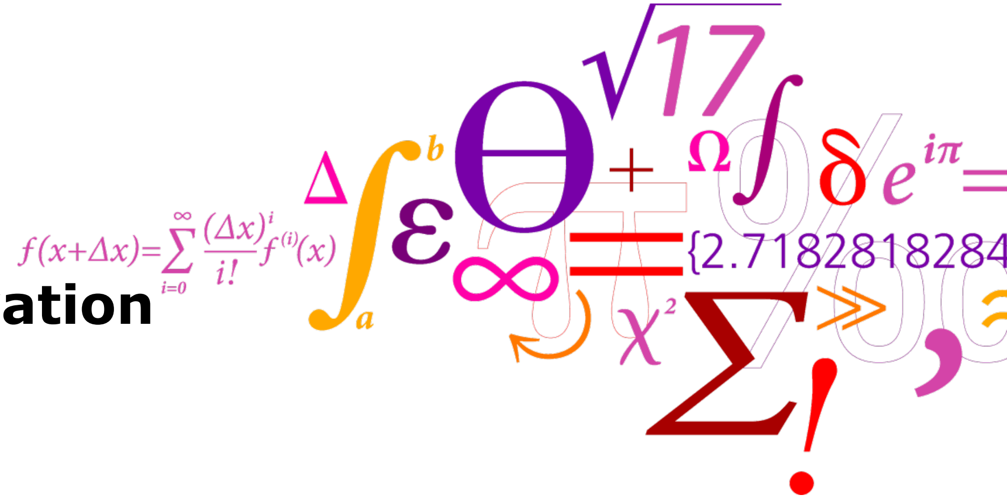
30552 – Lecture 10.

Time varying changes in Geodesy
Ocean Tides,
Sea level change,
Vertical Land Movement

Prof Ole B. Andersen,
DTU Space,
Geodesy and Earth Observation



DTU Space
National Space Institute

Before we start:

If you feel ill, go home
Keep your distance to others
Wash or sanitize your hands
Disinfect table and chair
Respect guidelines and restrictions

Outline

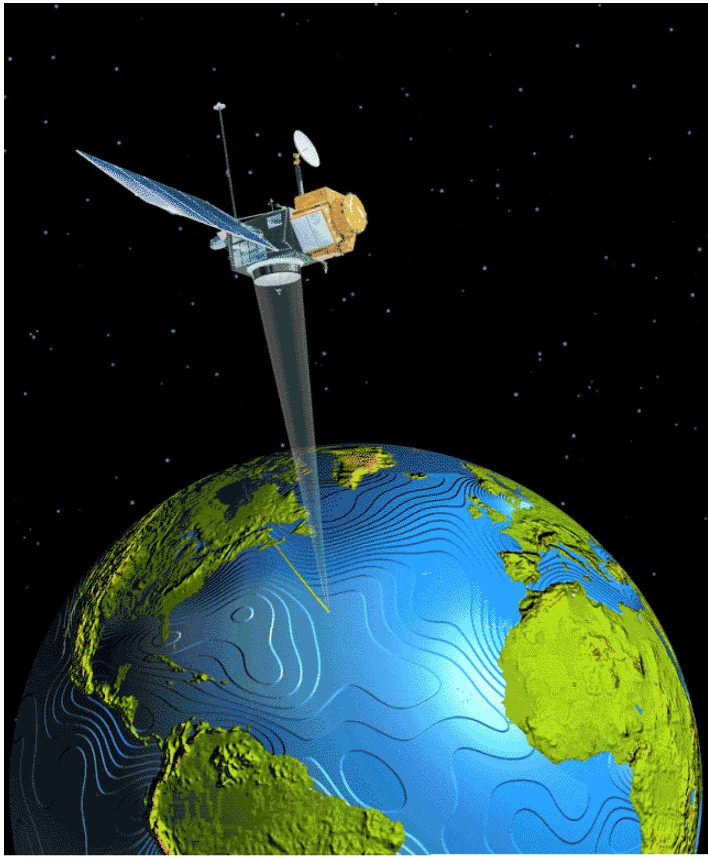
Timevariable changes in geodesy.

Earth and Ocean Tides

- Sea level change (Tadea)

- Global Isostatic Adjustment +
- Present Day ice loading (Carsten)

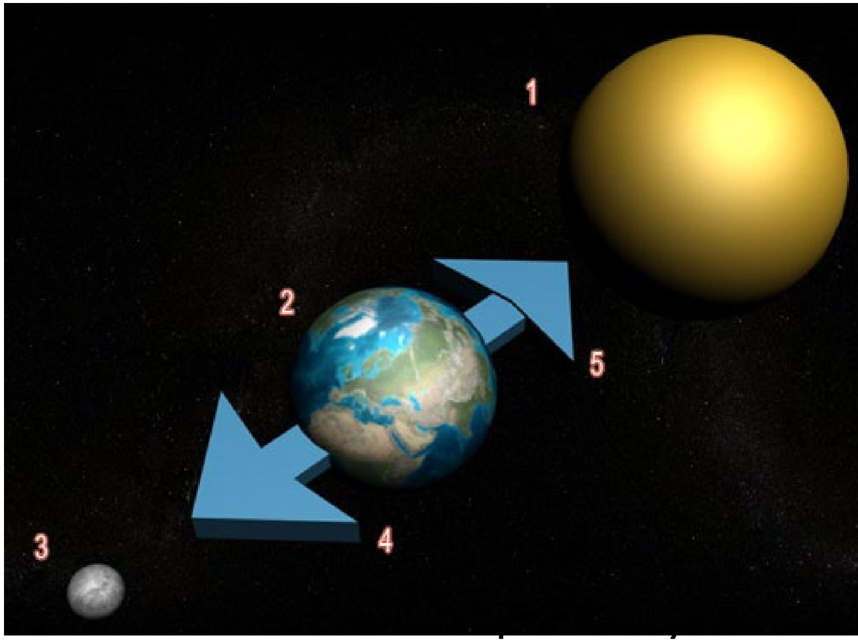
Assignment



READING MATERIAL FOR TODAY.

Torge, Wolfgang, Geodesy 4th Edition, pp 349 – 367,
Veng and Andersen, Consolidating sea level acceleration, ASR 2019

External and Internal



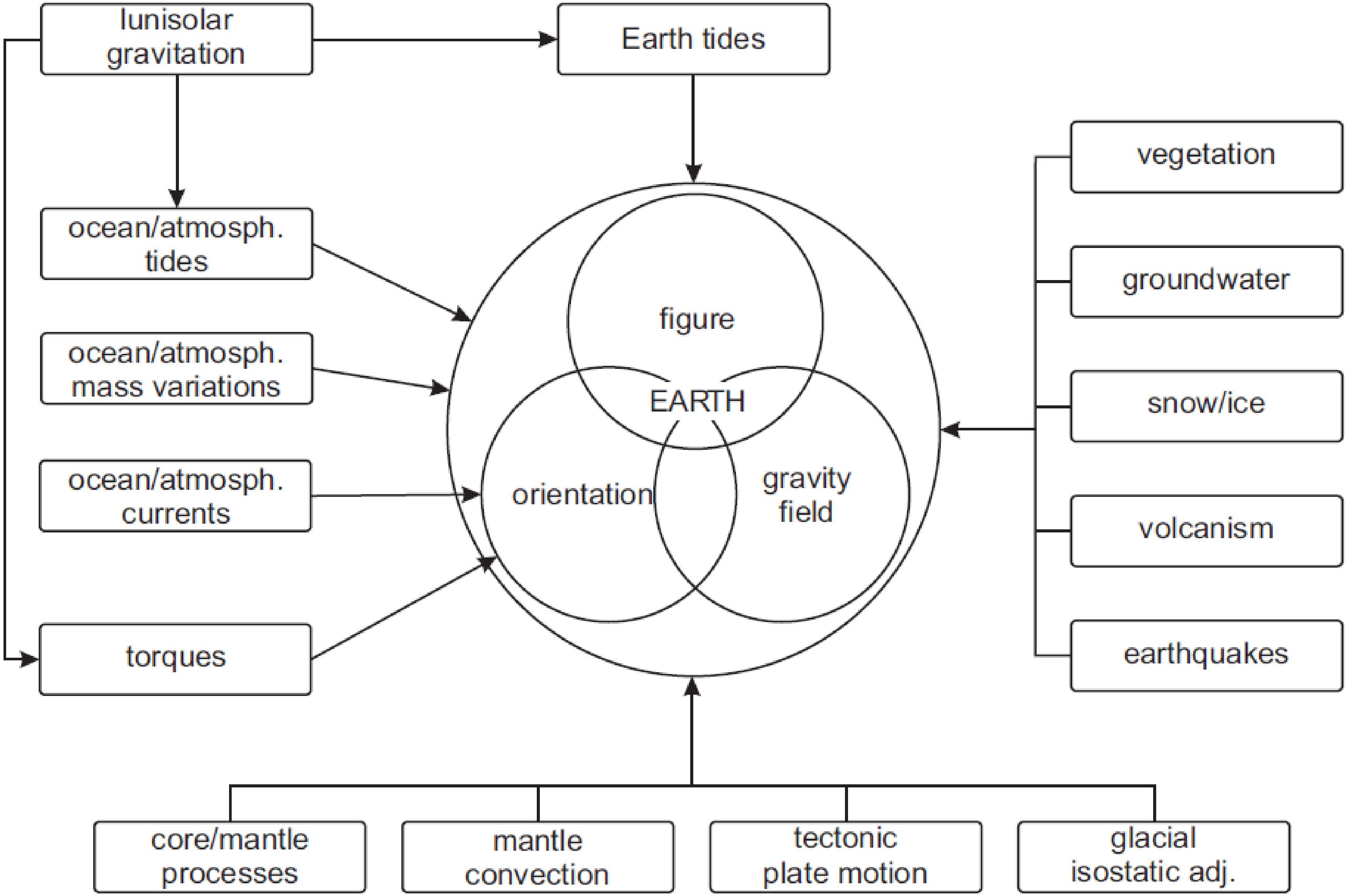


Fig. 8.14: Astronomical and geophysical processes and effects on the Earth's figure, gravity field and orientation.

Torge Geodesy, 4th edition, De Grüyer

Timevarying changes affect fundamental geodetic parameters:

Earth Orientation / Earth Rotation.

Changes to the orientation (affect Length of Day and Polar Motion.

Moving Mass h(t) changes angular momentum as

Figure of the Earth or Surface geometry (mostly today)

Changes to the Figure – Vertical and horizontal land movement

Change to the mean (sea surface)

Earthquakes - Plate tectonics. Interior Earth Processes

Changes to the Gravity field. (deal with this in Lecture 13)

Earthquakes - Plate tectonics.

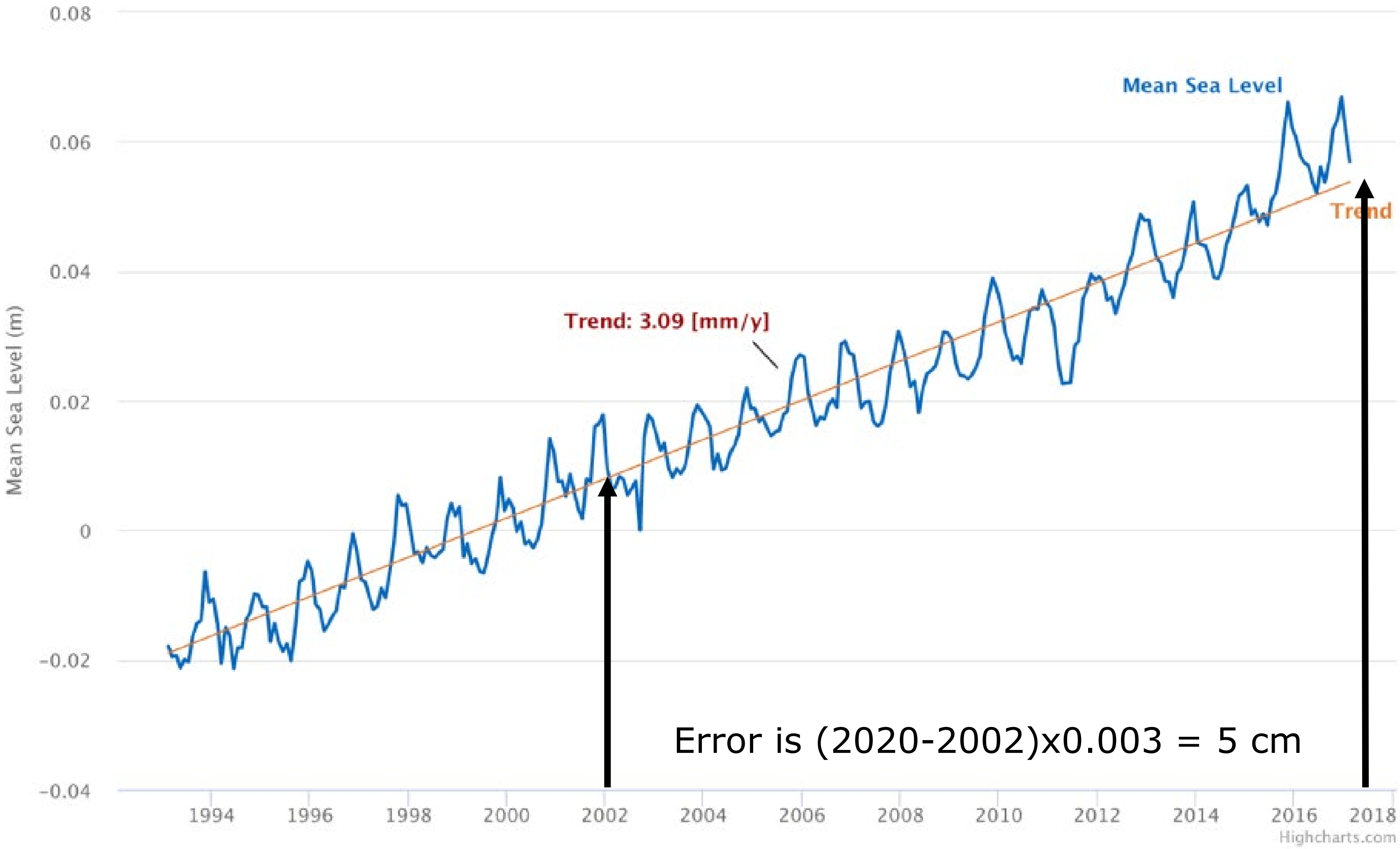
Cryospheric and hydrospheric changes => impact on sea level.

Time varying changes in geodesy

- Fundamental geodetic Earth Parameters changes due to:

- Periodic signals
  - **Earth and Ocean Tides (short scale)**
  - **Annual circulation of fluids on the Earth's surface (oceans and atmosphere)**
    - **Length og Day variations -> Earth Rotation Variation**
- Non periodic signals.
  - Glacial isostatic adjustment (GIA)/Post-glacial rebound (PGR)
  - Present day geodynamic changes.
    - Ice sheet changes / Ice Loading changes
    - Sea level change
    - Solid earth motion (plate tectonics, volcanoes, earthquakes)
  - Internal changes in the Earth (mantle convection)

Global mean sea level (GMSL) time series and estimated trend from multi-mission satellite altimetry (Jan 1993-Feb 2017)



Error is (2020-2002)x0.003 = 5 cm

Satellite Geodesy

- Ice sheet and sea level changes can be measured with altimetry
  - Radar altimeters (ERS-1, ERS-2, Envisat, Cryosat-2)
  - Laser altimeters (ICESat, ICESat-2)

- Vertical crustal motion can be measured by GNSS

- Mass changes via changes in gravity and the geoid can be measured by gravity missions (GRACE)

Satellite geodesy is TODAY the most important data sources for monitoring (and understanding) present day climate change

Outline

Timevariable changes in geodesy.

Earth and Ocean Tides

- Sea level change (Tadea)

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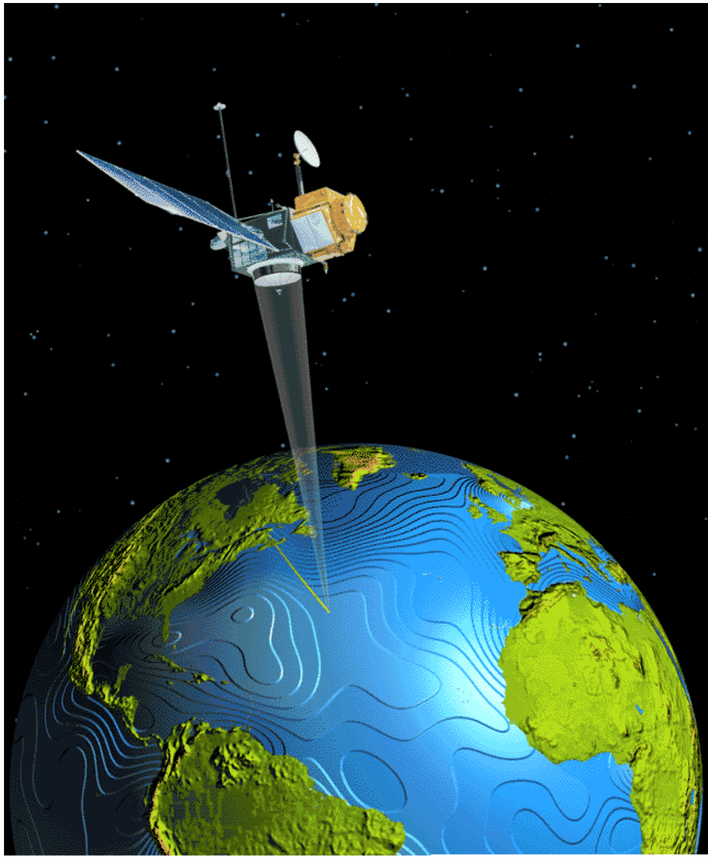
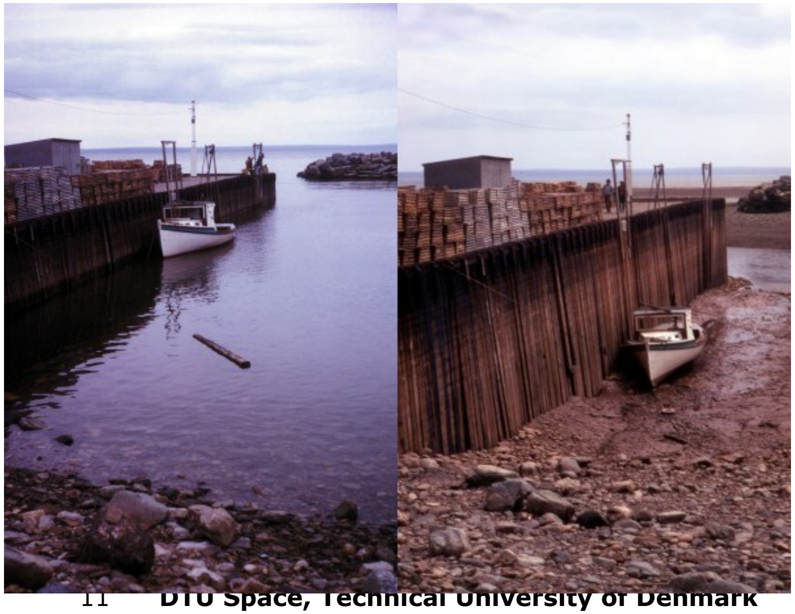
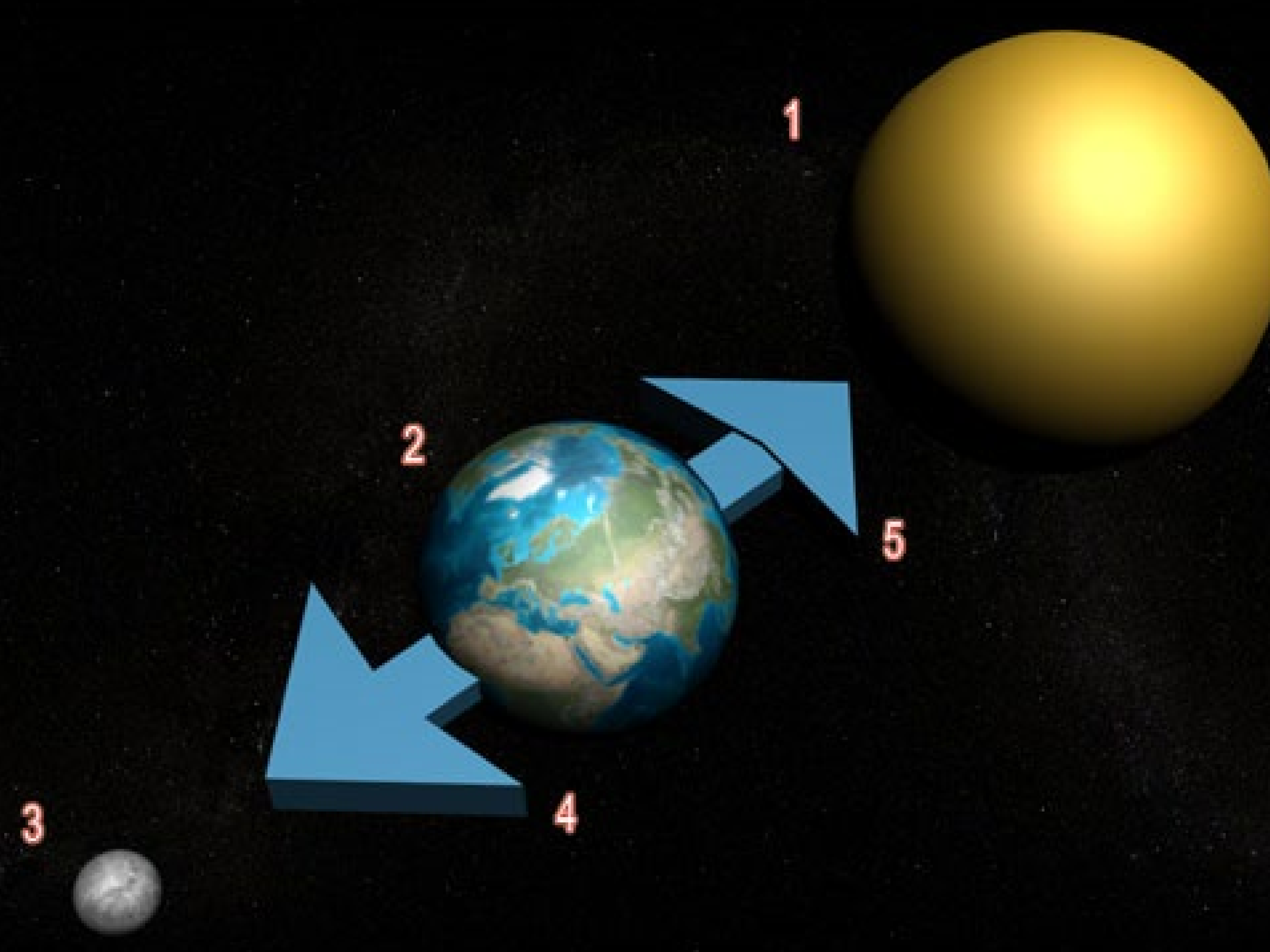
Assignment

