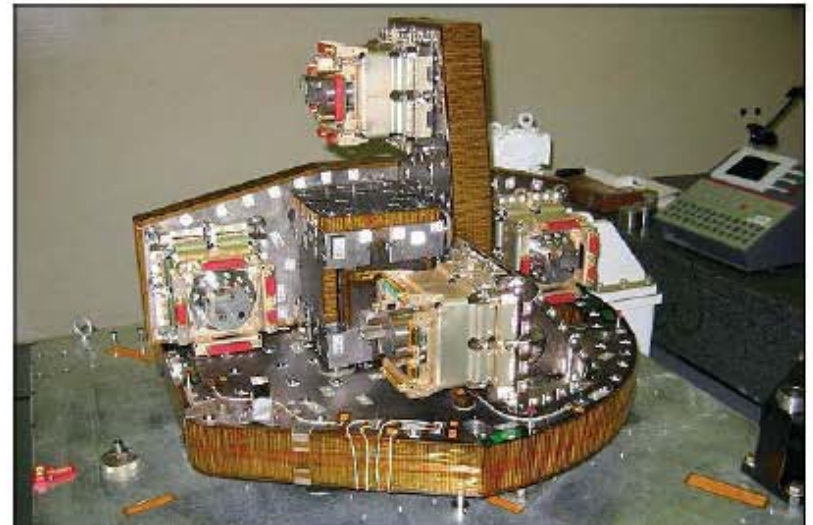
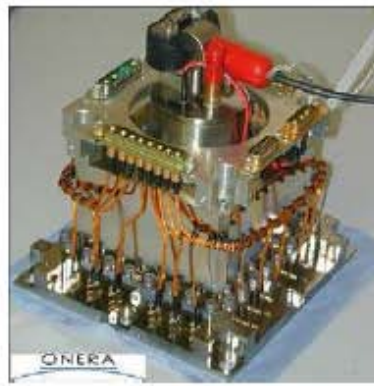
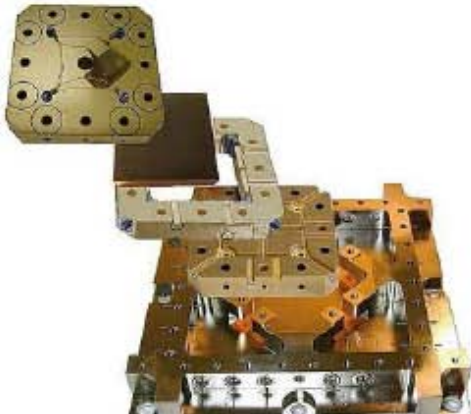


GOCE Gradiometer



Gradiometer = 6 Accelerometers

- 100 times more sensitive than any accelerometer previously flown
- Proof-mass is kept levitated at the center of a slightly larger cage by applying control-voltages to electrodes on the inner walls of the cage
- Control-voltages are representative for accelerations experienced by accelerometer
- Dimensions of proof-mass = $4 \times 4 \times 1$ cm
- 2 ultra-sensitive axes + 1 less sensitive axis



GOCE accelerometer measurements

$$a = \underbrace{-[V] \cdot r}_{\text{gradient}} + \underbrace{(\omega \times (\omega \times r)) - (\dot{\omega} \times r) + (2\omega \times \dot{r})}_{\substack{\text{centrifugal acceleration} \\ \text{angular acceleration} \\ \text{Coriolis}}} + \underbrace{\ddot{r} + d - s}_{\substack{\text{cage} \\ \text{drag} \\ \text{self gravity} \\ \text{magnetic coupling}}} \times$$

Forming common and differential accelerations:

