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The Choice of Stage-Discharge Relationship for the Ganges and Brahmaputra Rivers in Bangladesh

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The Ganges and Brahmaputra are the two largest rivers in Bangladesh. Discharge estimations of these rivers from a stage-discharge relationship or rating curve are crucial for flood warning/control/mitigation and water resources development. So far, logarithmic rating curves have been widely used in Bangladesh. The suitability of semi-logarithmic, polynomial and quadratic rating curves has not been investigated. In this study, all four recognised stage-discharge relationships were examined for the Ganges and Brahmaputra rivers. Unbiased least squares estimators were determined for the segmented logarithmic and semi-logarithmic rating curves. This enhanced their efficiency in inter-and extrapolating discharges from the given river stages. Based on detailed analysis and goodness-of-fit criteria, segmented logarithmic and third order polynomial rating curves were found to be the best for the Ganges and Brahmaputra rivers, respectively.

Introduction

In the discharge estimation of a river, the stage-discharge relationship or rating curve is a fundamental technique (ISO 1982; Herschy 1985; Sefe 1996). The quality of the stage-discharge relationship or rating curve determines the accuracy of the computed discharge data (Mosley and McKerchar 1993). This is particularly important for the rivers which carry huge volumes of flood water and may experience morphological changes (for example, the Ganges and Brahmaputra rivers in Bangladesh). The Ganges is a meandering river while the Brahmaputra is one of the largest braided river systems in the world (Coleman 1969; RPT *et al.* 1989).

The peak discharges of the Ganges and Brahmaputra rivers can cause severe floods in Bangladesh. Therefore, the stage-discharge relationship is particularly useful for flood forecasting and warning *interalia*, water resources assessment and environmental monitoring. Extrapolation of the stage-discharge relationship is also needed for the planning of flood control and other development projects. The actual practice of developing and applying rating curves varies between agencies (Mosley and McKerchar 1993). A logarithmic stage-discharge relationship is widely used (ISO 1982; Herschy 1985; McKerchar and Henderson 1987) for discharge estimation for a given water level. For the Ganges and Brahmaputra rivers, the Bangladesh Water Development Board (BWDB) uses a logarithmic stage-discharge relationship with a fixed offset value (gauge height at zero discharge). The China-Bangladesh Joint Expert Team (CBJET) (1991) has recommended a segmented logarithmic stage-discharge relationship with fixed offset value. The Flood Action Plan (FAP) 24 (1993) suggested using a similar type of rating curve, but with variable offset values. During the feasibility study of the Jamuna Multipurpose Bridge[1], Randel, Palmer and Tritton (RPT) *et al.* (1989) proposed a quadratic rating curve for the Brahmaputra River.

Tests for goodness-of-fit of these relationships are not well documented. Furthermore, suggestions for logarithmic and quadratic stage-discharge relationships were made without examining other recognised stage-discharge relationships (polynomial and semi-logarithmic) (Herschy 1985; ISO 1982). This article examines the suitability of logarithmic (segmented), semi-logarithmic, polynomial and quadratic stage-discharge relationships for the Ganges and Brahmaputra rivers, based on goodness-of-fit criteria. Finally, the best stage-discharge relationship is recommended for these two rivers for future use.

Extrapolation of the rating curve in both directions is often necessary for design purposes (Herschy 1985). Caution should be taken into account when carrying out such extrapolations. Large errors can result if the stage-discharge function is extrapolated beyond the range of gauged discharges without consideration of the cross-section geometry and controls (Mosley and McKercha 1993). Where cross-section is stable, a simple method is to extend the stage-area and stage-velocity curve for given stage values. Beyond the stage values that have been gauged, discharge is calculated by taking the product of velocity and cross-section area (ISO 1983). Extrapolation of the stage-velocity curve requires an understanding of the high stage control. Where there is a channel control, the Manning equation can be used to assist in the extrapolation. However, this is not reliable where Manning roughness varies with stage (Mosley and McKerchar 1993).

1. The Brahmaputra River is known as Jamuna in Bangladesh. A 5.5-kilometer bridge was opened for traffic in June, 1998 which connected the eastern and western halves of the country, separated by the Jamuna River.

The Study Area

The Ganges and Brahmaputra are the two largest rivers in the world. The *Ganges* rises south of the main Himalayan divide near Gangotri at a height of 4,500 metres in the Uttar Pradesh (UP) region of India. The 1.095 million km^2 basin area of the Ganges River is distributed over China, India, Nepal and Bangladesh. Mean annual runoffs of the Ganges River at Farakka and Hardinge Bridge are estimated to be $415x10^3$ million cubic metres (mcm) and $352x10^3$ mcm, respectively. The highest water level and peak discharge was recorded to be 14.8 m and 76,000 m^3/sec, respectively in 1987 (BWDB 1987; Mirza 1997). Apart from the water discharge, the Ganges also carries huge amounts of sediment. Coleman (1969) estimated that the Ganges River carries 479 million tons of suspended sediment annually, while the Master Plan Organisation (MPO) (1986) is estimated at about 212 million tons.

Statistical analyses indicate that there are no increasing or decreasing trends for low and medium water levels of the Ganges River. However, a slightly decreasing trend was reported for high water levels. This was perhaps caused by changes in local morphology at or downstream of the Hardinge Bridge (FAP 24 1993). Overall, the Ganges appears to be in dynamic equilibrium in terms of its cross-sectional and planform geometric. However, this does not preclude aggradation/degradation, or progressive channel shifting (FAP 4 1993).

The Brahmaputra is one of the world's largest braided river systems in terms of discharge, sediment transport, and channel processes (Coleman 1969; RPT *et al.* 1989). The river originates in a large glacier mass in the Kailash range of the Himalayas, south of Laka Kanggu Tso, very close to Lake Manassarovar, at an elevation of 5,150 m. The area of the Brahmaputra basin is 0.58 million km^2, distributed over Tibet (China), India, Bhutan and Bangladesh. The mean annual runoff of the Brahmaputra at Bahadurabad is estimated to be $643x10^3$ mcm (Mirza 1997). The highest observed water level occurred in 1988. The highest peak discharge estimated from the rating curve during the 1998 monsoon was 102,534 m^3/sec. Annually, the Brahmaputra also carries a huge sediment load. The available estimates vary between 499-608 million tons (Coleman 1969; CBJET 1991). According to one estimate, the amount of sediment transported annually by the Brahmaputra River is actually 721 million tons, one of the highest in the world (Islam *et al.* 1999).