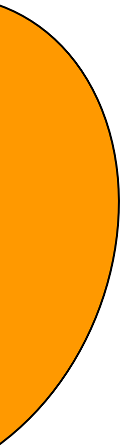


Tides



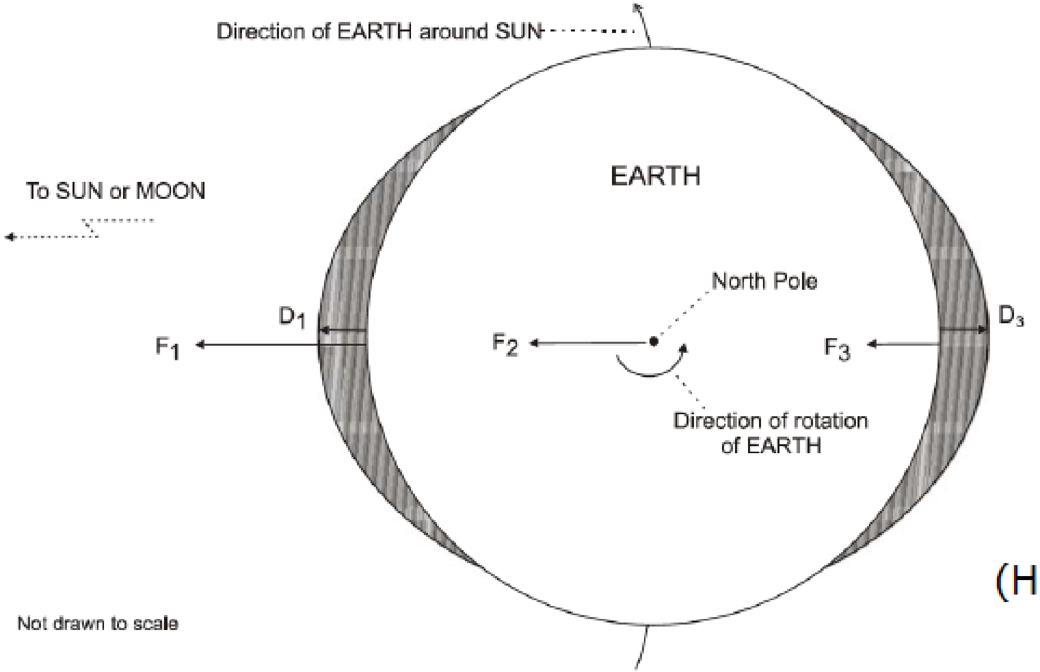

1
2
5
4
3

Tidal forces



$$D_1 = F_1 - F_2$$

$$D_3 = F_3 - F_2$$

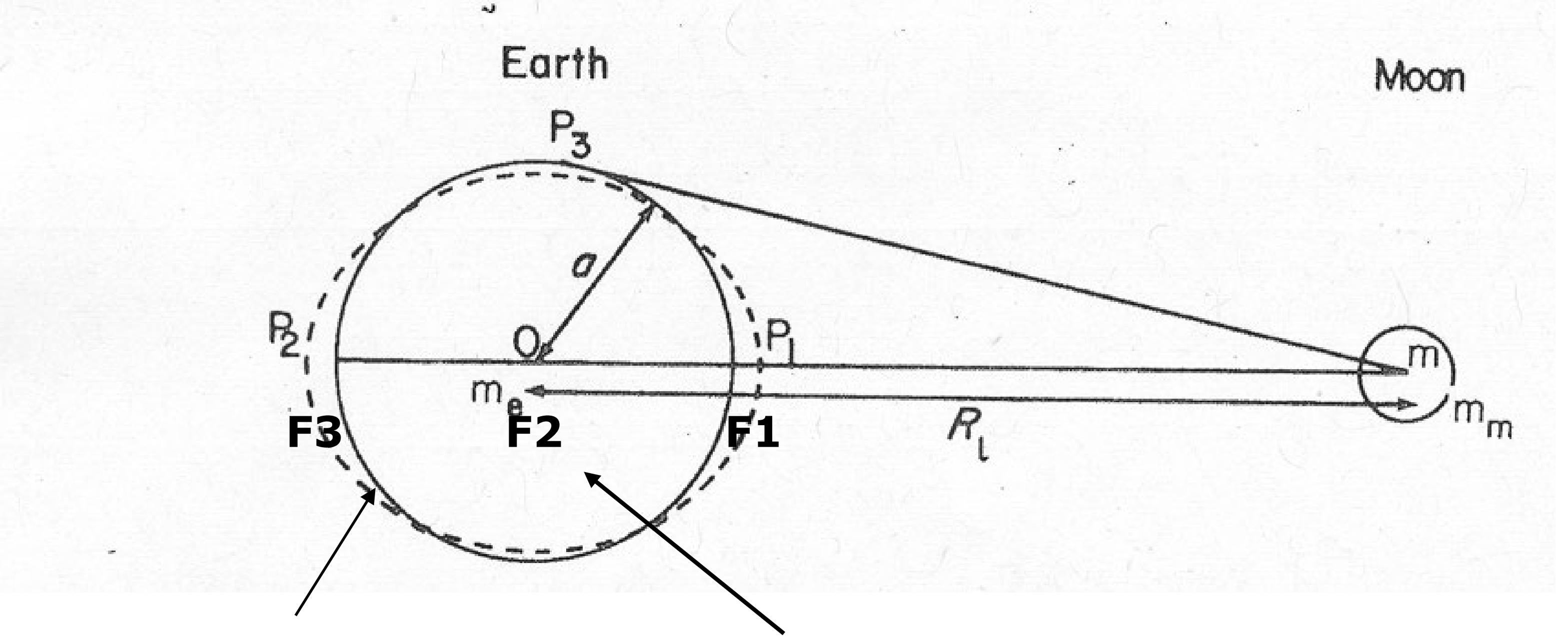


(Hicks, 2006)

At the center of mass (CM) of the Earth, the gravitational attraction from the Sun has the value necessary to keep the Earth in orbit (F_2).

At a point on the Earth surface facing the Sun, attraction is larger because the distance is smaller (F_1). Vice versa at the opposite side (F_3).

Tide generating forces can be computed as vector differences between attraction at the Earth CM and attraction at the surface.



Force $= G\frac{m_e\, m_m}{(R_1+a)^2}$ $\quad G\frac{m_e\, m_m}{(R_1)^2}$ $\quad G\frac{m_e\, m_m}{(R_1-a)^2}$

Difference (F1-F2) is the Tide producing force

$$Gm_em_m\left[\frac{1}{(R_1-a)^2}-\frac{1}{(R_1)^2}\right] \qquad \frac{Gm_em_m}{R_1^2}\left[\frac{1}{(1-\frac{a}{R_1})^2}-1\right]$$

Tidal Forces.

$\left(\frac{a}{R_1}\right)^2 \langle\langle 1$ As a/R = 1/60

Expanding $(1/(1-a)^2 = 1 + 2a$ yields

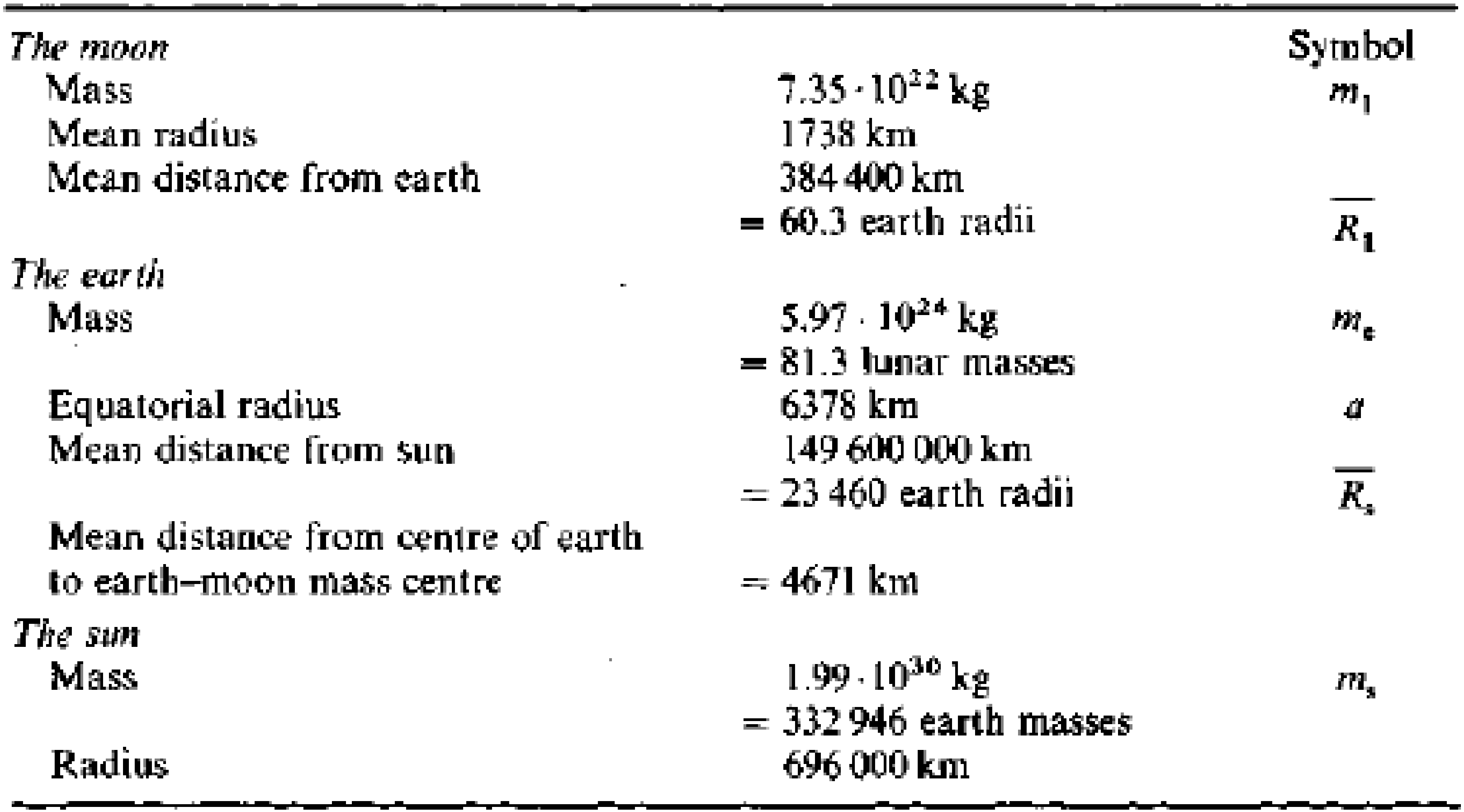
Difference $(P_1\text{-}O)$ is the Tide producing force at F1 $2G\frac{am_e m_m}{(R_1)^3}$

At F3 the force away from the moon is = $-2G\frac{am_e m_m}{(R_1)^3}$

Using $\sin(OMP_3) = a/R_1$

At P3 the force is directed towards O $G\frac{am_e m_m}{(R_1)^3}$

Astronomical Constants



		Symbol
The moon		
Mass	$7.35 \cdot 10^{22}$ kg	m_1
Mean radius	1738 km	
Mean distance from earth	384 400 km	
	= 60.3 earth radii	$\overline{R}_1$
The earth		
Mass	$5.97 \cdot 10^{24}$ kg	m_e
	= 81.3 lunar masses	
Equatorial radius	6378 km	a
Mean distance from sun	149 600 000 km	
	= 23 460 earth radii	$\overline{R}_s$
Mean distance from centre of earth to earth–moon mass centre	= 4671 km	
The sun		
Mass	$1.99 \cdot 10^{30}$ kg	m_s
	= 332 946 earth masses	
Radius	696 000 km	

Tidal Forces (Semi diurnal).

Force by the Moon is 1.4 Force by the Sun.

This is because Moon is much closer.

Semi dirunal tides dominate:

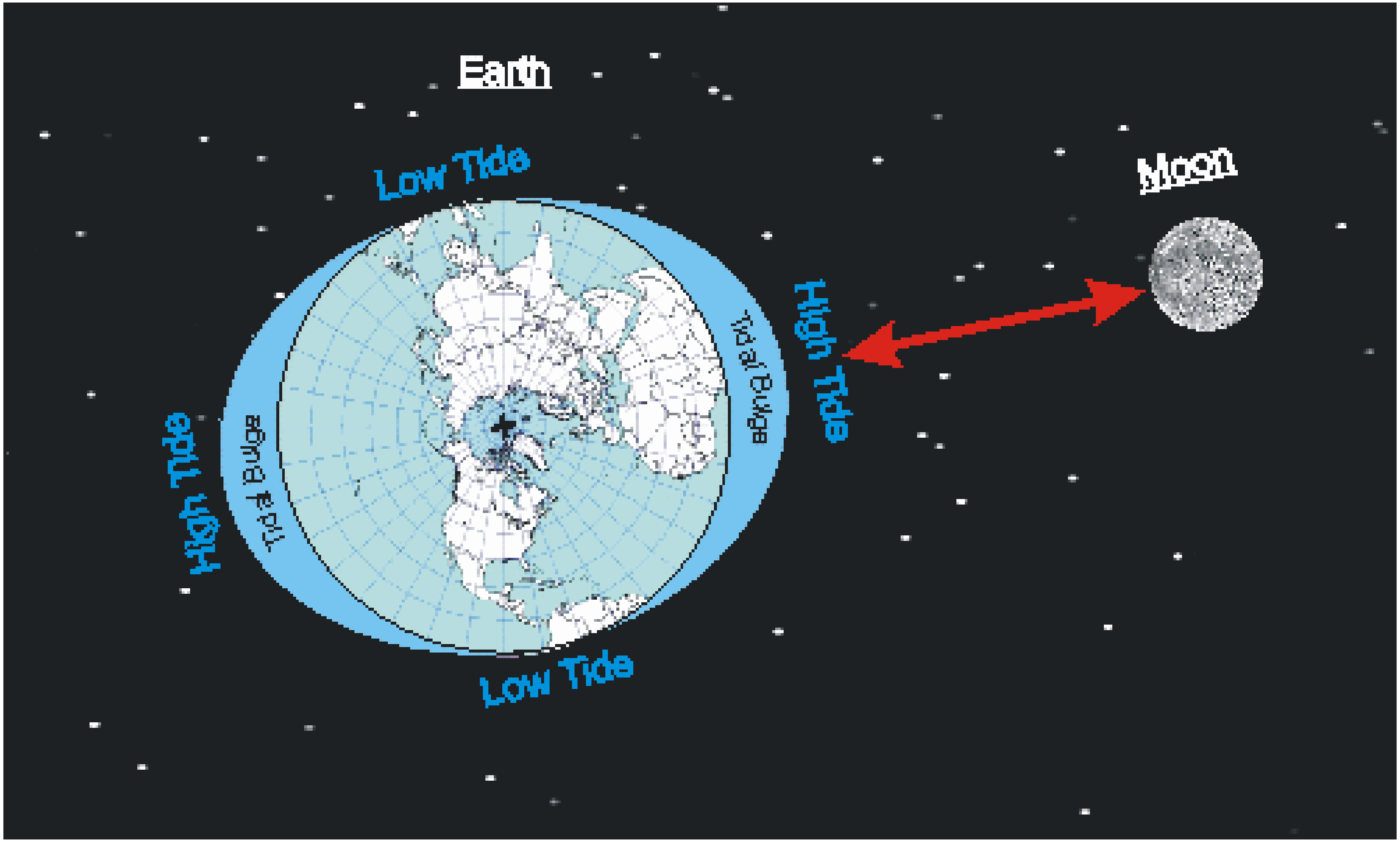
When gravitational forces from sun and moon are added together the relative motion of the Sun-Earth-Moon system causes a much more sophisticated system