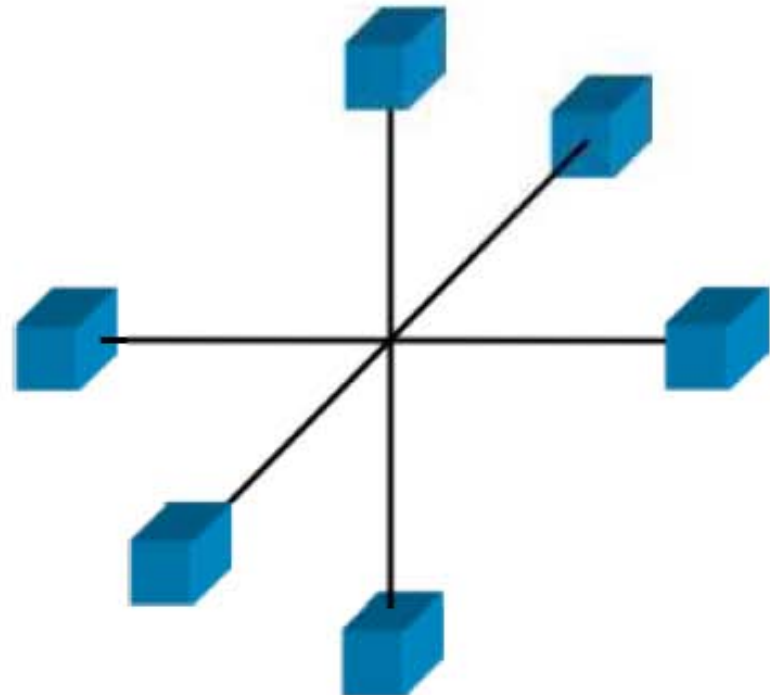
- Common relates to drag and winds
- Differential relates to gravity gradients rotating around the Earth

-> use information from star trackers

Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \Omega^2 - \dot{\Omega})r_i + d$$

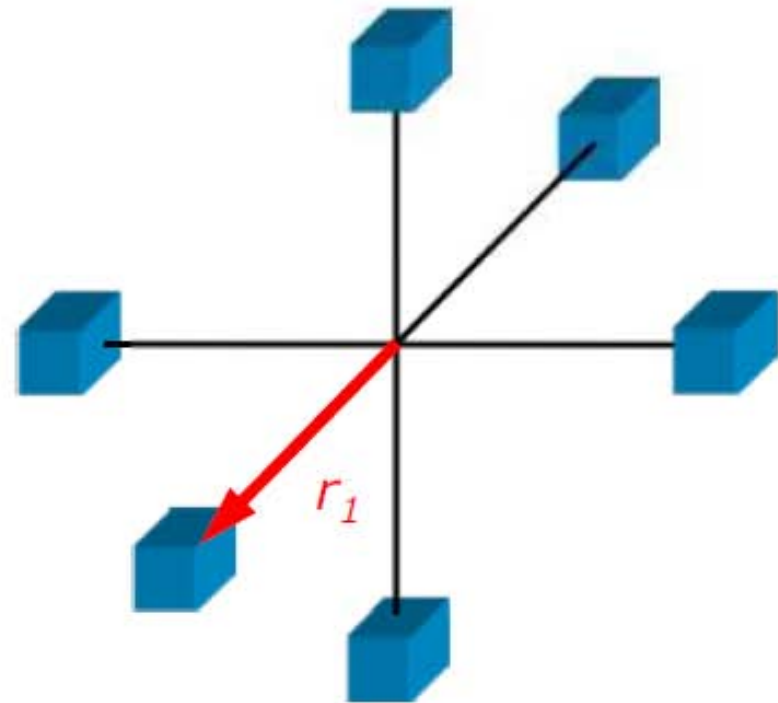


Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \Omega^2 - \dot{\Omega}) \boxed{r_i} - d$$