No long-term trends in water levels could be detected in the Brahmaputra River. However, some discharges showed increasing water levels; while some others demonstrated decreasing levels instead (FAP 24 1993). Based on cross-section analysis, RPT *et al.* (1989) concluded that over the last few decades, the Brahmaputra was stable in both horizontal and vertical directions. Recently Barua (1994) also reported that the Brahmaputra was in a state of dynamic equilibrium.

The Data

Computerised measured discharge and water level data for the Ganges at Hardinge Bridge and Brahmaputra at Bahadurabad (Fig. 1) were collected from the BWDB (BWDB 1995). At Hardinge Bridge, discharge is currently measured daily. Prior to 1992, measurements were taken once a week during May to November at Bahadurabad and once in every fortnight during rest of the water year. In Bangladesh, a water year is defined from 1 April to 31 March. At these stations, water levels are recorded five times a day on a three hourly basis from 6 AM to 6 PM (CBJET 1991; FAP 24 1993). Public Work Department (PWD) datum is used for water levels. Standard equipment, acceptable methods and specifications are reported to be used in discharge and water level measurements. However, changing bed forms, velocity measurements taken from non-anchored boats and inaccurate measurement of depths for current meters may cause as much as 20 per cent uncertainty in discharge and water level measurements (Sir William Halcrow and Partners Ltd. 1991; FAP 24 1993).

For this study, measured stage and discharge data for the Ganges and Brahmaputra rivers were used. Stage-discharge curves were fitted for four years; 1966, 1974, 1988 and 1992. These years were chosen for three reasons. *First*, they are distributed over various decades; *second*, any progressive change over the years can be compared; and *third*, 1974 and 1988 represent two major flood years.

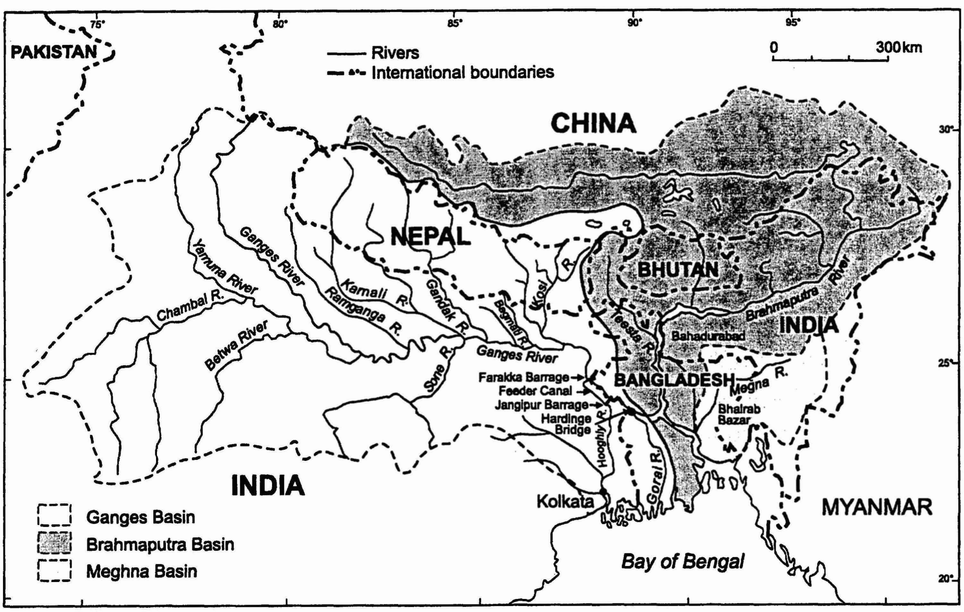


Fig.1. The gauging stations for the Ganges River (at Hardinge Bridge) and Brahmaputra River (at Bahadurabad).

The Method

In order to select the best stage-discharge relationship for the Ganges and Brahmaputra Rivers, the following relationships were examined.

(i) *Logarithmic relationship* - (Herschy 1985; ISO 1982; Mosley and McKerchar 1993)

The general stage-discharge relation is expressed by the following equation. The relationship is correct from a hydraulic point of view if the velocity height is disregarded.

$$Q = C(h+a)^N \tag{1}$$

where

Q = discharge
C and N = constants; C reflects the scales being used for stage and discharge and N denotes the degree of curvature or slope of the estimated relationship
h = stage or gauge height
a = stage at zero flow

For the analytical fitting, logarithmic transformation of Eq. (1) may be made to

$$\log Q = \log C + N \log(h+a) \tag{2}$$

Eq. (2) is now in the form of a straight line ($y \equiv Nx + C$).

Herschy (1985) proposed various numerical methods for determining the value of a if it is not accurately determined from the field. They are: the trial and error procedure, graphical method, and using computer programme and arithmetic procedures. However, a suggestion was made to compare the calculated value by field investigation. The value of a is positive if the curve (log$h$ *vs* log$Q$) is "concave down" and takes a negative sign if the curve is "concave up".

The method of least squares may be applied in order to calculate the values of C and N. It involves minimising the sum of the squared deviations between the logarithms of the measured discharge and the estimated discharge from the fitted curve. The values of C and N can be determined from the following two equations

$$\Sigma Y - n(\log C) - N\Sigma(X) \equiv 0 \tag{3}$$

$$\Sigma(XY) - \Sigma(X)(\log C) - N\Sigma(X^2) = 0 \tag{4}$$

where

$\sum(Y)$ is the sum of all the values of log$Q$
$\sum(X)$ is the sum of all the values of log$(h+a)$
$\sum(X^2)$ is the sum of all the values of (X)
$\sum(XY)$ is the sum of all the values of the product of (X) and (Y) and n is the number of observations.

For this study, the value of a was chosen via the trial and error method. In order to determine the break points, h and Q were plotted on the ordinate and abscissa, respectively on a logarithmic scale as suggested by (ISO 1982) and Herschy (1985).

(ii) *Semi-logarithmic Relationship* - (Herschy 1985)
The relation between stage and discharge is determined by plotting h linearly on the ordinate and log$Q$ on the abscissa. The equation for Q can be determined by taking three coordinates (h_2, $Q2$; h, Q; and h_2, $Q2$) on a straight line. However, unlike the relationship in (i), the equation for Q can be determined by the least squares method which takes the form

$$Q = C*(N)^{h} \tag{5}$$

$$\log Q = \log C + h\log N \tag{6}$$

Particularly, it has one specific advantage over the logarithmic relationship. It identifies break points more efficiently. The semi-logarithmic stage-discharge relationship is rarely used, but may have some advantages in exceptional cases (Herschy 1985).