This is because (viewed from the Earth) that the sun and moon orbits

- with different periods
- with different distance
- with different declination

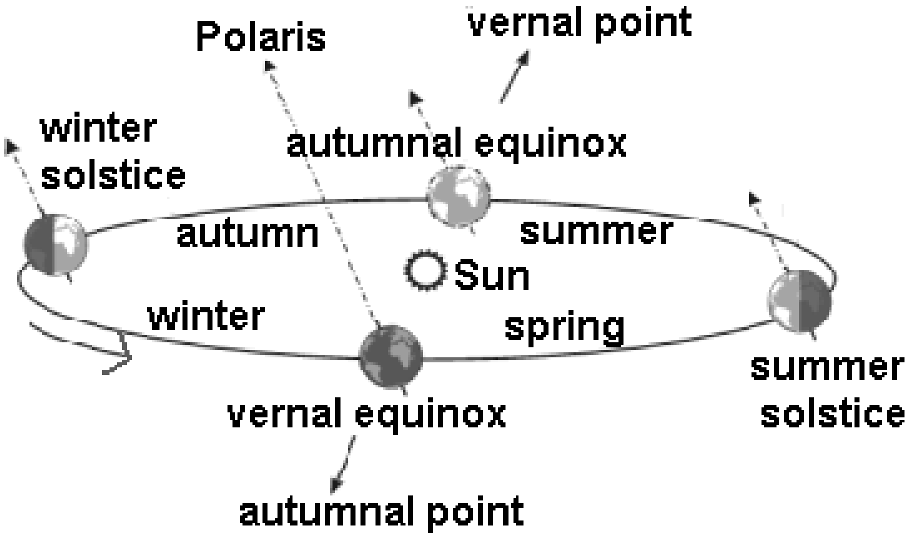
However Forces are ADDITIVE

- Major tidal constituent M_2 is created by the attraction of the Moon.
- Major solar tidal constituent is called S_2


Earth
Moon
Low Tide
Low Tide
High Tide
High Tide
Tidal Bulge
Tidal Bulge

Period

Looking from above the North Pole, th
lunar day (24.84 solar hours). With
24 solar hours. Together, it takes a
the same Sun-Moon-Earth configurat



Distance

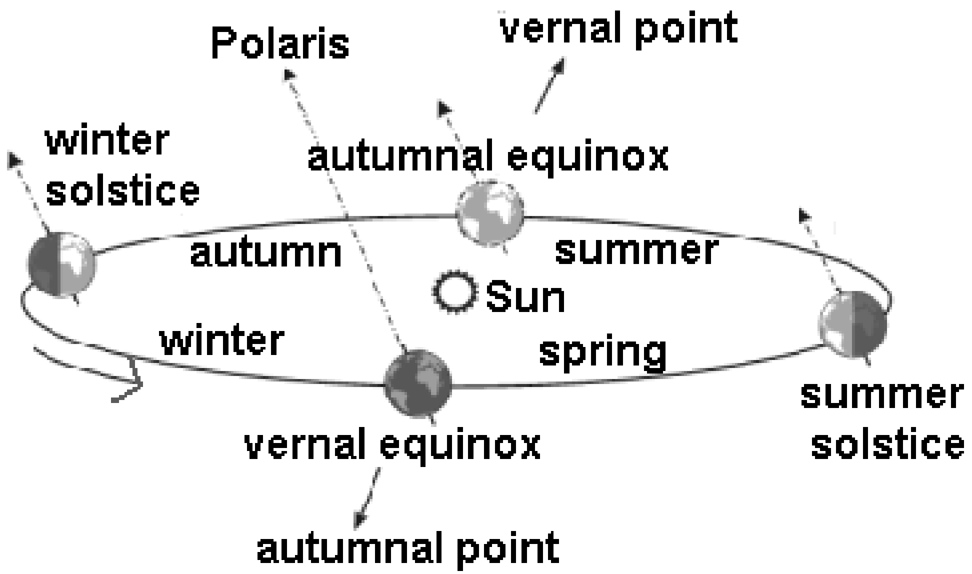
The amplitude of tides is also affected by the distance of Sun and Moon: largest when the Earth is in perihelion (closer to the Sun, 1 in 365.2596 days, or 1 per anomalistic year) and when the Moon is in perigee (closer to the Earth, 1 in 27.5546 days, or 1 per anomalistic month).

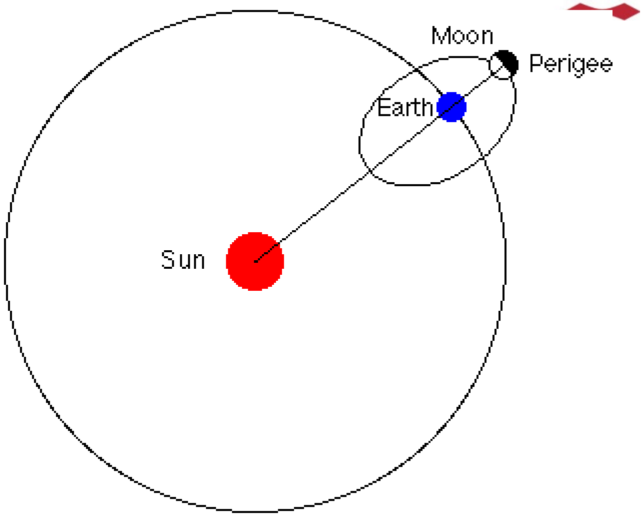
Declination

The Earth rotation axis is inclined by 23.452° to the ecliptic (rotation plane of the Earth around the Sun), with a period of 365.2422 days (tropical year).

The orbital plane of the Moon is inclined 5.145° to the ecliptic, with a period of 27.2122 days (nodical month).

Basic Frequencies

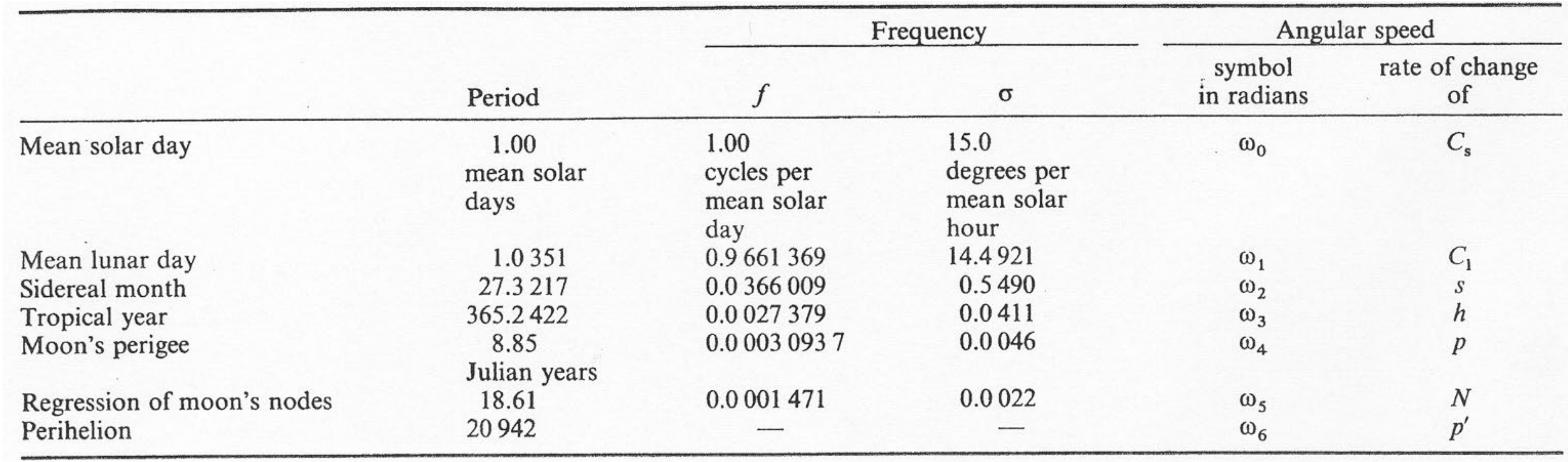




The angular speed can be described using a series of predefined frequencies

$$\omega_n = i_a\omega_1 + i_b\omega_2 + i_c\omega_3 + i_d\omega_4 + i_e\omega_5 + i_f\omega_6$$

The I's are small integer (Doodson numbers).



	Period	Frequency f	Frequency σ	Angular speed symbol in radians	Angular speed rate of change of
Mean solar day	1.00 mean solar days	1.00 cycles per mean solar day	15.0 degrees per mean solar hour	ω_0	C_s
Mean lunar day	1.0 351	0.9 661 369	14.4 921	ω_1	C_1
Sidereal month	27.3 217	0.0 366 009	0.5 490	ω_2	s
Tropical year	365.2 422	0.0 027 379	0.0 411	ω_3	h
Moon's perigee	8.85 Julian years	0.0 003 093 7	0.0 046	ω_4	p
Regression of moon's nodes	18.61	0.0 001 471	0.0 022	ω_5	N
Perihelion	20 942	—	—	ω_6	p'

Tidal constituents (Semidiurnal)

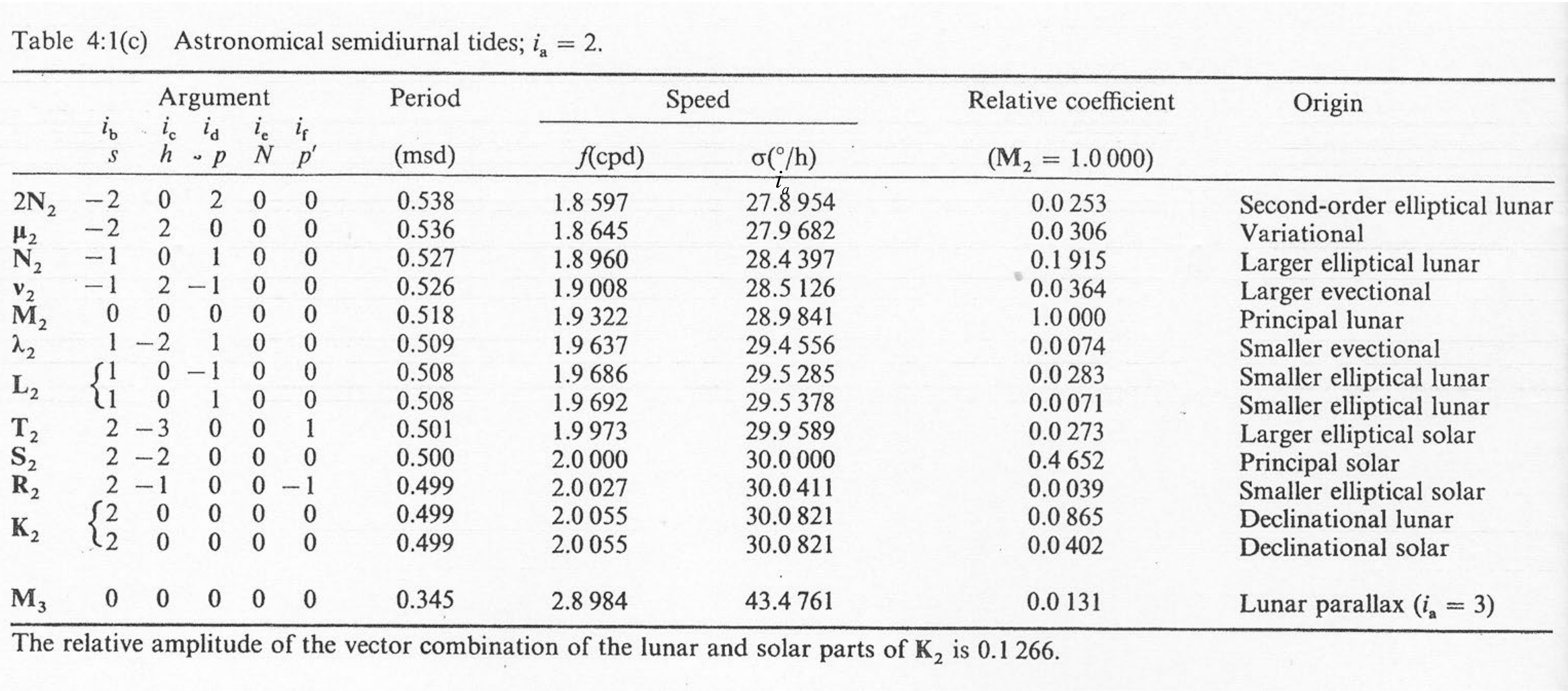


Table 4:1(c) Astronomical semidiurnal tides; $i_a = 2$.

	Argument					Period	Speed		Relative coefficient	Origin
	i_b s	i_c h	i_d p	i_e N	i_f p'	(msd)	f(cpd)	σ(°/h)	(M_2 = 1.0 000)	
$2N_2$	−2	0	2	0	0	0.538	1.8 597	27.8 954	0.0 253	Second-order elliptical lunar
μ_2	−2	2	0	0	0	0.536	1.8 645	27.9 682	0.0 306	Variational
N_2	−1	0	1	0	0	0.527	1.8 960	28.4 397	0.1 915	Larger elliptical lunar
ν_2	−1	2	−1	0	0	0.526	1.9 008	28.5 126	0.0 364	Larger evectional
M_2	0	0	0	0	0	0.518	1.9 322	28.9 841	1.0 000	Principal lunar
λ_2	1	−2	1	0	0	0.509	1.9 637	29.4 556	0.0 074	Smaller evectional
L_2	1	0	−1	0	0	0.508	1.9 686	29.5 285	0.0 283	Smaller elliptical lunar
	1	0	1	0	0	0.508	1.9 692	29.5 378	0.0 071	Smaller elliptical lunar
T_2	2	−3	0	0	1	0.501	1.9 973	29.9 589	0.0 273	Larger elliptical solar
S_2	2	−2	0	0	0	0.500	2.0 000	30.0 000	0.4 652	Principal solar
R_2	2	−1	0	0	−1	0.499	2.0 027	30.0 411	0.0 039	Smaller elliptical solar
K_2	2	0	0	0	0	0.499	2.0 055	30.0 821	0.0 865	Declinational lunar
	2	0	0	0	0	0.499	2.0 055	30.0 821	0.0 402	Declinational solar
M_3	0	0	0	0	0	0.345	2.8 984	43.4 761	0.0 131	Lunar parallax ($i_a = 3$)

The relative amplitude of the vector combination of the lunar and solar parts of K_2 is 0.1 266.

Tidal Forces (diurnal).

Diurnal tides:

Diurnal tides are created because the Earth

Rotational plane is enclined and the semi-diurnal bulge is will typically be higher on one side that on the other side.P

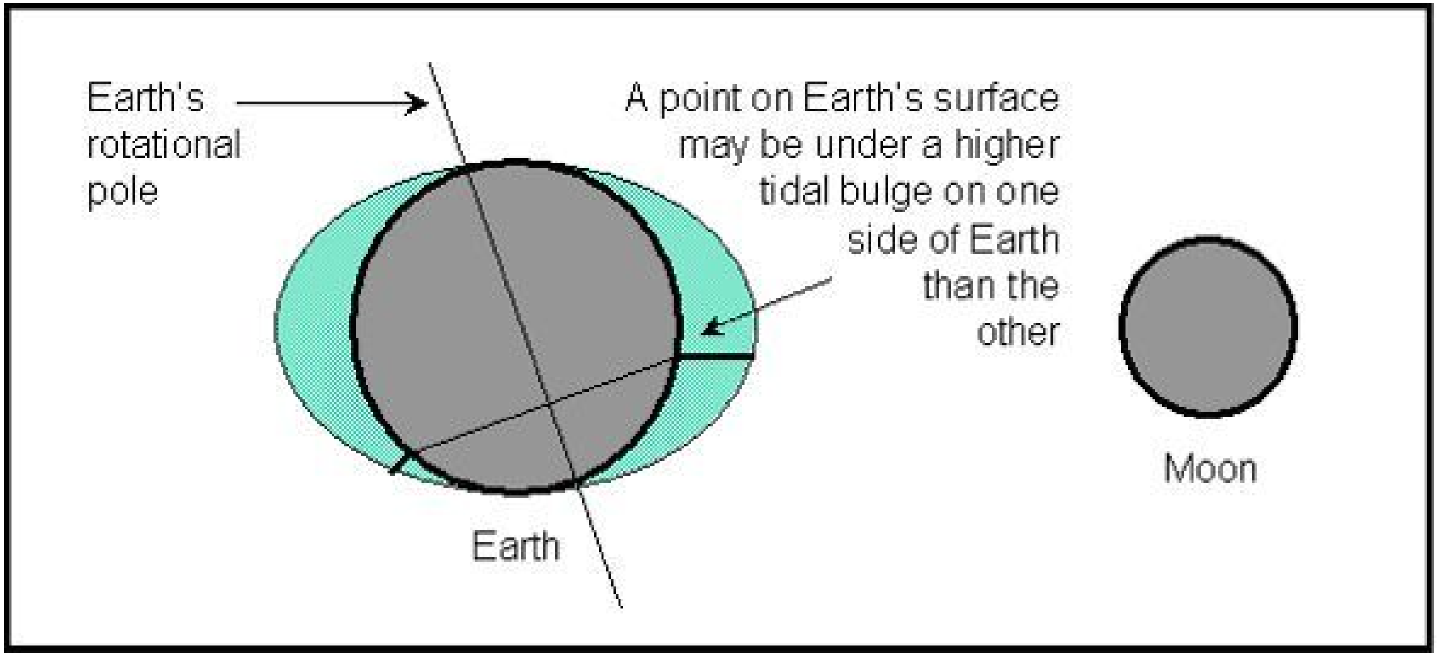
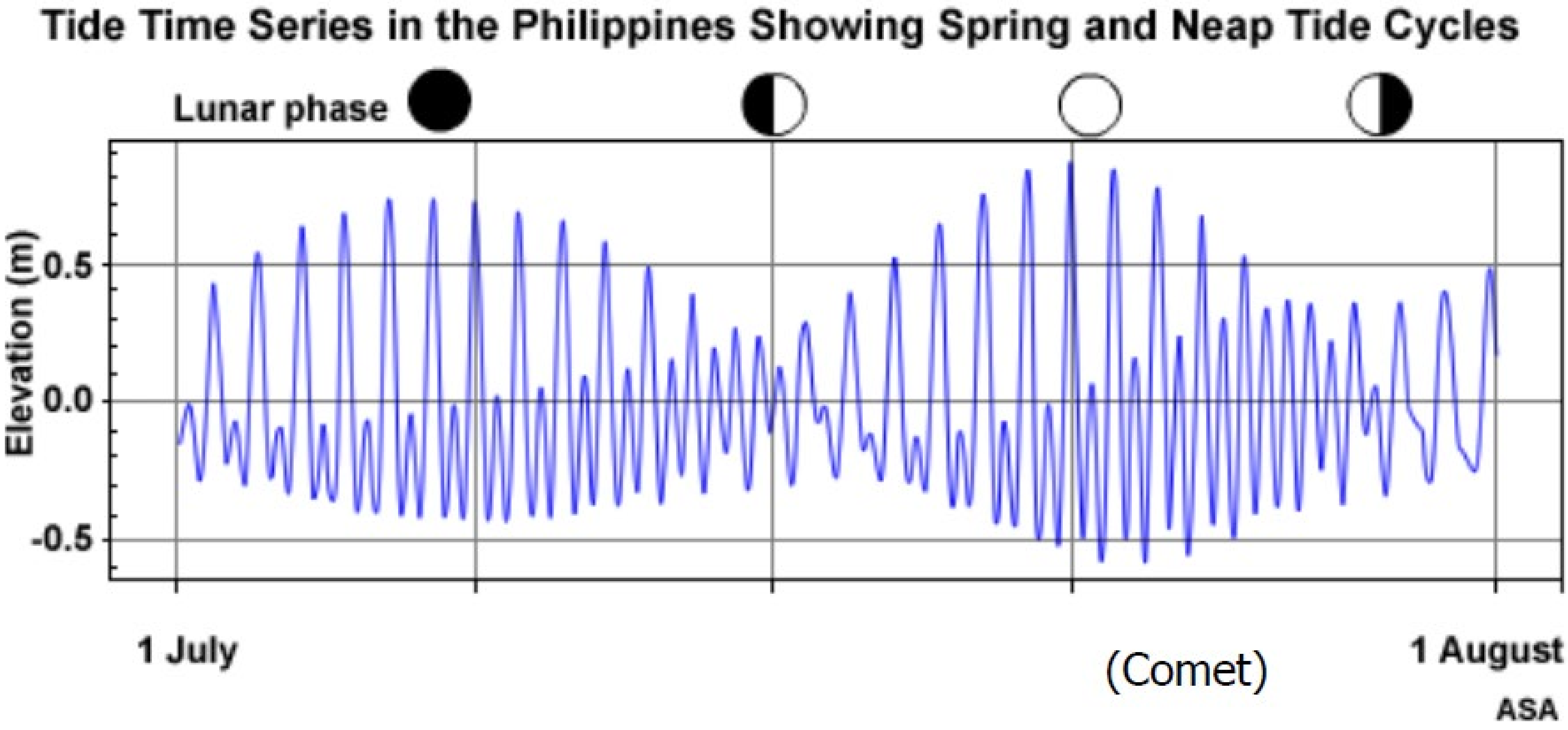