position of accelerometer

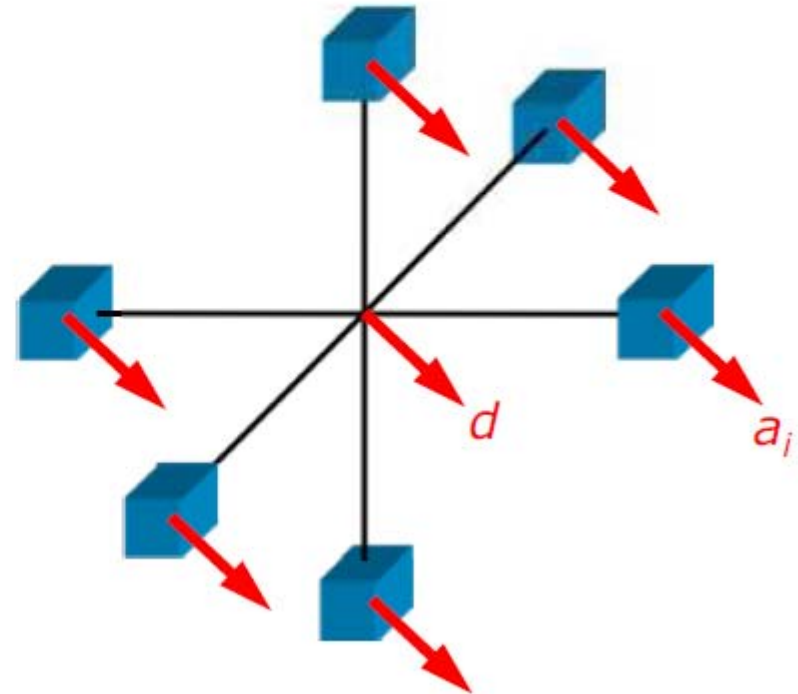
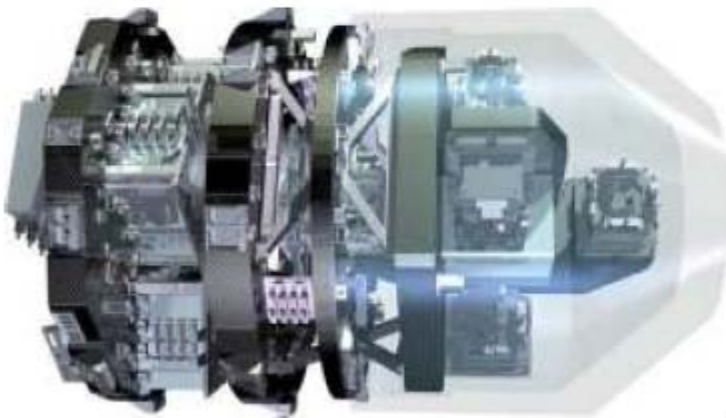


Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \Omega^2 - \dot{\Omega})r_i - d$$

linear acceleration of satellite

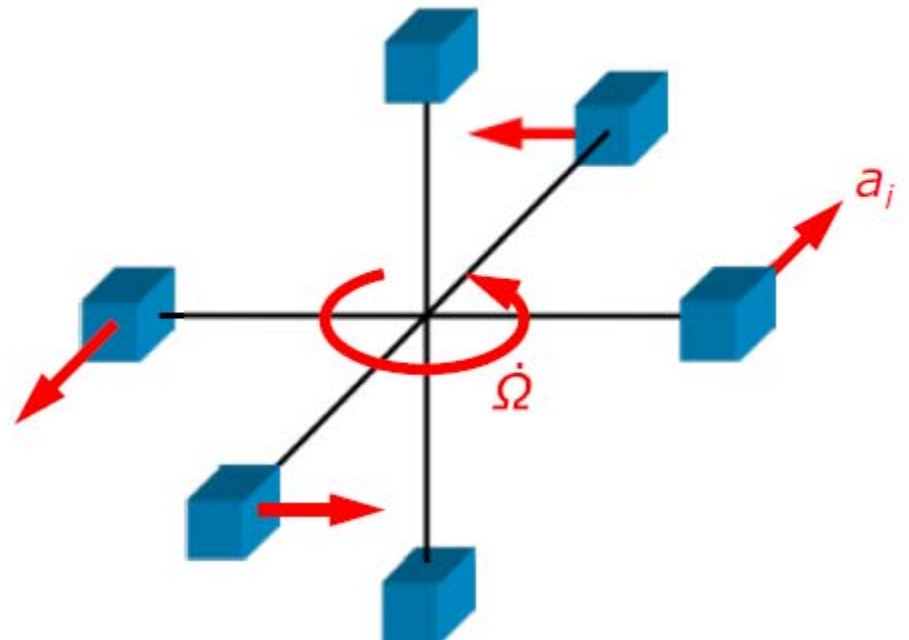


Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \Omega^2 - \dot{\Omega})r_i + d$$