(iii) *Polynomial Relationship* - (ISO 1988; Herschy 1985)
The general polynomial equation for stage (h) and discharge (Q) is

$$Q = b_0 + b_1h + b_2h^2 + b_3h^3 + \ldots + b_mh^m \quad \text{m}^3/\text{sec} \tag{7}$$

Replacing Q and h by Y and X, the least-squares procedure may be applied to a set of m+1 simultaneous equations, commonly known as the 'normal equations'.

$$\Sigma Y_i = nb_0 + \Sigma(X_i)b_1 + \Sigma(X_i^2)b_2 + \Sigma(X_i^3)b_3 + \ldots + \Sigma(X_i^m)b_m \quad \text{m}^3/\text{sec} \tag{8}$$

$$\Sigma(X_iY_i) = \Sigma(X_i)b_0 + \Sigma(X_i)^2b_1 + \ldots + \Sigma(X_iX_i^m)b_m \tag{9}$$

$$\Sigma(X_i^mY_i) = \Sigma(X_i^m)b_0 + \Sigma(X_i^mX_i)b_1 + \ldots + \Sigma(X_i^m)^2b_m \tag{10}$$

Eqs. (7)-(9) can then be solved for the m+1 unknowns $b_0, b_1, \ldots, b_m$. The polynomial procedure may have some advantages in fitting stage-discharge relationships having break points or inflexions which cannot be treated satisfactorily by other methods (Herschy 1985).

(iv) *Quadratic Relationship* (RPT *et al.* 1989)
For the Brahmaputra River, RPT *et al.* (1989) proposed the following quadratic stage-discharge relationship.

$$Q = C*(h-h_0)^2 + b \tag{11}$$

C and b are the two coefficients, which can be determined by applying the method of least squares.

Goodness-of-Fit and Selection of the Best Stage-Discharge Relationship

Previously, two types of stage-discharge relationships (logarithmic and quadratic) were applied for the Ganges and Brahmaputra rivers. However, an examination of goodness-of-fit of these relationships was not well documented. Only in the case of the quadratic relationship for the Brahmaputra River, the RPT *et al.* (1989) did consider the coefficient of determination (R^2) as a criterion of goodness-of-fit. R^2 should not be the only goodness-of-fit criterion used for selecting a regression model (Bowerman and O'Connell 1990). In addition to R^2, constant variance, independence, normality of residuals and mean squared error (RMSE) are also goodness-of-fit requirements of a linear or non-linear regression model (Cook and Weisberg 1982; Maidment 1993; Gawne and Simonovic 1994).

The assumption of the *constant variance* is that the residual plot should not 'fan out' or 'funnel in' with increases in the horizontal plot criterion (Bowerman and O'Connell 1990).

The *independence* criterion is that any one value of the error term is statistically independent of any of its other values. In other words, if the successive signs of the residuals are independent of each other, the sequence of the differences may be considered as distributed according to a binomial law $(p + q)n$, where n is the number of observations, and p and q, the probabilities of occurrence of positive and negative values, are $1/2$ each (ISO 1982).

For any regression model, residuals are expected to be *normally* distributed (Bowerman and O'Connell 1990). Normality of residuals can be tested in a simple way: by determining its third and fourth moments (Gawne and Simonovic 1994; Sefe 1996). Other alternative methods of examining normality of residuals include a normal probability plot and application of statistical test. In the normal probability plot, if the data come from a normal distribution, the standardised values of the residuals and observed residuals should fall onto a straight line. If the residuals are not normally distributed, then they will deviate from the line (Vogel 1986). The Shapiro-Wilk's W test is the preferred test of normality because of its good power properties compared to a wide range of alternative tests (Shapiro *et al.* 1968; Cook and Weisberg 1982).

W is calculated by ordering the observations in the sequence $x_1 \leq x_2 \leq ... \leq x_n$. Then S^2 and b are determined from the following relationship:

$$S^2 = \sum_{i=1}^{n} x_i^2 - \frac{1}{n} \left(\sum_{i=1}^{n} x_i \right)^2 \qquad (12)$$

$$b = \sum_{i=1}^{k} a_{n-i+1} (x_{n-i+1} = x_i) \qquad (13)$$

where $k = n/2$ if n is even, $k = (n-1)/2$ if n is odd and a_{n-i+1} are found from a table. Thus a test of normality for small samples ($3 \leq n \leq 50$) is defined as:

$$W = \frac{b^2}{S^2} \tag{14}$$

If the calculated value of W is found to be less than the critical value at a certain level of significance, then the hypothesis of normality is rejected.

The RMSE is a statistic based on the sum of squares of the residuals (SSE) and is given as:

$$RMSE = \sqrt{(SSE/n-k)} \tag{15}$$

where k =2 for a simple linear regression. For the polynomial relationship, the denominator in Eq. (15) is replaced by $(n\text{-}m\text{-}1)$.

$$SSE = \sum_{i=1}^{n} (Q_i - Q_c)^2 \tag{16}$$

A minimum value of *RMSE* is expected, as it represents the magnitude of error inherent in a model.

For intrinsically non-linear models (polynomials), in addition to the above described goodness-of-fit criteria, Student's t-test with $(n - m - 1)$ degrees of freedom is suggested in order to determine whether the coefficients (for example, in Eq.(7)) differ significantly from zero (ISO 1988).

Results and Discussion

The observed stage-discharge points for the Ganges and Brahmaputra rivers were selected randomly and are plotted in Figs. 2 and 3. For the Ganges River, the selected observation numbers were 30 each for 1966, 1974 and 1988 and 26 for 1992. Note that 1974 and 1988 were two severe flood years. The pattern of *Q-h* points of 1966 and 1974 are not different, except in the discharge range of 10,000-20,000 m^3/sec. In this range, the *h* points for the year 1966 show slightly higher water levels than those for 1974, indicating some aggradation. On the other hand, in 1988, the *h* points at high discharges (>30,000 m^3/sec) show comparatively lower water levels, indicating a slight degradation. The pattern of *h* points for 1992 is quite different from the remaining three years. Degradation at low discharge values is highly pronounced and so are the break points.