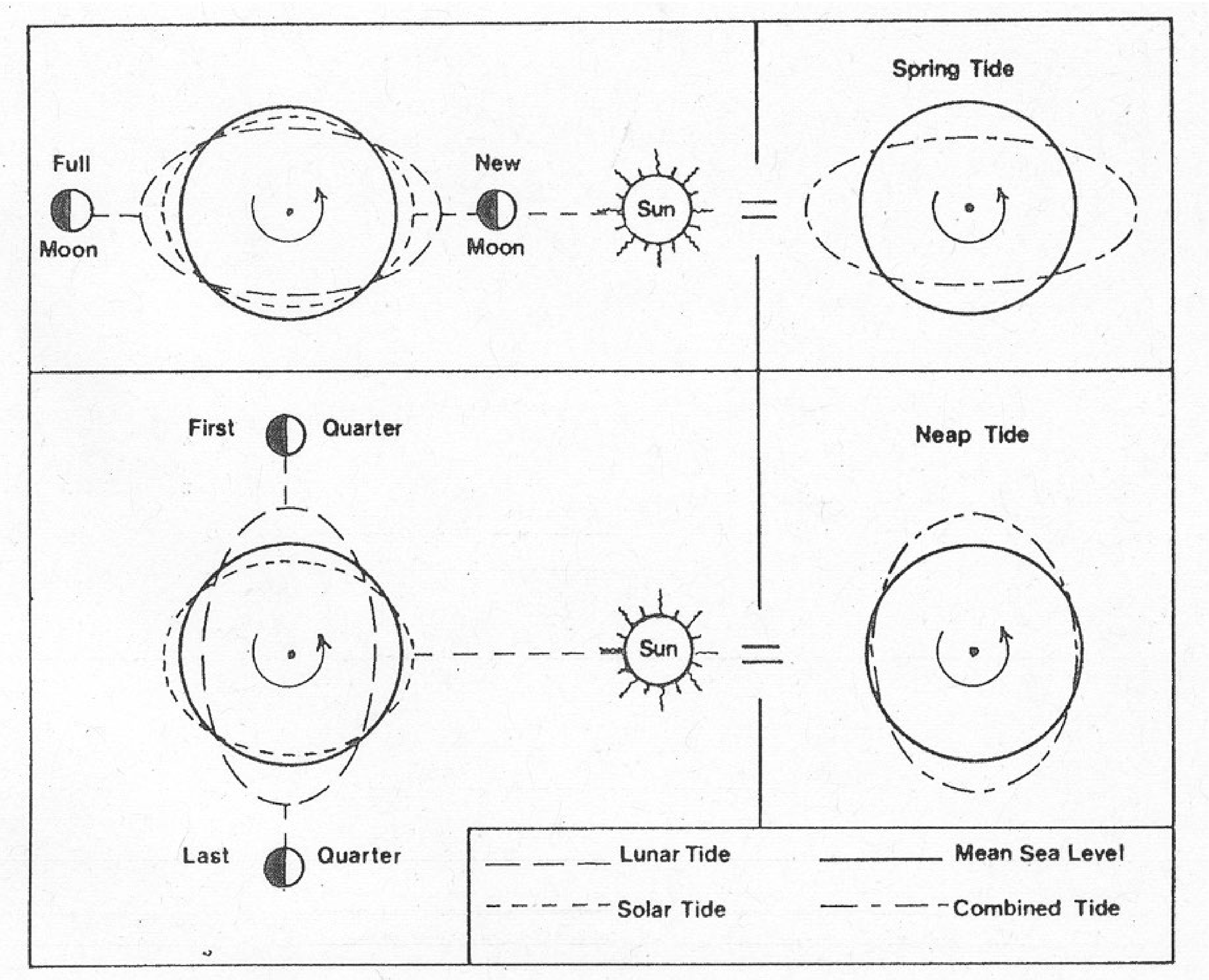
Figure 8. Diurnal (Daily) Tide Level Differences

Example



(Comet)

Tidal Forces (Long term).



Tidal constituents (Diurnal+Long)

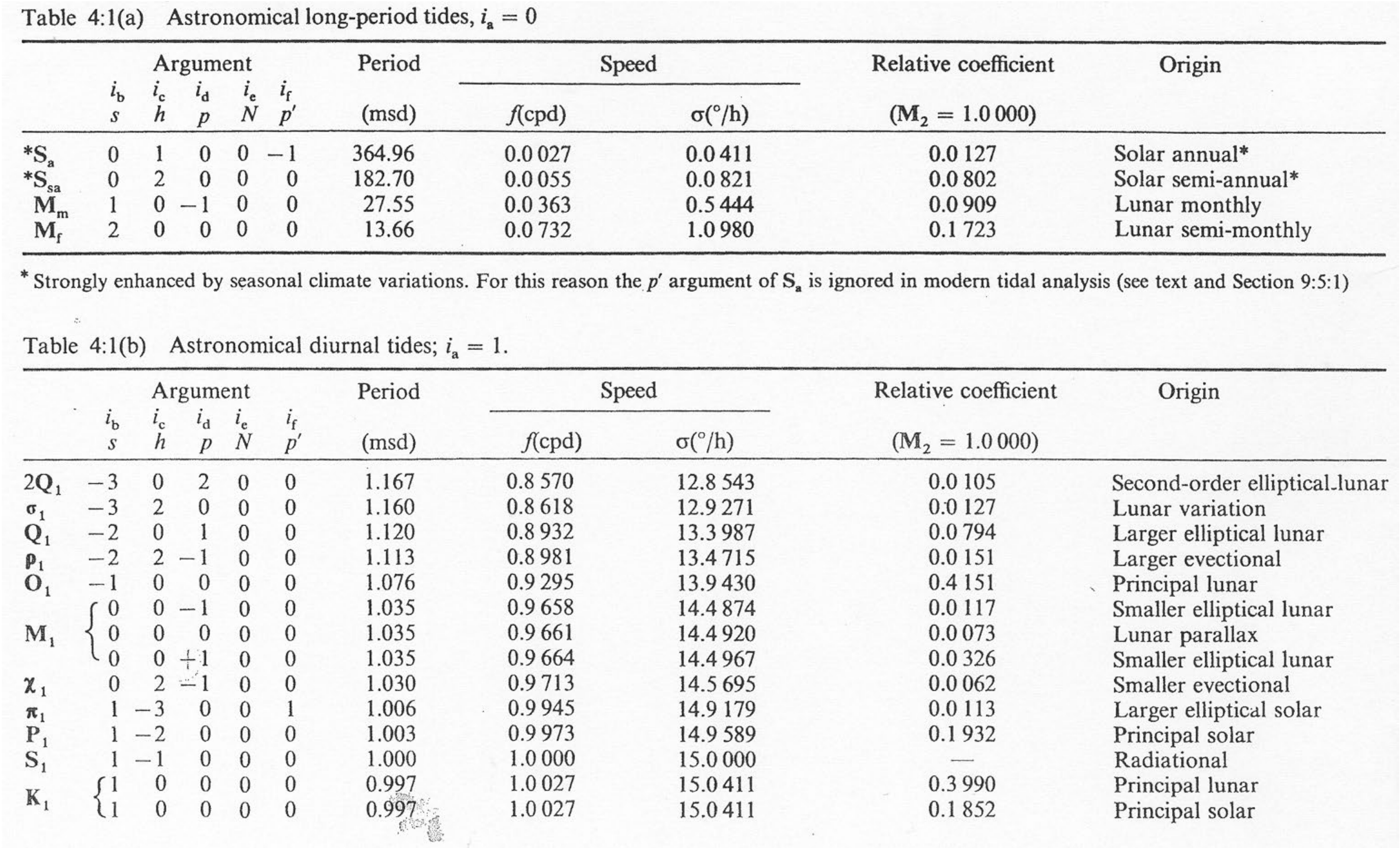


Table 4:1(a) Astronomical long-period tides, $i_a = 0$

	Argument					Period	Speed		Relative coefficient	Origin
	i_b s	i_c h	i_d p	i_e N	i_f p'	(msd)	f(cpd)	σ(°/h)	(M_2 = 1.0000)	
*S_a	0	1	0	0	−1	364.96	0.0027	0.0411	0.0127	Solar annual*
*S_{sa}	0	2	0	0	0	182.70	0.0055	0.0821	0.0802	Solar semi-annual*
M_m	1	0	−1	0	0	27.55	0.0363	0.5444	0.0909	Lunar monthly
M_f	2	0	0	0	0	13.66	0.0732	1.0980	0.1723	Lunar semi-monthly

* Strongly enhanced by seasonal climate variations. For this reason the p' argument of S_a is ignored in modern tidal analysis (see text and Section 9:5:1)

Table 4:1(b) Astronomical diurnal tides; $i_a = 1$.

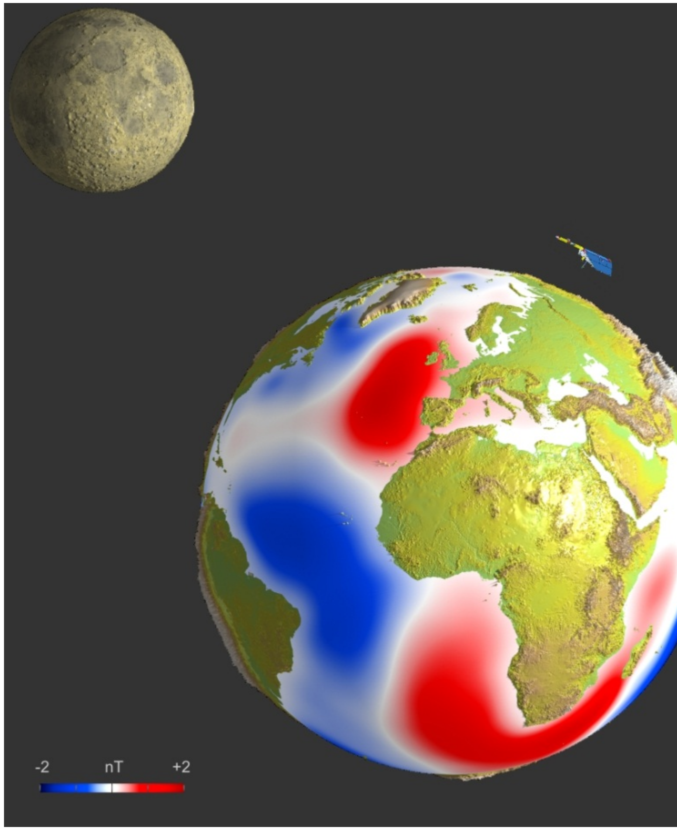
	Argument					Period	Speed		Relative coefficient	Origin
	i_b s	i_c h	i_d p	i_e N	i_f p'	(msd)	f(cpd)	σ(°/h)	(M_2 = 1.0000)	
$2Q_1$	−3	0	2	0	0	1.167	0.8570	12.8543	0.0105	Second-order elliptical lunar
σ_1	−3	2	0	0	0	1.160	0.8618	12.9271	0.0127	Lunar variation
Q_1	−2	0	1	0	0	1.120	0.8932	13.3987	0.0794	Larger elliptical lunar
ρ_1	−2	2	−1	0	0	1.113	0.8981	13.4715	0.0151	Larger evectional
O_1	−1	0	0	0	0	1.076	0.9295	13.9430	0.4151	Principal lunar
M_1	0	0	−1	0	0	1.035	0.9658	14.4874	0.0117	Smaller elliptical lunar
	0	0	0	0	0	1.035	0.9661	14.4920	0.0073	Lunar parallax
	0	0	+1	0	0	1.035	0.9664	14.4967	0.0326	Smaller elliptical lunar
χ_1	0	2	−1	0	0	1.030	0.9713	14.5695	0.0062	Smaller evectional
π_1	1	−3	0	0	1	1.006	0.9945	14.9179	0.0113	Larger elliptical solar
P_1	1	−2	0	0	0	1.003	0.9973	14.9589	0.1932	Principal solar
S_1	1	−1	0	0	0	1.000	1.0000	15.0000	—	Radiational
K_1	1	0	0	0	0	0.997	1.0027	15.0411	0.3990	Principal lunar
	1	0	0	0	0	0.997	1.0027	15.0411	0.1852	Principal solar

Solid Earth Tides and Ocean Tide

Tides acts on the Earth surface through acting on the

solid earth and the ocean on the Earth

$$T_{total} = \mathrm{Tearth} + T\ \mathrm{ocean}\ (+ T_{\mathrm{loading}})$$



Solid Earth reacts instantaneous and is described accuratelyThorough mathematical (the tides moves in equilibrium)

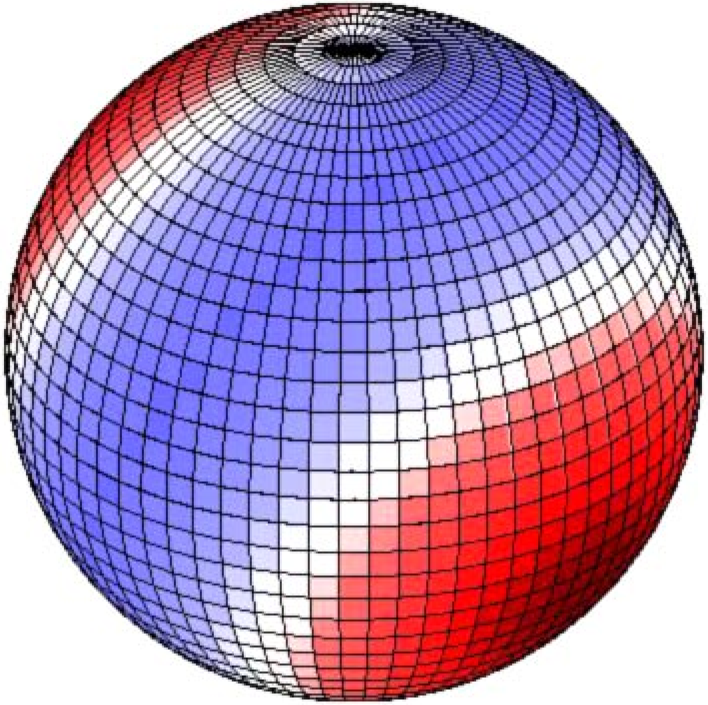
Ocean Tides has to follow hydrodynamic equations as they can not move freely due to the continents. They become complex to describe

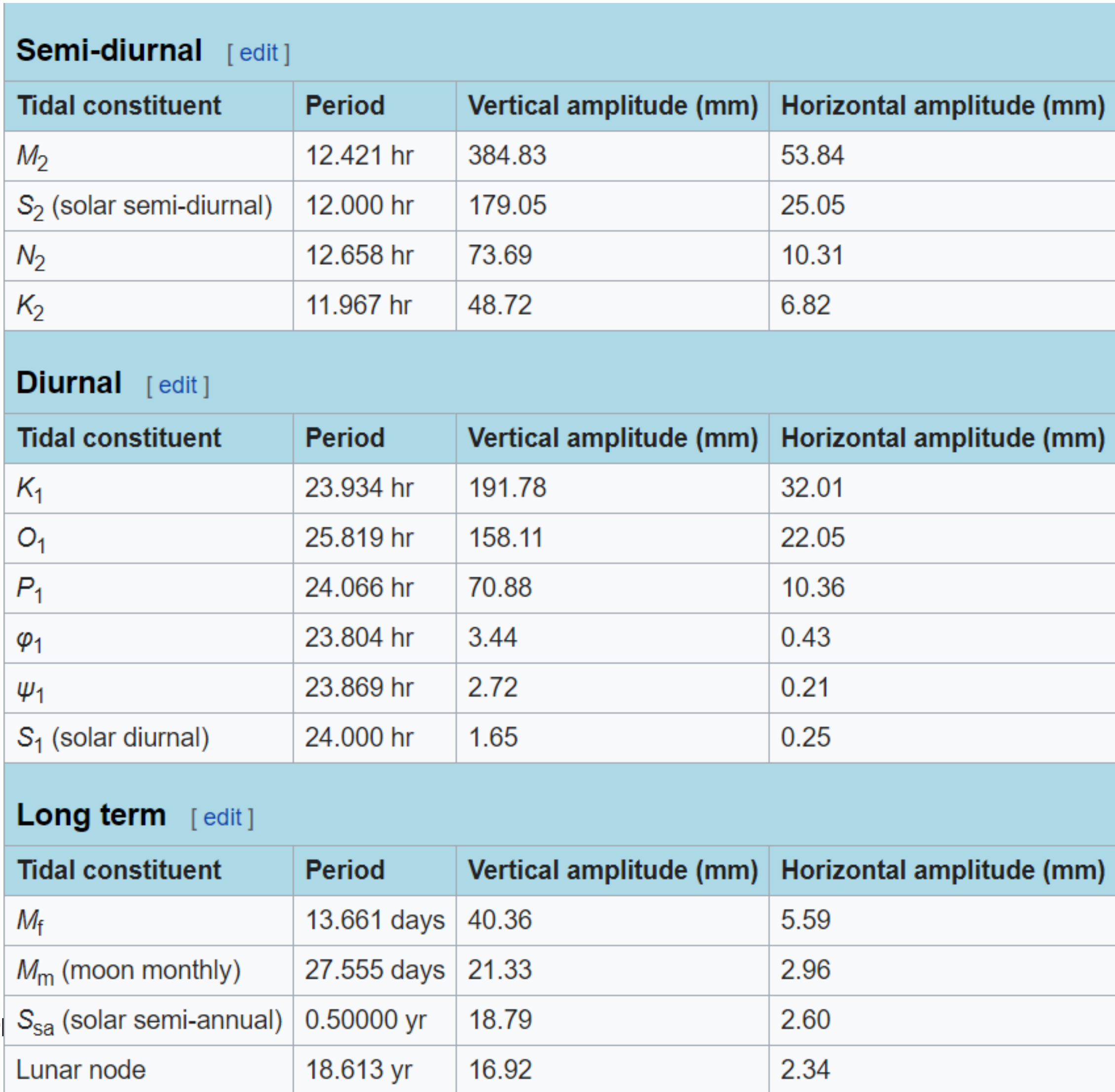
smaller Ocean Loading

Loading Tides are due to the weight of the ocean tides pressing down the underlaying crust.

Solid Earth Tides.