angular acceleration of satellite

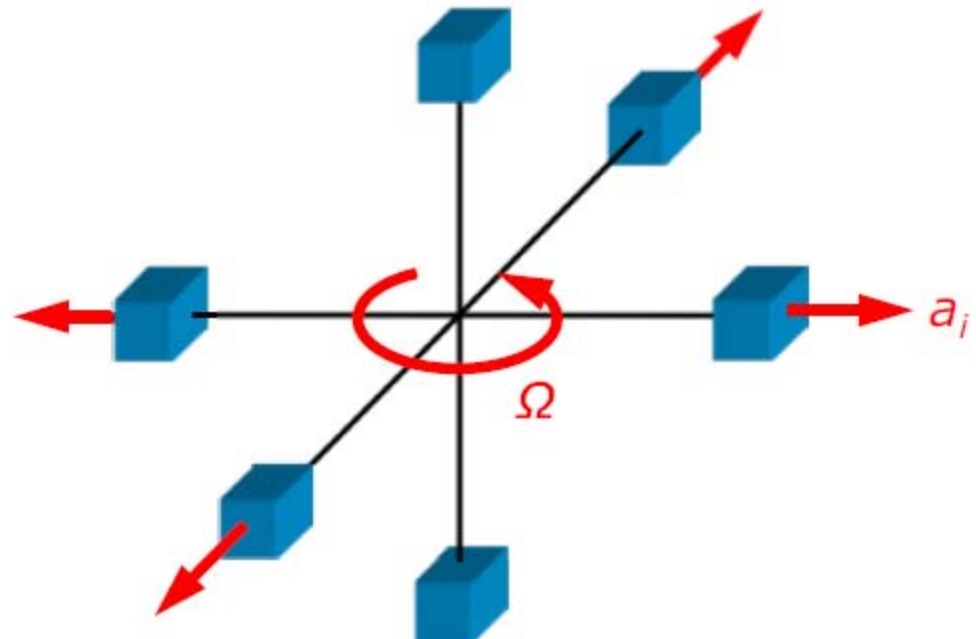
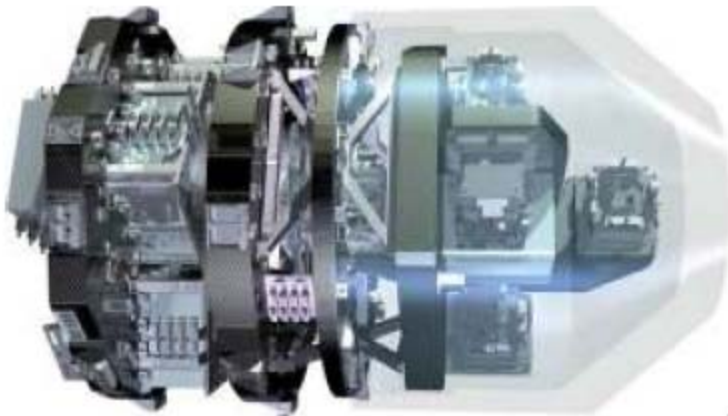


Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \Omega^2 - \dot{\Omega})r_i + d$$

*centrifugal acceleration
caused by satellite rotation*

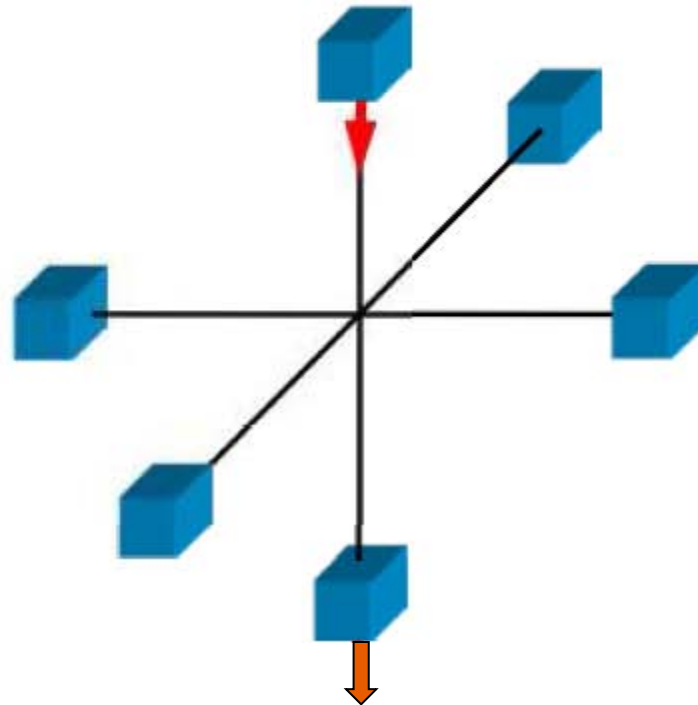
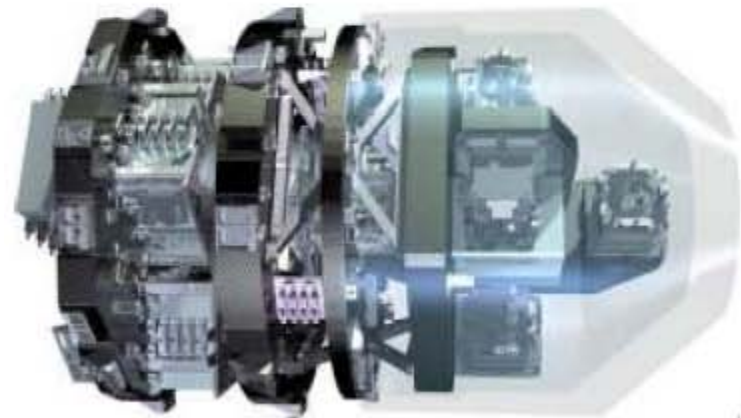


Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(\boxed{V} - \Omega^2 - \dot{\Omega})r_i + d$$

*gravity gradient
between accelerometers*



Form common sums and differences....

- 1. Common mode rejection** (for accelerometer pair on one gradiometer arm)

$$a_{cij} = \frac{1}{2}(a_i + a_j) \approx d \quad \text{common mode acceleration}$$

$$a_{dij} = \frac{1}{2}(a_i - a_j) = -\frac{1}{2}(V - \Omega^2 - \dot{\Omega})(r_i - r_j) \quad \text{differential mode acceleration}$$

- 2. Separation of symmetric and anti-symmetric parts of**

$$A_d = \begin{bmatrix} a_{d14} & a_{d25} & a_{d36} \end{bmatrix} \quad L = \begin{bmatrix} r_1 - r_4 & r_2 - r_5 & r_3 - r_6 \end{bmatrix}$$

$$A_d L^{-1} - L^{-T} A_d^T = \dot{\Omega} \quad \text{angular acceleration}$$

$$A_d L^{-1} + L^{-T} A_d^T = -V + \Omega^2 \quad \text{gravity gradient} \\ \text{+ centrifugal acceleration}$$

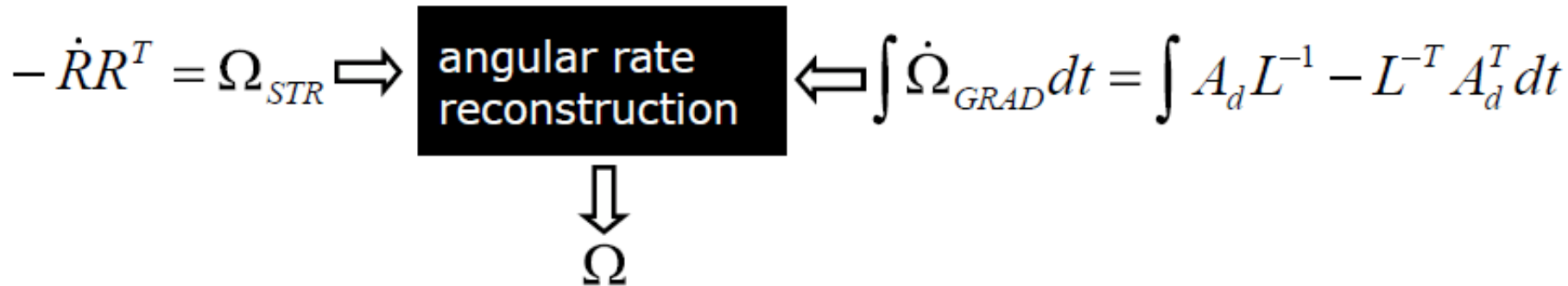
Getting the gravity potential

3. Separation of centrifugal accelerations

$$V = \boxed{\Omega^2} - (A_d L^{-1} + L^{-T} A_d^T)$$

Star trackers provide angular rate as first derivative in time of inertial attitude

Gradiometer provide the angular acceleration





Summarizing

Satellite gradiometry gives us the long wavelength of the gravity field as they measure the gravity field / potential in space hundreds of km away

Satellite altimetry gives us detailed information on the shorter wavelength of the gravity field by measuring the sea surface, but can not give us information about longer wavelength due to sea surface variations....

We still have a long way to go.....

GRACE Follow On or GRACE 2 will use lasers that are 100 times more accurate.