For the Brahmaputra River at Bahadurabad, the selected *Q-h* observation numbers are 34, 39, 37 and 29 for the years 1966, 1974, 1988 and 1992, respectively. At low to medium discharge, ≤ 30,500 m^3/sec, the pattern of water levels is found to be similar for the years 1966, 1988 and 1992. However, water levels for this range of discharge for 1974 are slightly higher, which indicates an occurrence of aggradation. The break points are different for all years considered for development of rating curves. For stage-discharge relationships (i), offset value *a* was chosen by trial and error until log *(h-a)* and log*Q* was linear, as much as possible. The selected offset

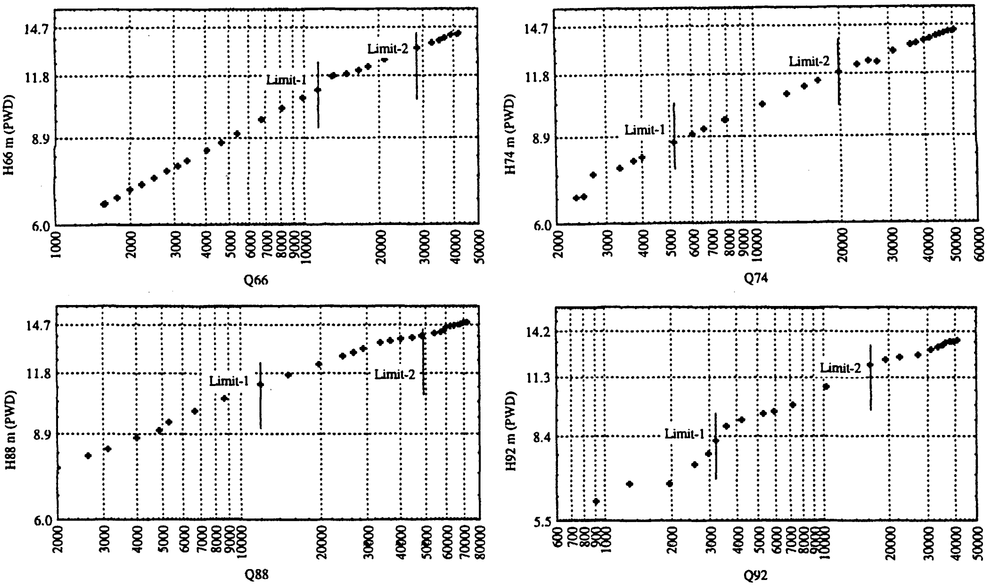


Fig.2. Measured stage-discharge points for the Ganges River.

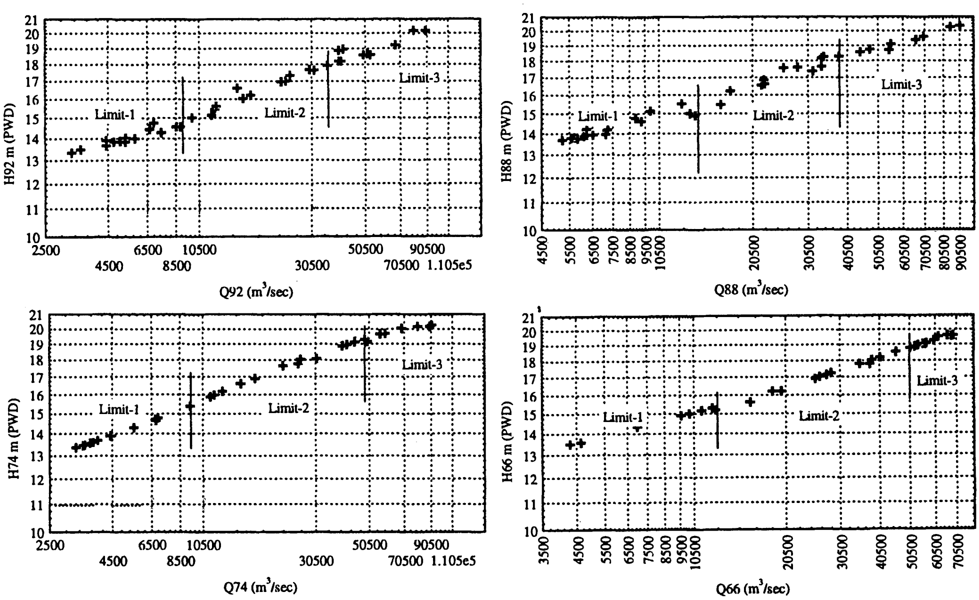


Fig.3. Measured stage-discharge points for the Brahmaputra River.

values were also used for the semi-logarithmic rating curves. Offset values were negative for all eight curves as they are "concave upward" (Fig. 2 and Fig. 3).

Bangladesh Water Development Board (BWDB) and some other consultants produced and recommended logarithmic stage-discharge curves for the non-tidal rivers in Bangladesh. The rating curves were developed either by applying the least squares method via a grapical method (mentioned in the relevant section). Development of such rating curves through logarithmic transformation produces biased least square model estimators (intercept and slope) (McCuen *et al.* 1990; Ferguson 1986 and 1987; Cohn *et al.* 1989). Uses of ordinary least squares in Eqs.(2) and (6) produce unbiased estimates of log$Q$ that have the minimum expected variation in the log$Q$-space: Thus, the correlation coefficient reflects the accuracy of the unbiased estimates of log$Q$. However, when the least square estimators are determined from the logarithmic transformation in Eqs. (1) and (5), they are not unbiased and do not have constant error variance in the coordinate-space. In Bangladesh, so far, biased least square estimators were used for estimating discharges from the rating curves.

Bias in the least square estimators should be corrected in order to increase the prediction power of a rating model derived through transformation. Unless corrected, the mean error and mean square error, the two important goodness-of-fit indicators, will be biased. The sum of the unbiased residuals should be zero. This is defined as

$$\bar{e} = \sum_{i=1}^{n} (Q_i - \hat{Q}) \tag{17}$$

In case of the logarithmic and semi-logarithmic rating curves for the Ganges and Brahmaputra rivers, the sum of the residuals was not found to be zero. Instead, they showed positive values, which indicated the possibility of over-estimation. Correction of bias is particularly important for the high flows of these two rivers. The prediction power of logarithmic and semi-logarithmic rating curves are relatively less precise in cases of the high flows. In most of the years, the segmented rating models underestimated the high flow value. They produced positive residuals. This may be the result of two causes. *One*, the least square estimators are biased; and *two*, there are some measurement errors inherent in the high flow values. The possibility of measurement errors cannot be overruled, but they are largely unknown.