Instantaneous - ELASTIC
Moon/sun not in Zenit
so also minor horizontal
Ignore thin crust on Earth





Semi-diurnal [edit]

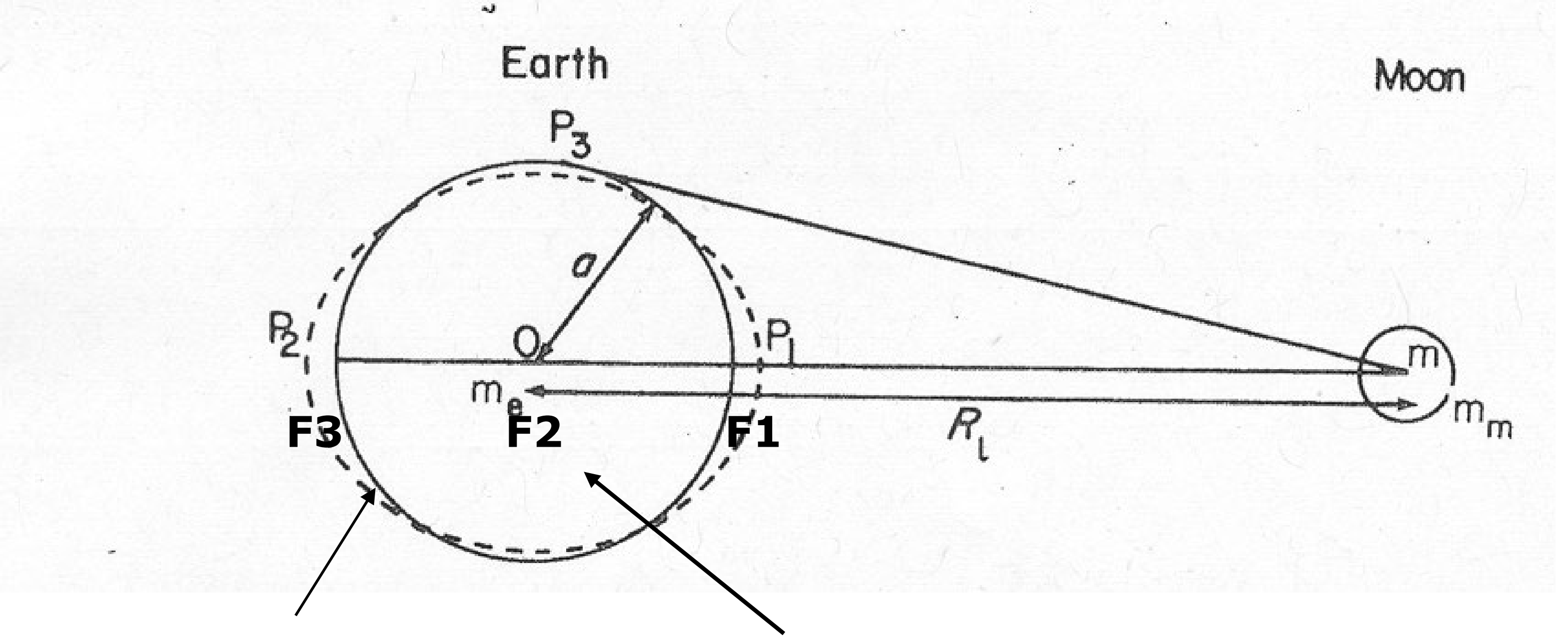
Tidal constituent	Period	Vertical amplitude (mm)	Horizontal amplitude (mm)
M_2	12.421 hr	384.83	53.84
S_2 (solar semi-diurnal)	12.000 hr	179.05	25.05
N_2	12.658 hr	73.69	10.31
K_2	11.967 hr	48.72	6.82

Diurnal [edit]

Tidal constituent	Period	Vertical amplitude (mm)	Horizontal amplitude (mm)
K_1	23.934 hr	191.78	32.01
O_1	25.819 hr	158.11	22.05
P_1	24.066 hr	70.88	10.36
φ_1	23.804 hr	3.44	0.43
ψ_1	23.869 hr	2.72	0.21
S_1 (solar diurnal)	24.000 hr	1.65	0.25

Long term [edit]

Tidal constituent	Period	Vertical amplitude (mm)	Horizontal amplitude (mm)
M_f	13.661 days	40.36	5.59
M_m (moon monthly)	27.555 days	21.33	2.96
S_{sa} (solar semi-annual)	0.50000 yr	18.79	2.60
Lunar node	18.613 yr	16.92	2.34



Force $= G\dfrac{m_e\, m_m}{(R_1+a)^2}$ $\qquad G\dfrac{m_e\, m_m}{(R_1)^2}$ $\qquad G\dfrac{m_e\, m_m}{(R_1-a)^2}$

Difference (F1-F2) is the Tide producing force

$$Gm_em_m\left[\frac{1}{(R_1-a)^2}-\frac{1}{(R_1)^2}\right] \qquad \frac{Gm_em_m}{R_1^2}\left[\frac{1}{(1-\frac{a}{R_1})^2}-1\right]$$

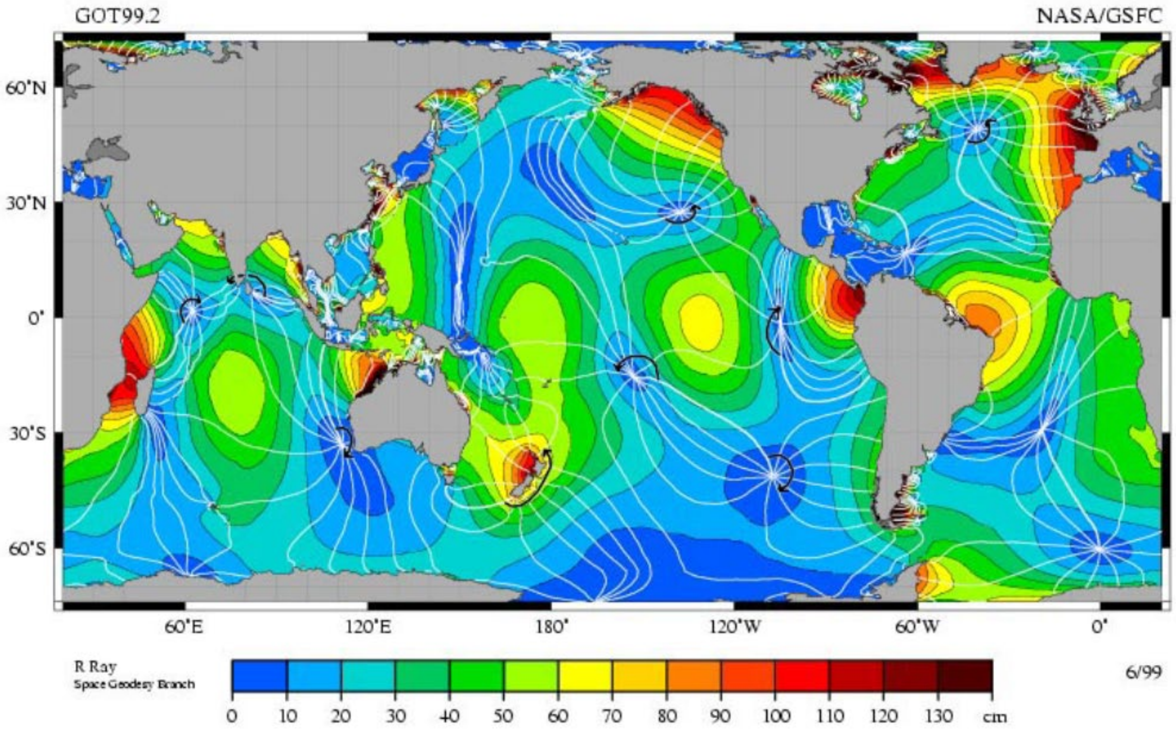
Ocean Tides

Ocean Tides has to follow hydrodynamic equations as they can not move freely due to the continents. They become complex to describe

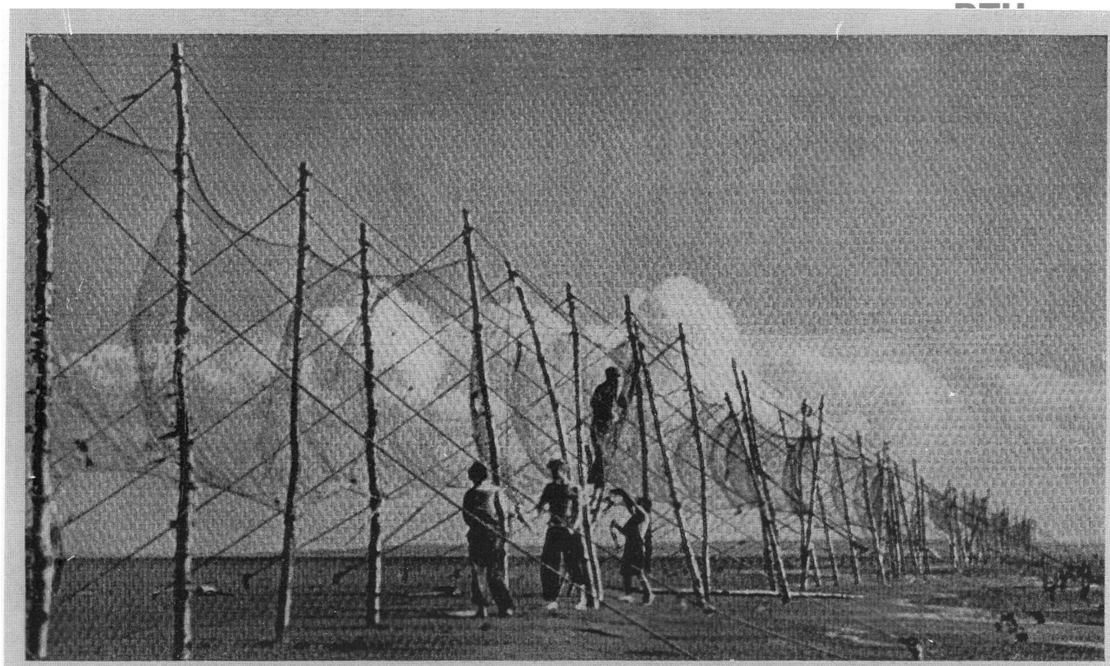
The propagation speed of a shallow water wave is

$$c = \frac{L}{T} = \sqrt{gh}$$

where L is the wavelength, T is the period, g is the gravitational acceleration and h the water depth.



Coriolis Force causes the tidal wave to be deflected



(*Canadian Information Service*)

PLATE XV—Graphic illustration of the Bay of Fundy tides. At high tide these nets are covered by deep water

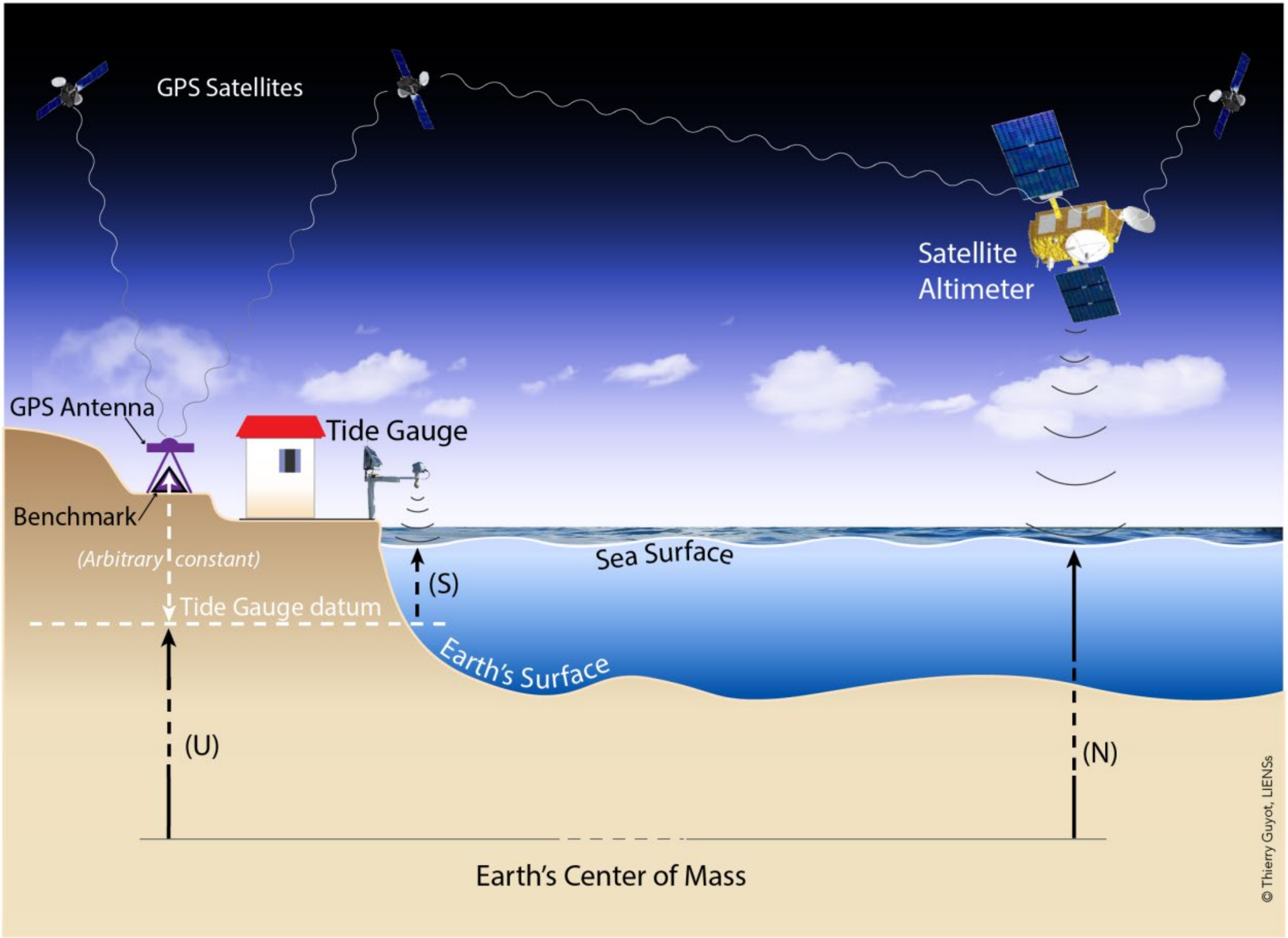
Bay of Funy (Canada): 11.7 m

Port of Bristol (UK): 9.6 m

Granville (France): 8.2 m

How to observe Tides.

- Satellite altimetry observe ALL: Earth+Ocean Tides + loading tide.
  Tide gauge only observe Ocean tide……



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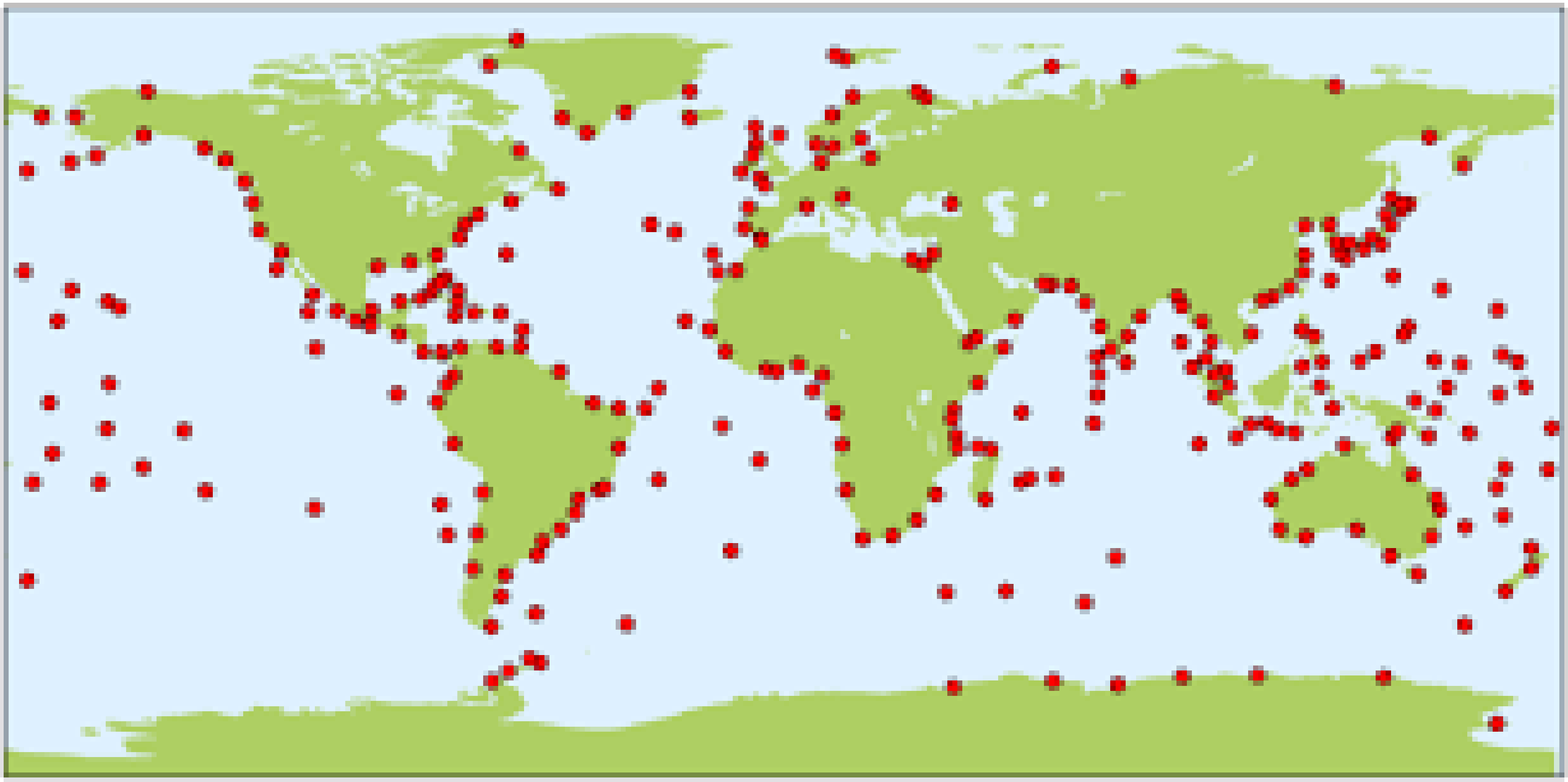
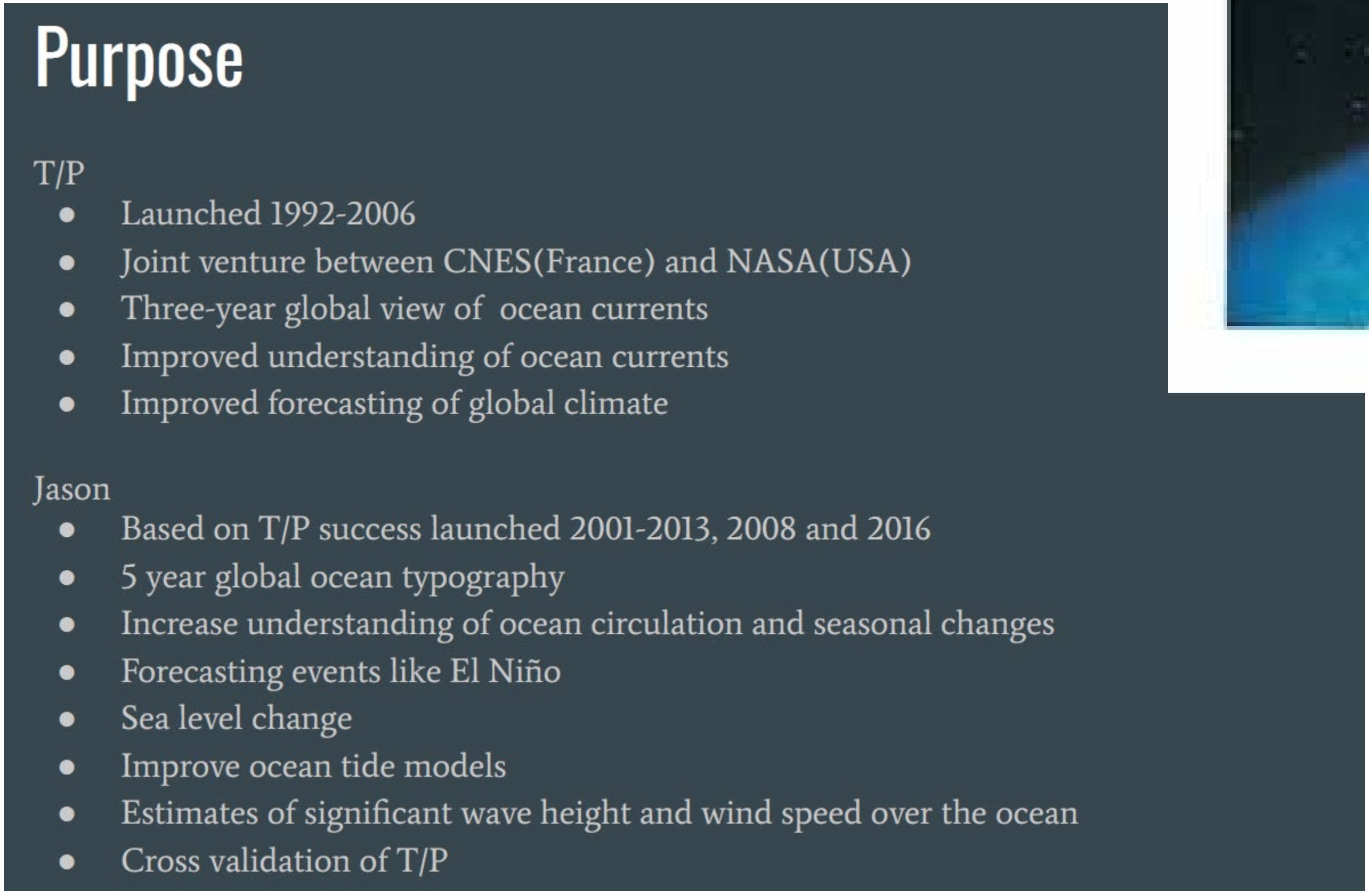
Global tide gauge network

- Its not relly global (80% on Northern Hemisphere).
- Only on coast (and giving relative sea level change).