In this research, two suitable methods of correction were applied to Eqs. (1) and (5) in order to determine unbiased least squares estimators. These methods are briefly discussed below.

The normal theory estimate of the biasing factor is $\exp(0.5S_b^2)$ where S_b = the standard error of estimate computed for the logarithmic transformed model Eqs.(2) and (6). Thus the unbiased intercept coefficient C_o is given by

$$C_o = C^* \exp(0.5\, S_b^2) \tag{18}$$

However, when residuals are not normally distributed, the bias-correction factor of Eq. (13) is inappropriate. Duan (1983) gave a non-parametric correction, which was termed the smearing estimator (d_o)

$$d_o = C^* \left(\frac{1}{n} \sum \exp(r_i)\right) \tag{19}$$

Table 1 – Segmented logarithmic stage-discharge relationship for the Ganges River at Hardinge Bridge.

Year	INTERVAL-1			LIMIT-1* (m)	INTERVAL-2			LIMIT-2**(m)	INTERVAL-3		
	h_o	N	C		h_o	N	C		h_o	N	C
1966	-3.1	2.31	90.24	11.07	-5.4	3.40	24.02	13.38	-8.0	2.40	492.05
1974	-1.2	2.54	29.55	8.67	-3.0	3.02	26.25	11.89	-6.5	2.41	339.20
1988	-3.5	2.81	36.70	11.18	-6.4	3.02	92.80	13.87	-9.5	1.88	3003.10
1992	-2.2	2.59	33.46	8.22	-5.0	2.41	147.21	12.01	-7.5	2.92	209.92

* Limit-1 is the upper limit stage (m) for interval-1
**Limit-2 is the lower limit stage (m) for interval-3

Table 2 – Segmented logarithmic stage-discharge relationship for the Brahmaputra River at Bahadurabad.

Year	INTERVAL-1			LIMIT-1* (m)	INTERVAL-2			LIMIT-2**(m)	INTERVAL-3		
	h_o	N	C		h_o	N	C		h_o	N	C
1966	-10.0	2.52	181.05	15.19	-12.0	1.97	1156.0	19.07	-14.0	1.71	3506.25
1974	-10.0	2.26	205.61	15.38	-11.0	2.74	137.81	19.30	-15.0	2.57	1172.27
1988	-10.0	2.38	245.02	16.63	-13.0	2.01	1376.20	18.72	-15.0	1.70	5162.92
1992	-10.0	2.74	126.00	15.13	-13.0	1.20	4207.62	17.31	-15.0	1.69	5791.47

* Limit-1 is the upper limit stage (m) for interval-1
**Limit-2 is the lower limit stage (m) for interval-3

In Eq.(1) and Eq.(5), C can be replaced by d_o whenever necessary. The corrected intercept values for the logarithmic and semi-logarithmic rating curves are shown in Table 1 and Table 2. Values were rounded up to two decimal places. However, for discharge estimation purposes, eight or more decimal places are recommended, especially for the intercept.

The unbiased intercepts of the logarithmic and semi-logarithmic rating models have improved their prediction power, particularly, for the high flows. Here, results from the logarithmic rating models are discussed. For example, for the year 1974, the highest recorded peak discharge was 50,700 m^3/sec. The estimated peak discharge from the logarithmic model was 49,410 m^3/sec, with a residual of 1,290 m^3/sec. However, with the corrected intercept, the estimated value increased to 49,430 m^3/sec. Therefore, the residual value decreased by 20 m^3/sec. Similarly, for the Brahmaputra River, the highest measured and estimated (from the rating curve) discharges were 90,800 m^3/sec and 89,313 m^3/sec, respectively. The corrected intercept has reduced the residual value by 25%. Cohn *et al.* (1989) argued that the bias correction might lead to overestimation in some cases. However, development of a rating model (by logarithmic transformation) will lead to violation of the principles of regression model building without bias correction.

Third order polynomials appear to be better fitted for the Brahmaputra River than those for the Ganges River (Fig. 4 and Fig. 6). For the Ganges River, the fitted polynomials underestimated the observations, particularly in the range of 5,000-25,000 m^3/sec for most of the years. One important aspect is that for all years, the polynomials have better estimated the high flows > 35,000 m^3/sec with the exception of 1988. For the Brahmaputra River, the polynomials show a very good fit over the whole range of observations for all years.

Unlike third order polynomials, the quadratic models appear to not fit very well for the Ganges River (Fig. 5). The quadratic models underestimated the low flows as well as the high flows for all years. However, they overestimated the medium flows (in the range of 10,000-25,000 m^3/sec). On the other hand, overall, for the Brahmaputra River, the quadratic models fit the observations much better than the Ganges River (Fig. 7). Unlike the Ganges River, quadratic models underestimated low flows for the Brahmaputra River.

For the Ganges River, the distribution of residuals for both logarithmic and semi-logarithimic rating curves shows similar patterns (scattered around the zero axis) for the years 1974, 1988 and 1992. However, for the semi-logarithimic rating curves, the distribution is "concave downward" for the year 1966 (Fig. 8). The residuals for these two types of rating curves are found overall to be homoscedastic. In other words, their variances are constant.

The polynomial residuals show similar patterns for the years 1966, 1974 and 1988 for the Ganges River. Up to 10-metre and approximately 10-12 metre of stages, the polynomials have a tendency to over- and under-estimate discharges. Residuals for high flows are found to be well scattered. For the quadratic rating curves, residuals