Topex Poseidon launched in 1992 to study tides.

- Repeat period is 9.9156 days.

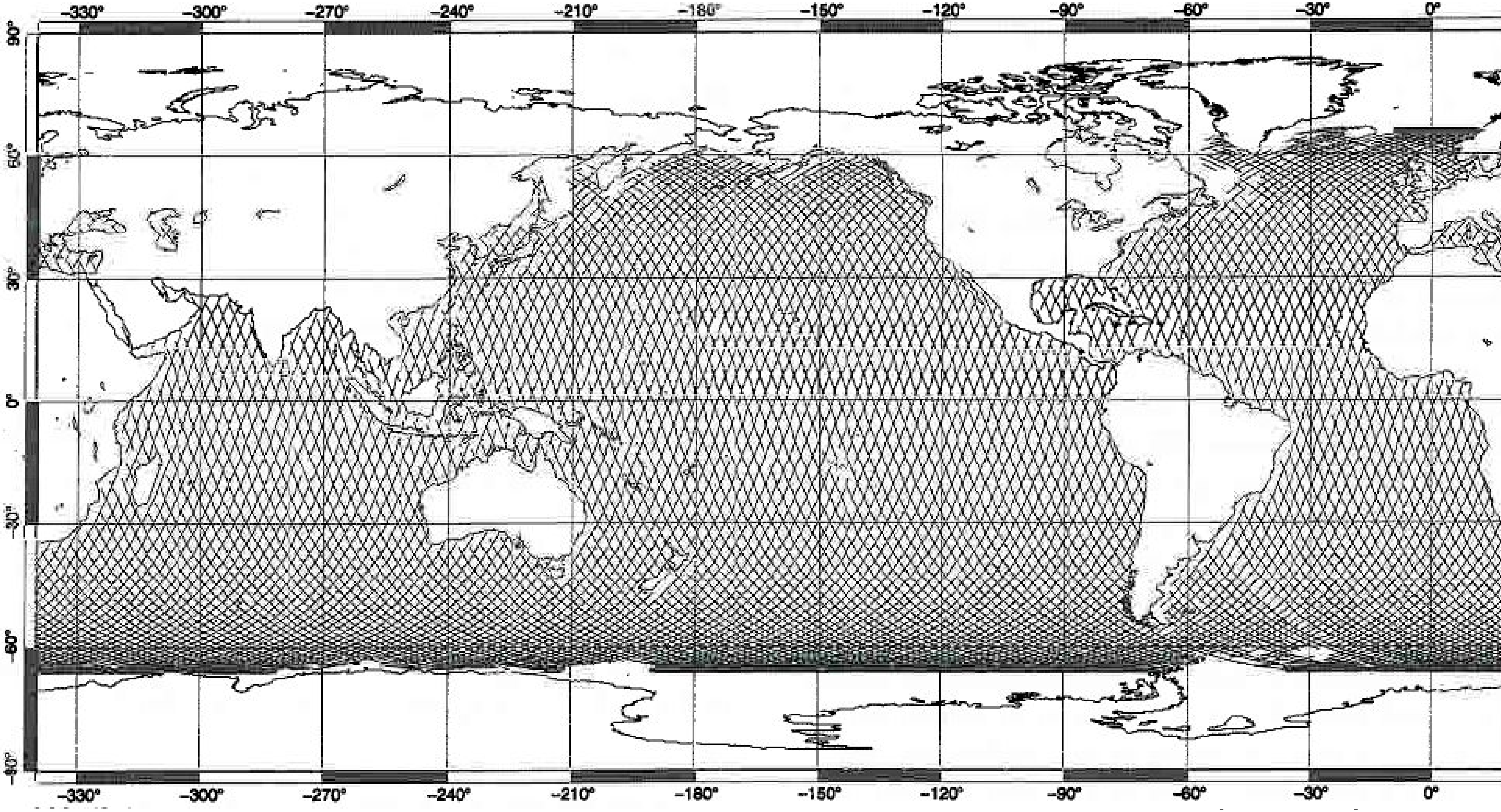


Purpose

T/P
- Launched 1992-2006
- Joint venture between CNES(France) and NASA(USA)
- Three-year global view of ocean currents
- Improved understanding of ocean currents
- Improved forecasting of global climate

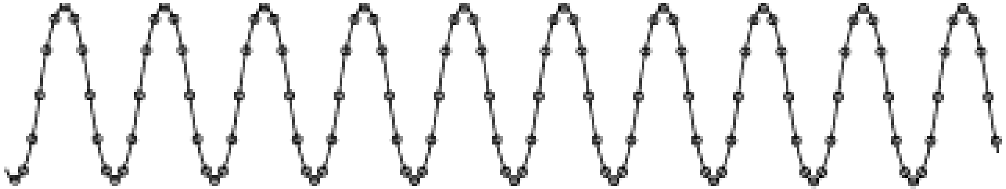
Jason
- Based on T/P success launched 2001-2013, 2008 and 2016
- 5 year global ocean typography
- Increase understanding of ocean circulation and seasonal changes
- Forecasting events like El Niño
- Sea level change
- Improve ocean tide models
- Estimates of significant wave height and wind speed over the ocean
- Cross validation of T/P

Regular sampling (9.9156) days



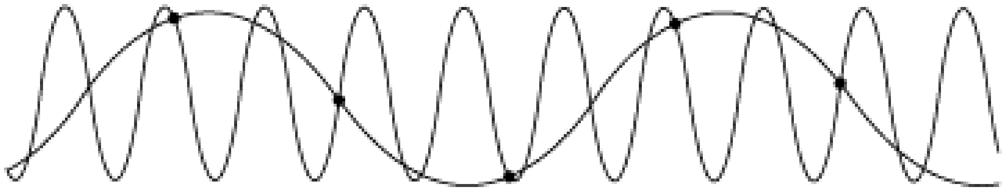
Sampling

Tide Gauge: High temporal sampling



Adequately Sampled Signal

Satellite altimetry: low temporal sampling =>Aliasing



Aliased Signal Due to Undersampling

Nyquist frequency of Satellite fN = fs / 2. Inversely it says that wavelength must be equal of longer than twice the sampling period

Basically you must sample a signal twice within one period to sample it.

So for Topes/Jason the period must be longer than 2 x 9.9156 days.

If you sample at 2 times the signal frequency you will get no signal.

P(S2) = 0.5 days. Topex/Poseidon samples at PS=9.9156 days

After 9.9156 days the signal is sampled by Ps-nP(S2) = -0.0844 d,

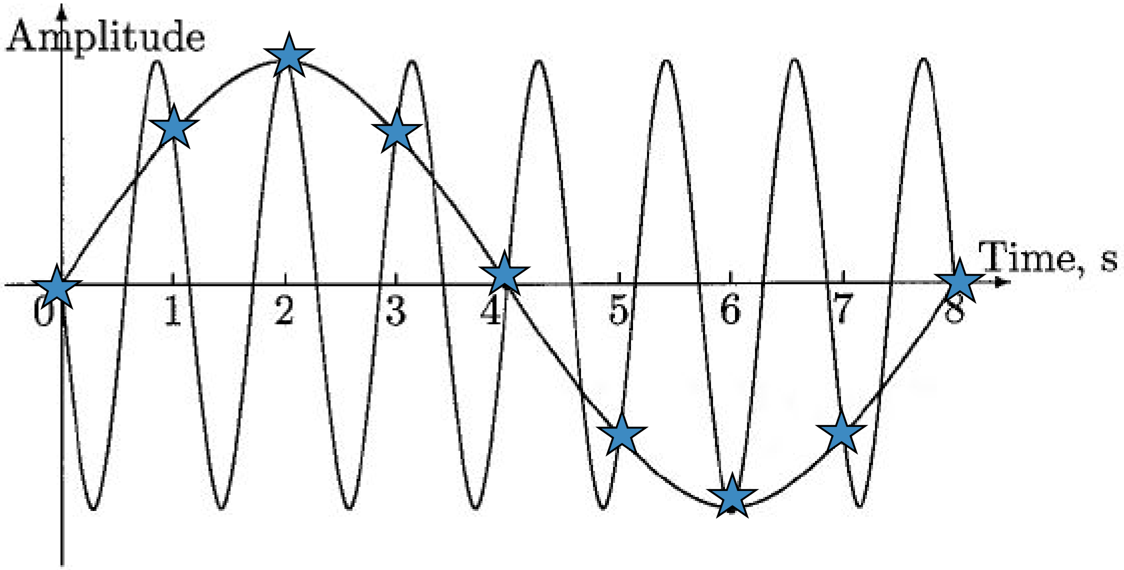
Where n is 20 full cycles of S2. In one day S2 phase change is 2 * 360 ° = 720 °

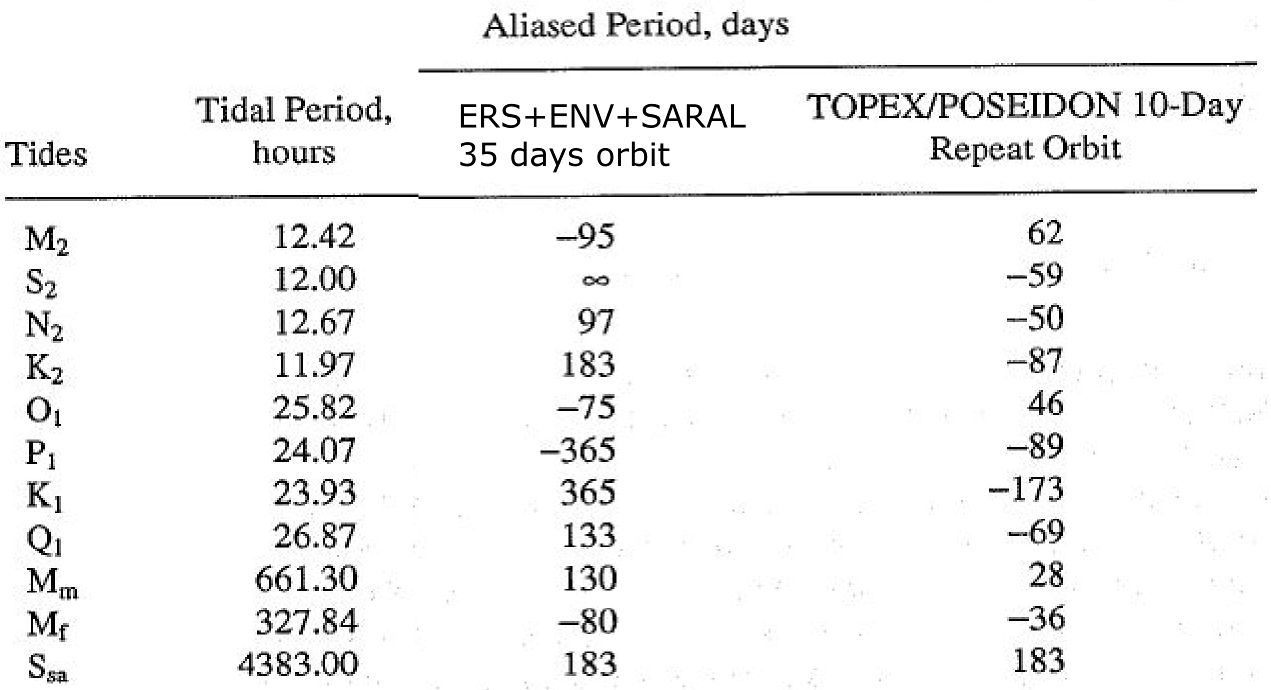
So 0.0844 days correspond to a phase change of 60.76°

So a full 360 ° signal is obtained after 360/60.76* 9.9156 d = 58.76days

Sampling:

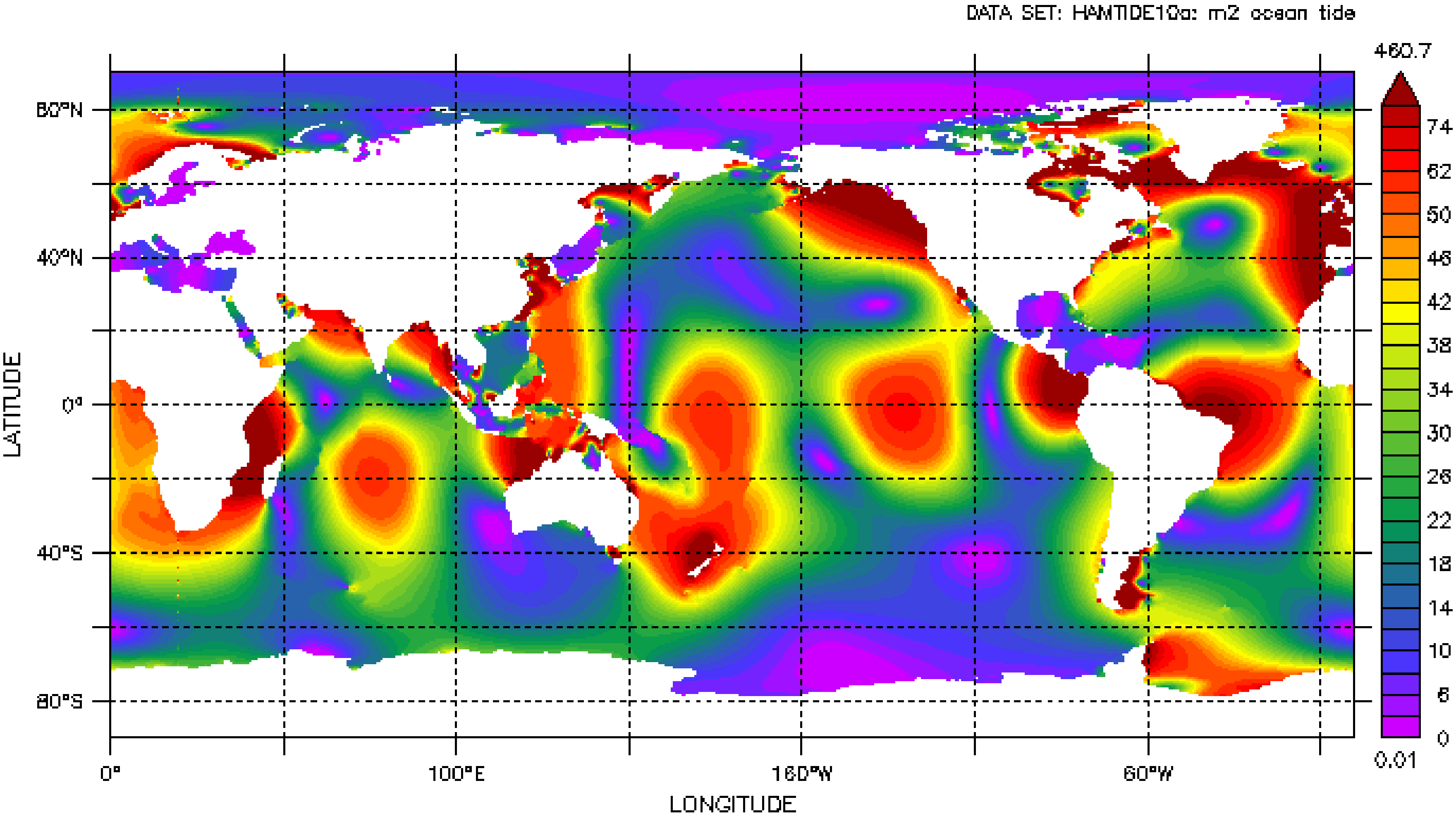
Satellite Altimetry Alias Periods



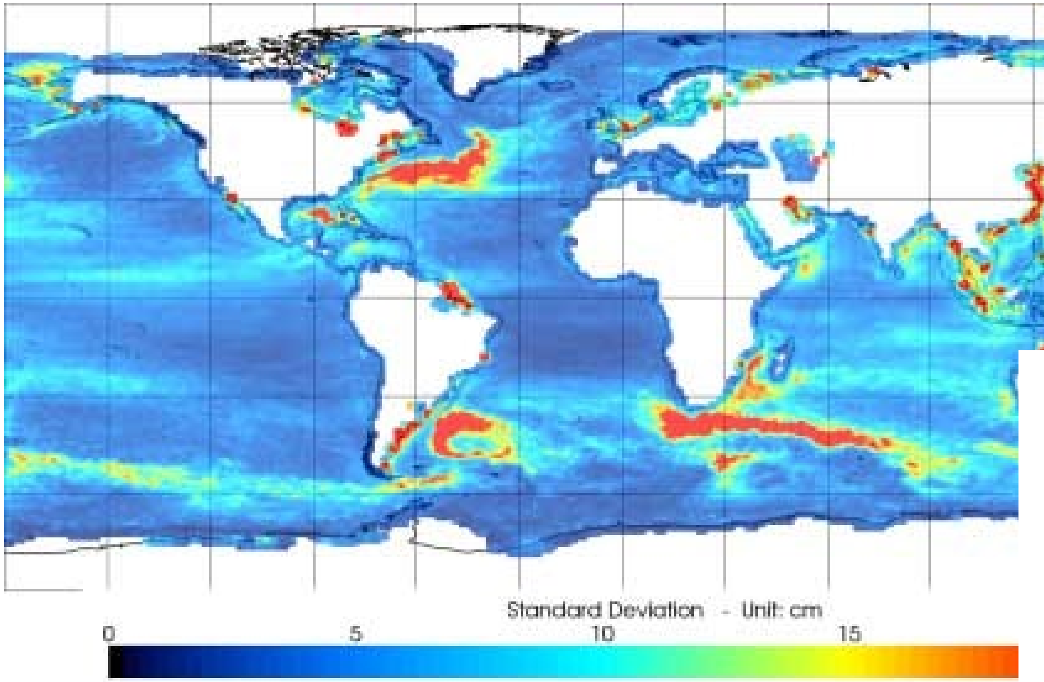


Tides	Tidal Period, hours	Aliased Period, days	
		ERS+ENV+SARAL 35 days orbit	TOPEX/POSEIDON 10-Day Repeat Orbit
M_2	12.42	–95	62
S_2	12.00	∞	–59
N_2	12.67	97	–50
K_2	11.97	183	–87
O_1	25.82	–75	46
P_1	24.07	–365	–89
K_1	23.93	365	–173
Q_1	26.87	133	–69
M_m	661.30	130	28
M_f	327.84	–80	–36
S_{sa}	4383.00	183	183

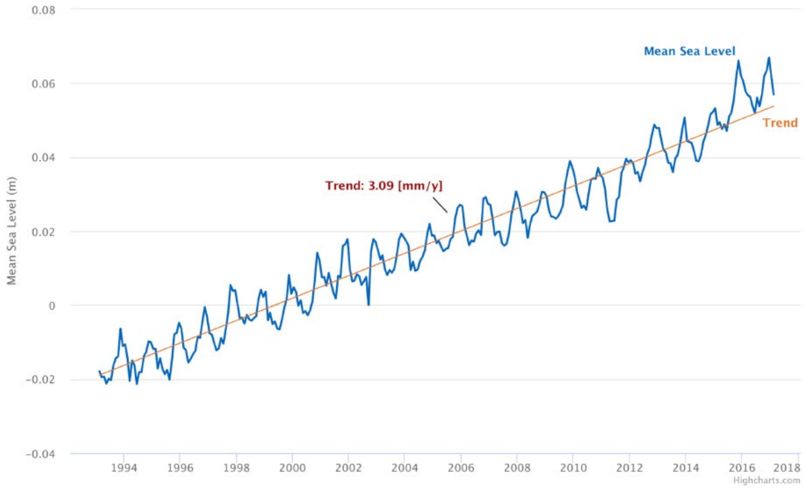
Today We have very Good tide models FES14, DTU10, GOT4.10. We use these as standard so we dont have to bother.