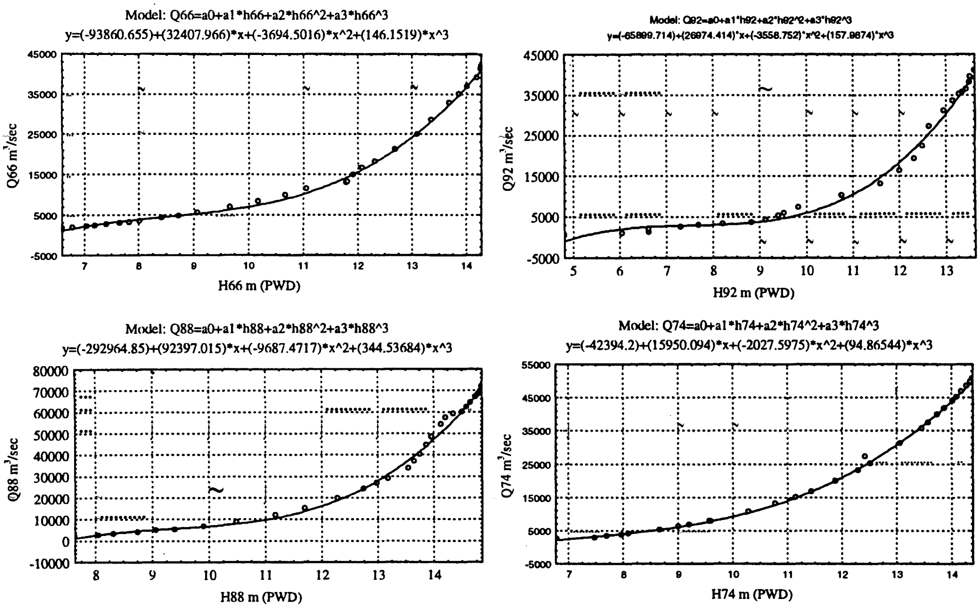


Fig.4. Third order polynomials for the Ganges River.

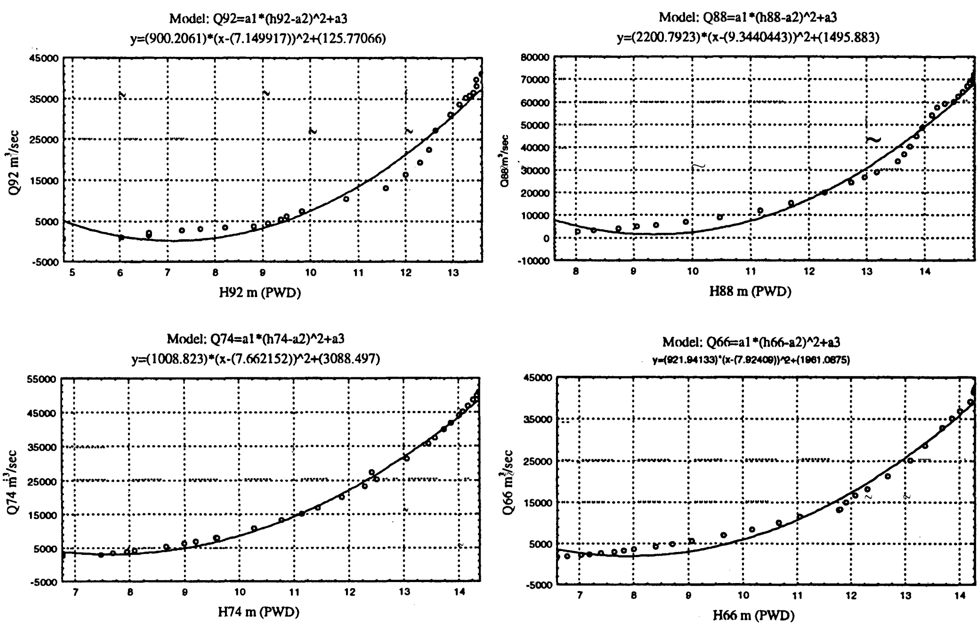


Fig.5. Quadratic rating curves for the Brahmaputra River.

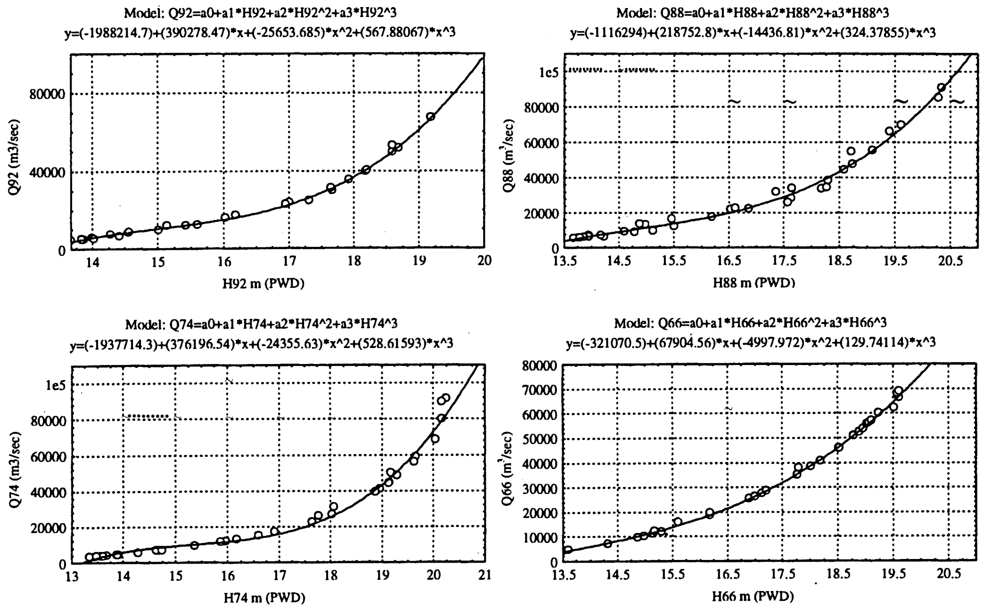


Fig.6. Third order polynomials for the Brahmaputra River.