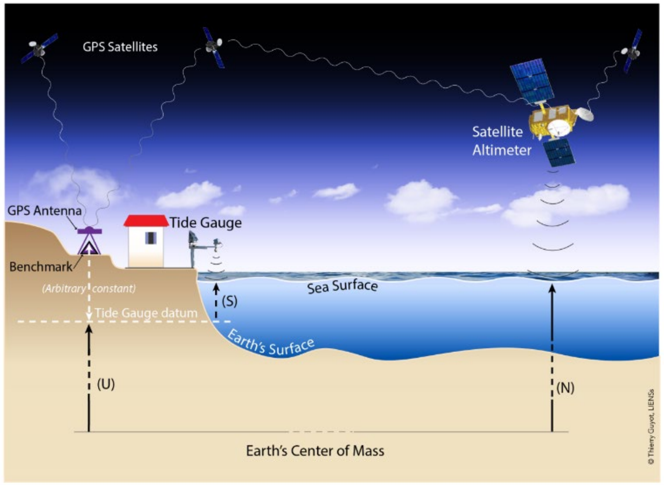
Take tides out and look at sea level variations and sea level change



Global mean sea level (GMSL) time series and estimated trend from multi-mission satellite altimetry (Jan 1993-Feb 2017)



Loading Tides



Earth reacts instantaneous (elastic) to load of water.

$$\mathbf{a}(\mathbf{r}) = \int_A \rho\, \mathbf{Z}(\mathbf{r}')\, G|\mathbf{r} - \mathbf{r}|\, dA$$

a is loading amplitude, Z is tides, rho is density of water, G is Greens function.
Greens function determins how much the earth deforms by loading it it with 1 kg.
Greens function is tabulated for different Earth models.

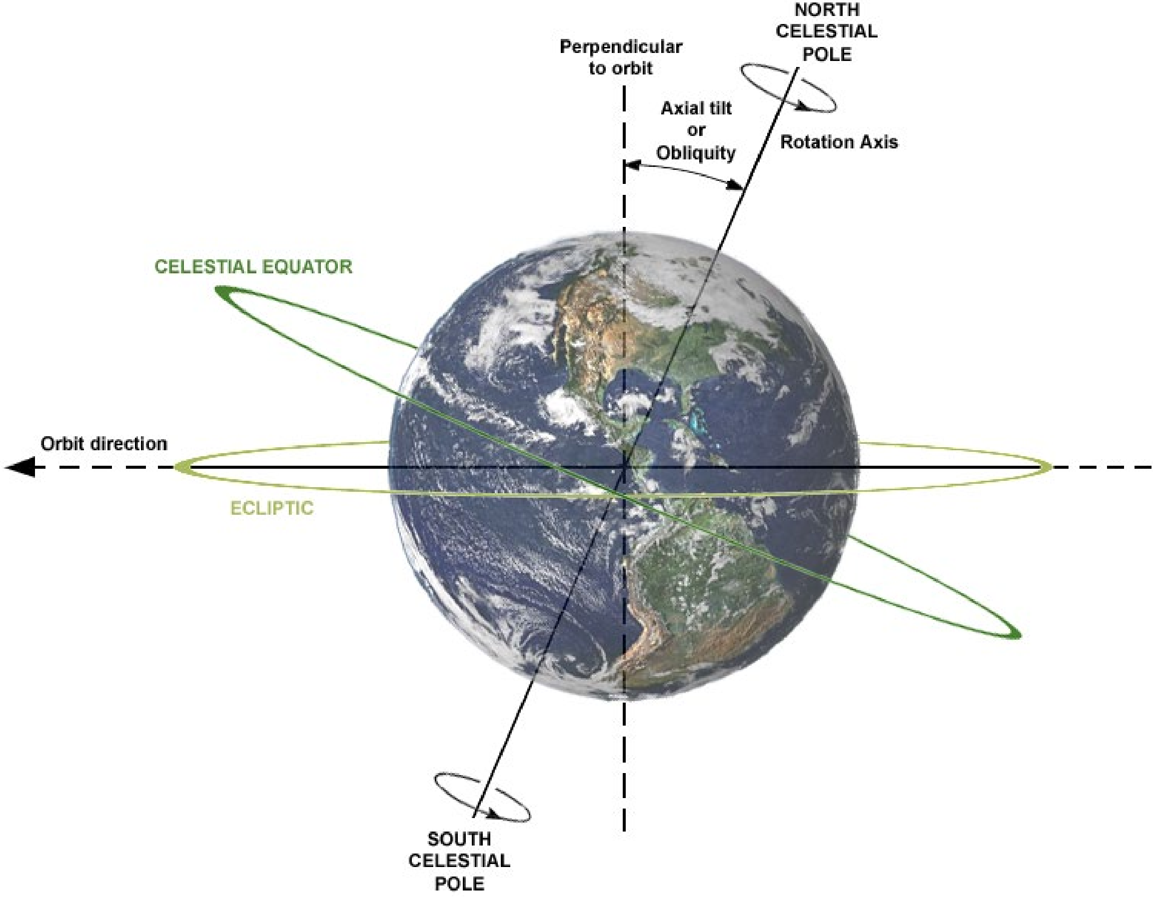
	Vertical Loading
Bay of Funy (Canada): 11.7 m	**- 45 cm**
Port of Bristol (UK): 9.6 m	**- 40 cm**
Granville (France): 8.2 m	**- 36 cm**

Tides (both) Add Torque on Earth - L(t)

Torge, Geodesy, 4ed, pp 351

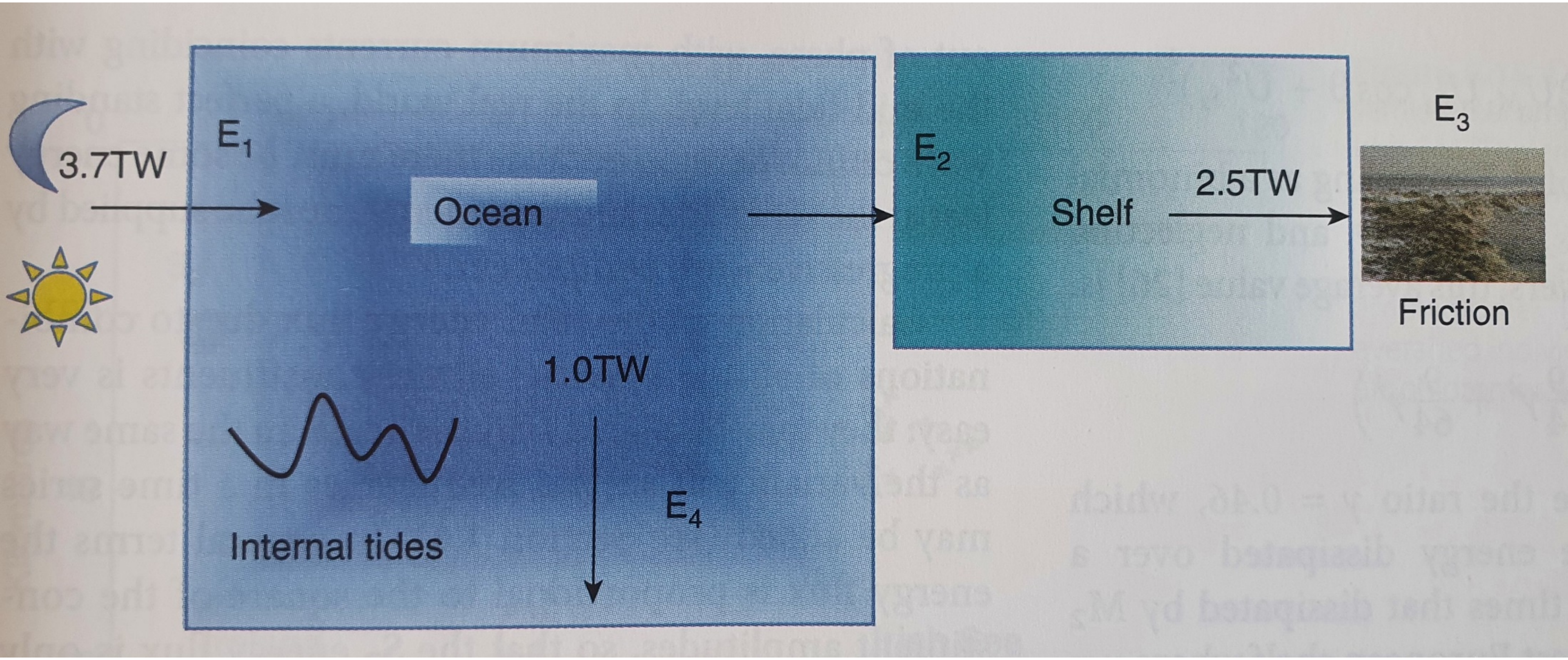
$$\frac{d}{dt}\mathbf{H}(t) + \boldsymbol{\omega}(t) \times \mathbf{H}(t) = \mathbf{L}(t)$$

$$\mathbf{H}(t) = \mathbf{I}(t) \cdot \boldsymbol{\omega}(t) + \mathbf{h}(t).$$



The first term $\mathbf{I}(t) \cdot \boldsymbol{\omega}(t)$ describes the angular momentum of a rigid body, where the tensor of inertia $\mathbf{I}(t)$ contains the time variable mass elements ("mass term"). The second term $\mathbf{h}(t)$ represents the angular momentum relative to the body rotation, and contains the mass elements' velocities with respect to the reference system ("motion term"). The equation (8.15a), (8.15b) is known as Euler-Liouville equation. It relates the – well-known – gravitational forces of moon, sun and planets, cf. [3.5.2], to mass redistributions and mass motions within the Earth's body. After linearization, the solution of (8.15) provides polar motion and length of day (LOD) variations as functions of their excitations, and allows the study of Earth's rotation variations.

Tides Add huge Friction on Earth.

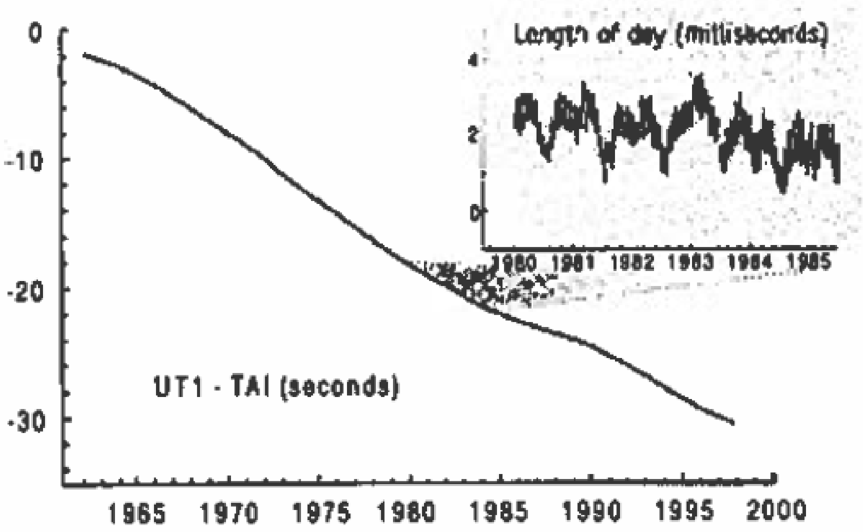


Must manifiest in some way?

Earth Rotation / Length of Day and Polar Motion.

Tides bulge exerts a force on the Earth changing Length of Day slightly. During the last 620 million years the period of rotation of the earth (length of a day) has increased from 21.9 hours to 24 hours; In this period the Earth has lost 17% of its rotational energy.

Tidal Friction increasing the LOD by 2ms / century.
Tides produce variations in LOD of 1 ms with tidal periods (annual and monthly periods.
Seasonal variations < 0.5 ms due to atmosphere, water on land and ice budget variations.

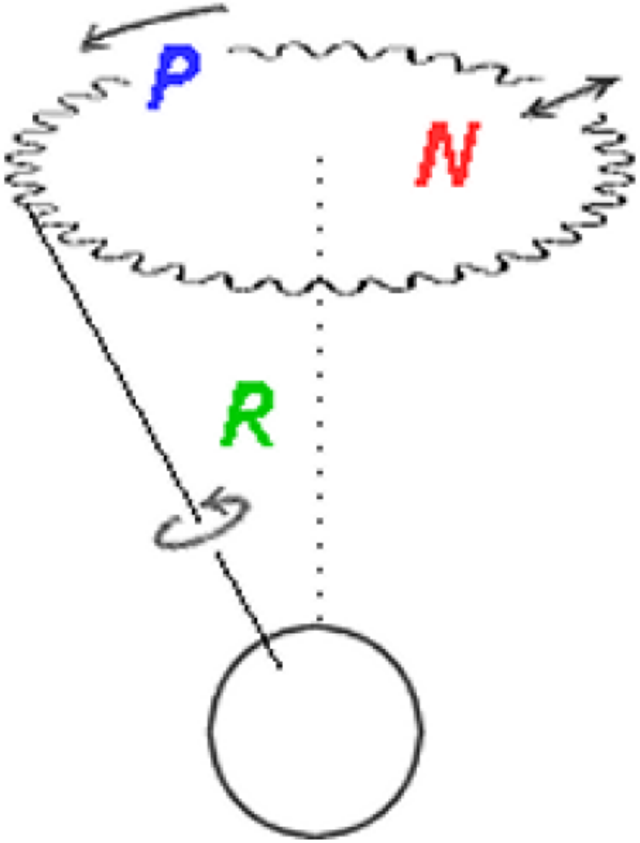


KLIMA

Dansk undergrund afslører: For 500 millioner år siden var et døgn to timer kortere

Precession and nutation

External forces cause the so-called *precession and nutation* – variations of the location of the rotational axis



- Figure from Wikipedia

Polar motion

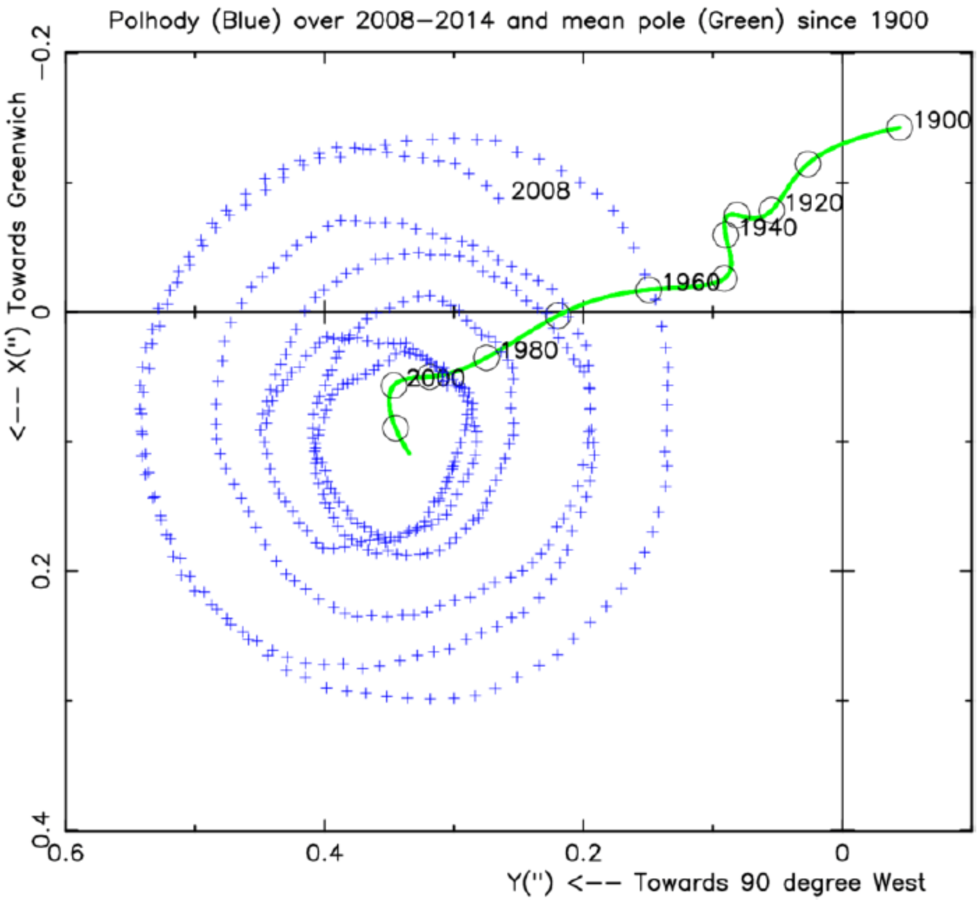


Figure: IERS Annual Report, 2013.

Anna Jensen (lecture 1)

Outline

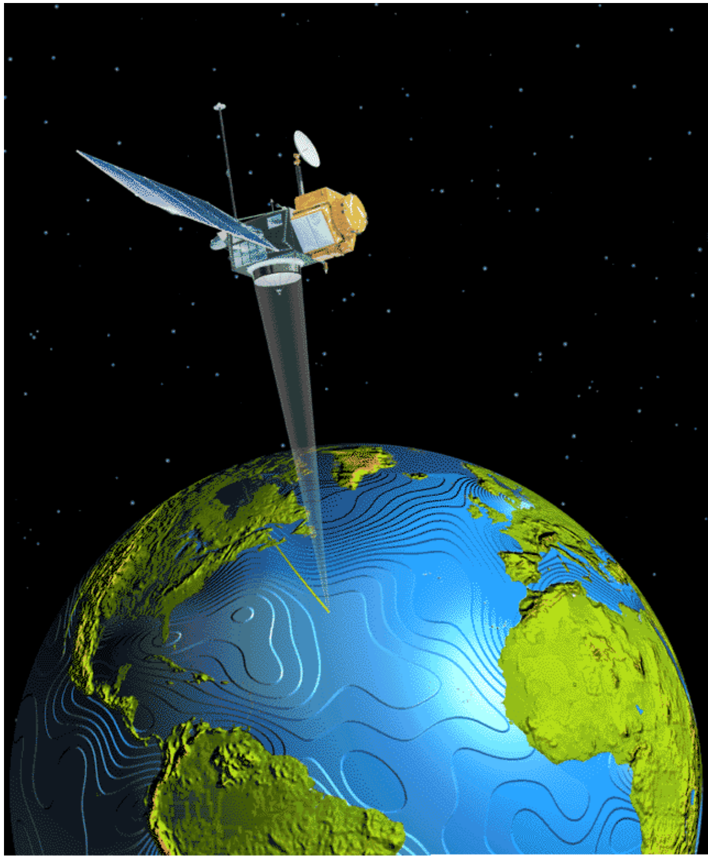
Timevariable changes in geodesy.