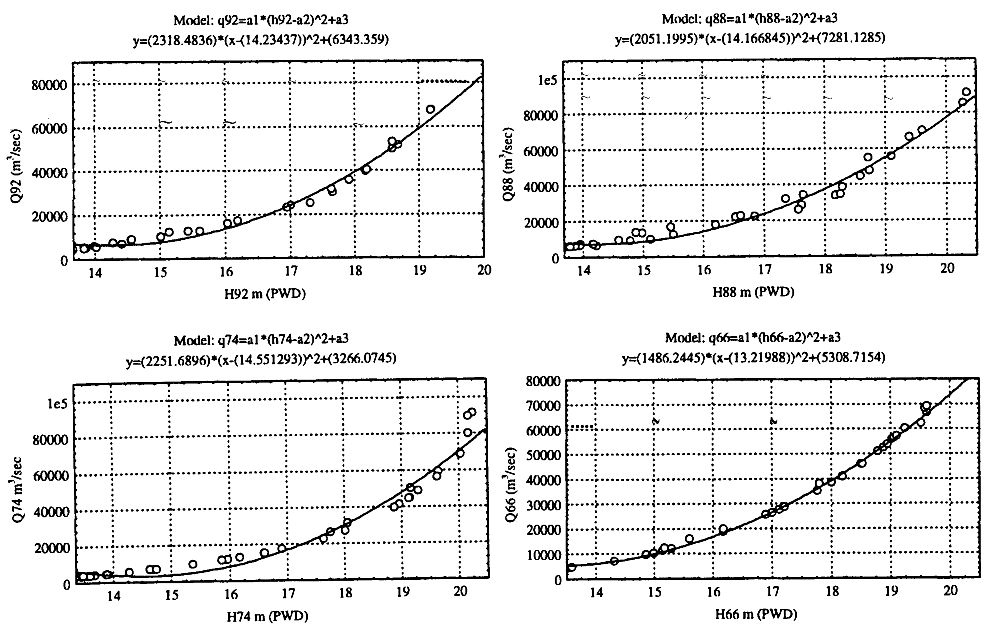
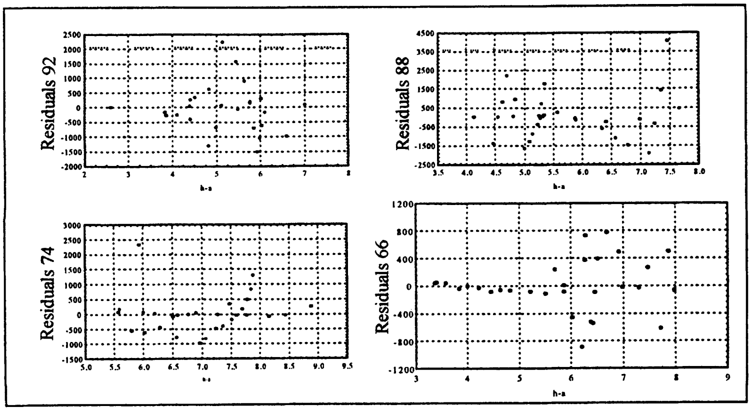


Fig.7. Quadratic rating curves for the Ganges River.



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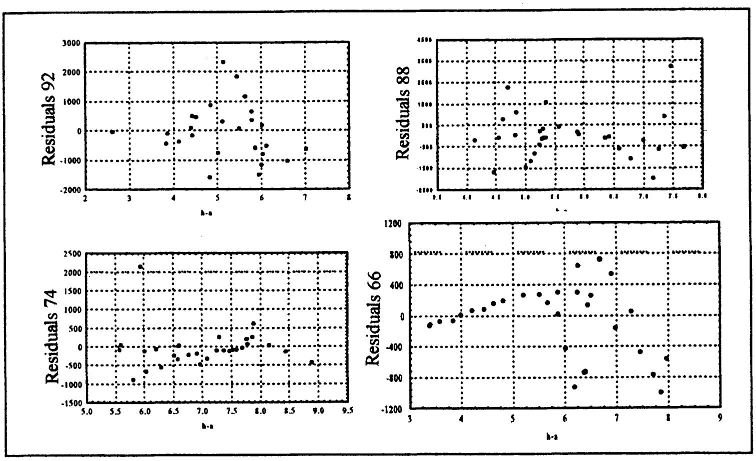
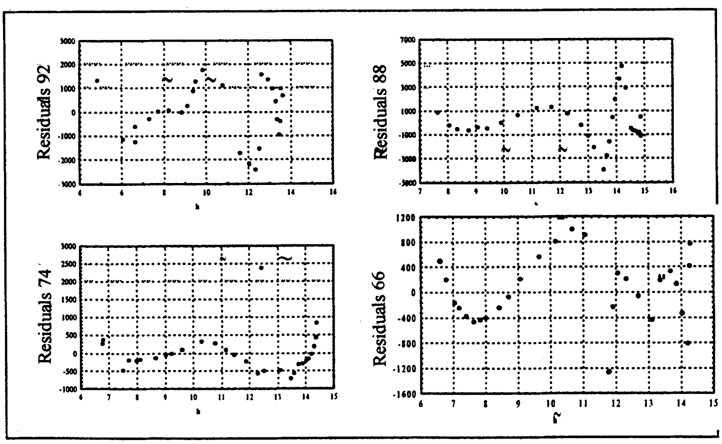


(8b)



(8c)

Fig.8. Residuals of the (a) segmented logarithmic, (b) semi-logarithmic, and (c) polynomial curves for the Ganges River.

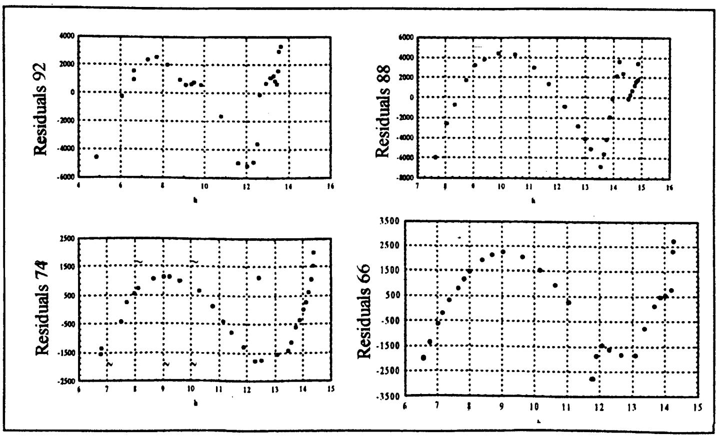


Fig. 8 cont. Residuals of the (d) quadratic curves for the Ganges River.

show a cyclic pattern, "concave downward" up to approximately 12-metre stages and then "concave upward" (Fig. 8). The patterns indicate that quadratic models have a tendency to generate cyclic patterns of residuals for the low flow to the high flow. Residuals were not found to be scattered on both sides of the zero axis for any segment of the rating curves. The distribution of residuals for the polynomials and quadratic rating curves cannot be said to be "homoscedastic".

Student's t-test was applied in order to examine the independence of the residuals. For the segmented logarithmic, polynomial and quadratic rating curves, residuals were found to be independent for the years 1966, 1974, 1988 and 1992. However, for the semi-logarithmic rating curve, residuals demonstrated independence for all years but 1974.