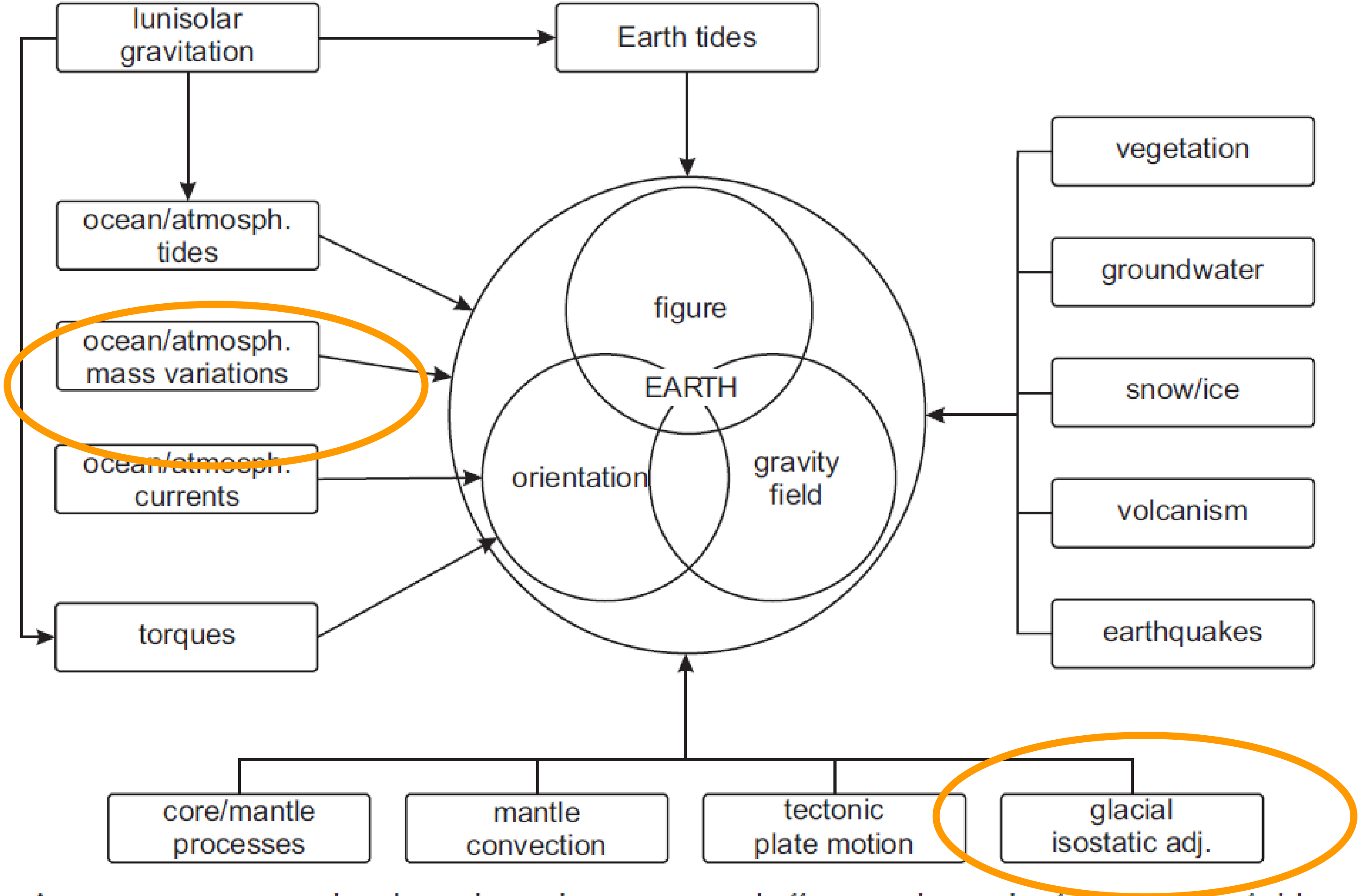
Earth and Ocean Tides

- Sea level change (Tadea)

- Global Isostatic Adjustment +
- Present Day ice loading (Carsten)

Assignment





lunisolar gravitation

Earth tides

ocean/atmosph. tides

ocean/atmosph. mass variations

ocean/atmosph. currents

torques

figure

EARTH

orientation

gravity field

vegetation

groundwater

snow/ice

volcanism

earthquakes

core/mantle processes

mantle convection

tectonic plate motion

glacial isostatic adj.

Fig. 8.14: Astronomical and geophysical processes and effects on the Earth's figure, gravity field and orientation.

Torge Geodesy, 4th edition, De Grüyer

Sea level change and crustal deformation

Recent sea level change (By Tadea Veng).

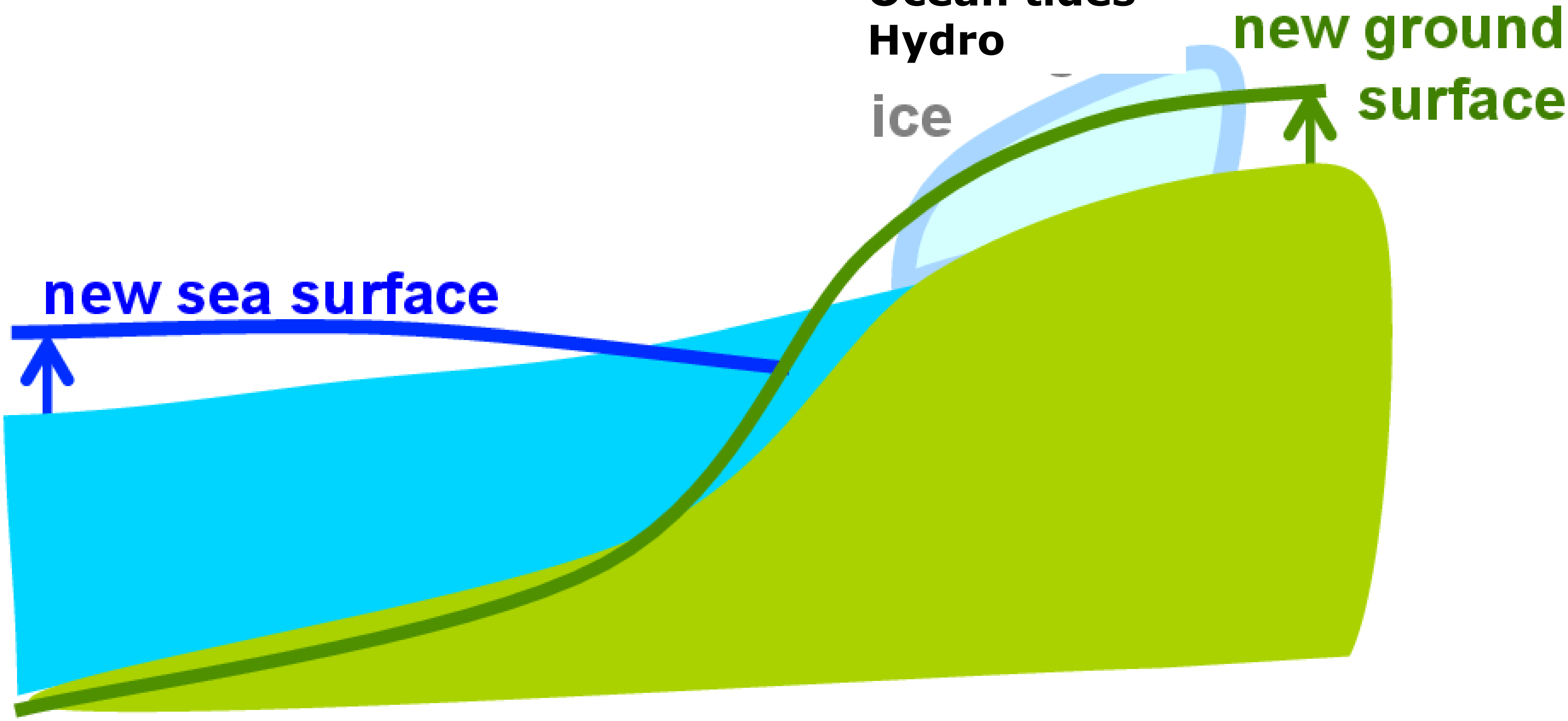
Vertical Crustal Deformation (by Carsten Ludwigsen).

End

backup

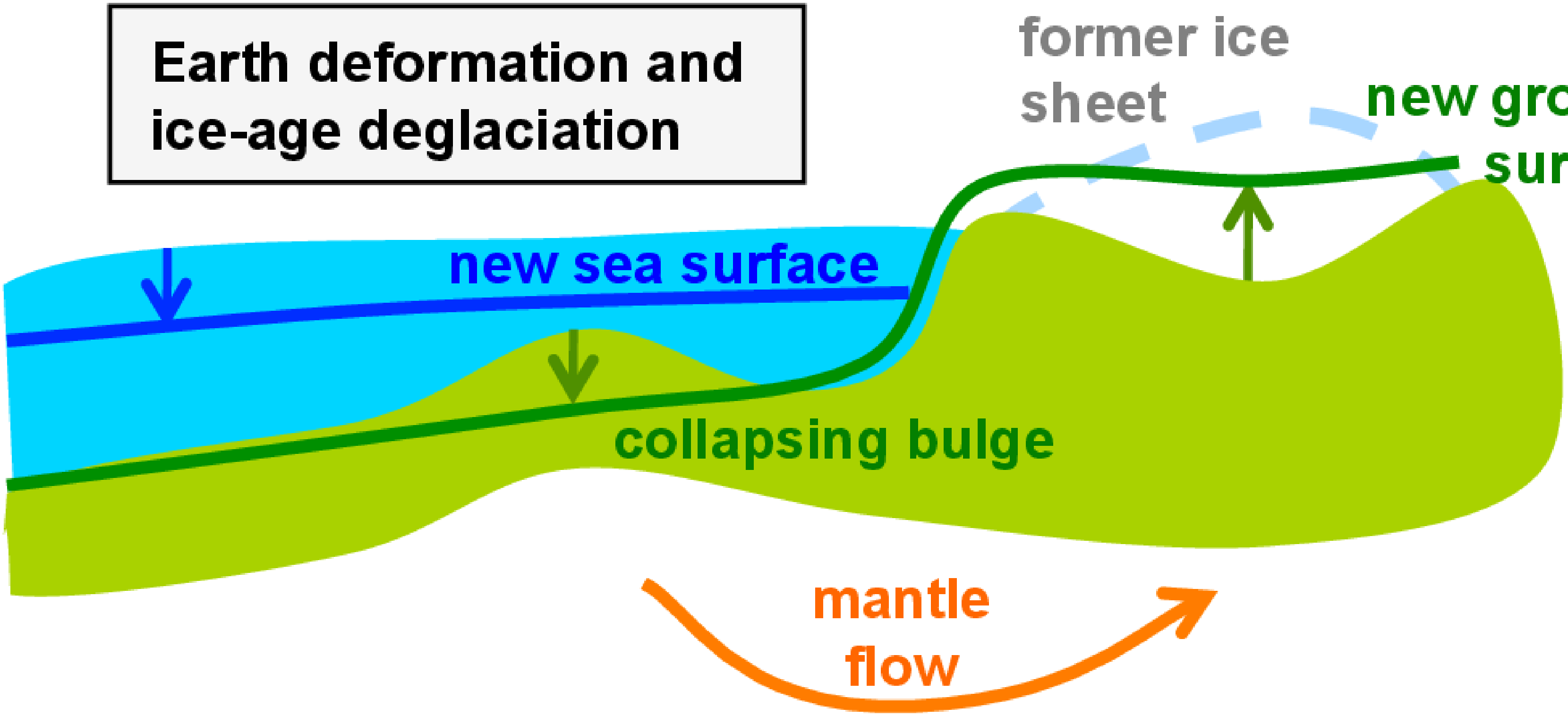
Changing Earth Figure
Elastic deformation (short term)

Load/unload
Atmosphere
Ocean tides
Hydro



Changing Earth Figure.
Visco-eastic Deformation - GIA (long term)

Earth deformation and ice-age deglaciation



Horizontal crustal deformation

Traditionally via GNSS monuments. Changes “figure of Earth” but also gravity.
IN-SAR (Lecture next time) is a new tool for this.
Typical changes 1-10 cm horizontal (few cm vertical).
It is important as it changes oceans (secondary effect) for sea level changes.

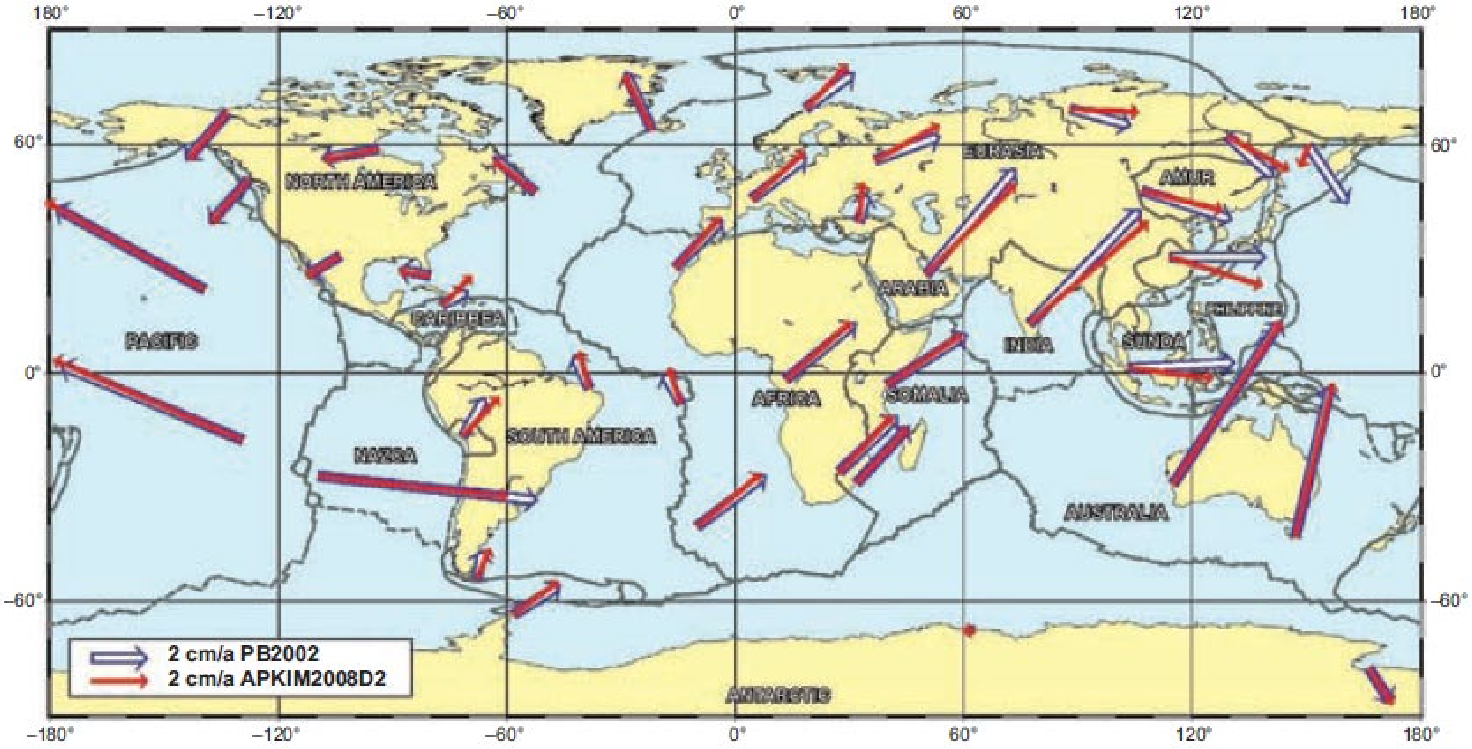


Fig. 8.21: Plate motions from geodetic observations (APKIM2008 model) and from the geophysical model PB2002 (Bird, 2003), DGFI Annual Report 2009, courtesy Deutsches Geodätisches Forschungsinstitut (DGFI), München.

Length of Day.

The circulation of fluids on the Earth's surface (oceans and atmosphere)

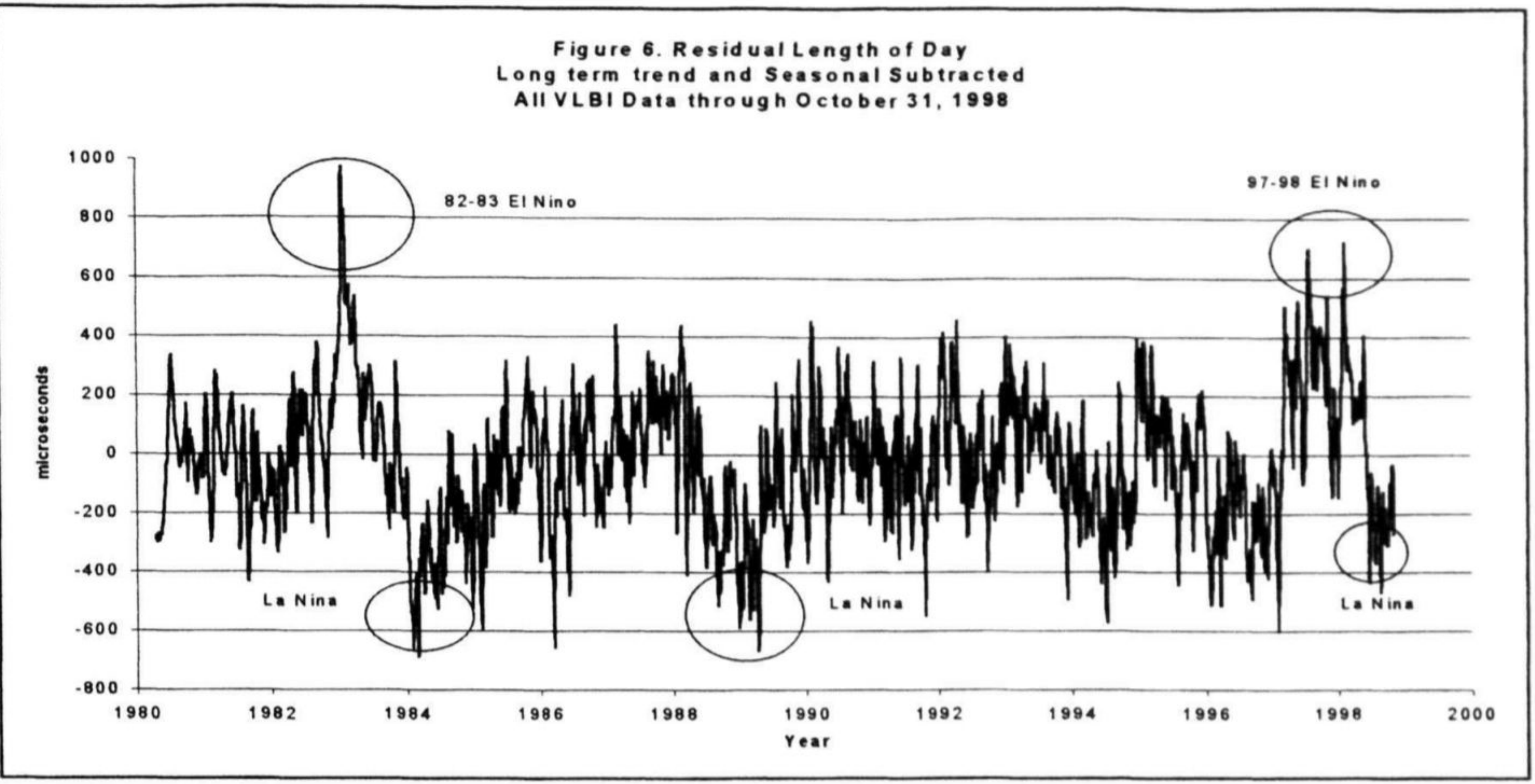


Figure 6. Residual Length of Day
Long term trend and Seasonal Subtracted
All VLBI Data through October 31, 1998