In order to examine the normality of the residuals, the Shapiro-Wilk test was conducted on the detransformed residuals, as this is more realistic. The residuals of the segmented logarithmic and semi-logarithmic rating curves were found to be normal for the years 1966 and 1992 for the Ganges River, two non-flood years. This indi-

Table 3 – Shapiro-Wilk *W* for testing normality of residuals for various stage-discharge models for the Ganges River.

Year	Segmented Logarithmic	Semi-logarithmic	Cubic Polynomial	Quadratic
1966	0.936	0.975	0.940	0.907
1974	0.846	0.749	0.772	0.877
1988	0.909	0.932	0.946	0.925
1992	0.942	0.966	0.951	0.847

Note: Shaded values have exceeded the W_{cr}. Value of $\alpha = 0.05$.

cates that the magnitude of residual for each of the Q points and the overall pattern of distribution of residuals are similar for these two types of rating curves. Residuals were found to be normally distributed for the years 1966, 1988 and 1992 for the polynomials. For all four quadratic rating curves, residuals were found to be non-normal, likely due to their pattern of distribution.

Note that normality of residuals may be highly influenced by outliers. As seen from Fig. 8, one high outlier is seen in the segmented logarithmic and semi-logarithmic rating curves for the years 1974 and 1988. Similarly, polynomial residuals for the year 1974 show a high outlier and this might have influenced its normality. The Q-h points related to the outliers were not removed due to inadequate physical evidence.

For the Brahmaputra River, the pattern of distribution of residuals for the two flood years 1974 and 1988 were found to be similar for the segmented logarithmic and semi-logarithmic rating curves. Overall, for both types of rating curves, the residuals show homoscedastic pattern (Fig. 9).

As indicated above, the polynomial family gives a better fit to the Q-h points of the Brahmaputra River than those for the Ganges River. The physical explanations for this difference are likely be many, but most probably it is because they are two different types of river in geographically and geologically, unique basins. Moreover, the process, pattern and magnitude of morphological changes of these two rivers also differ significantly.

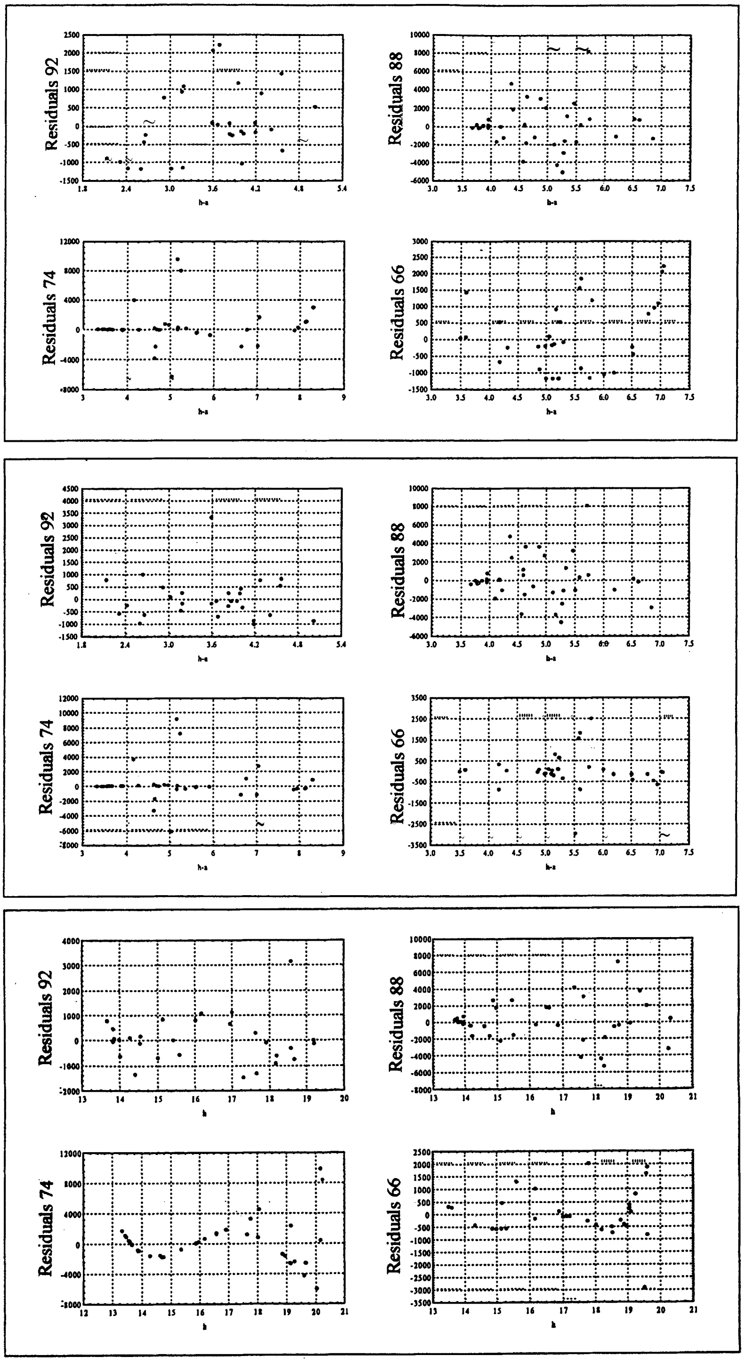
Unlike the Ganges River, residuals of the third order polynomials have not created any cyclic pattern for the Brahmaputra River (Fig. 9). While for the years 1966, 1988 and 1992, residuals are uniformly scattered, for 1974 with up to 17-metre stages, residuals show an upward "concave" pattern. Overall, the pattern of distribution of residuals is "homoscedastic".

For the quadratic rating curves, residuals created a distinguishable cyclic pattern (Fig. 9). However, this is not as prominent as for the Ganges River. The cyclic pattern of distribution of residuals clearly indicates that a quadratic rating curve is not suitable for the Brahmaputra River. While recommending it for the Brahmaputra River, RPT *et al.* (1989) did not investigate this phenomenon. The cyclic pattern was

Table 4 – Shapiro-Wilk W for testing normality of residuals for various stage-discharge models for the Brahmaputra River.

Year	Segmented Logarithmic	Semi-logarithmic	Cubic Polynomial	Quadratic
1966	0.964	0.807	0.910	0.948
1974	0.783	0.708	0.916	0.912
1988	0.935	0.937	0.966	0.972
1992	0.979	0.839	0.920	0.937

Note: Shaded values have exceeded the W_{cr}. Value of $\alpha = 0.05$.

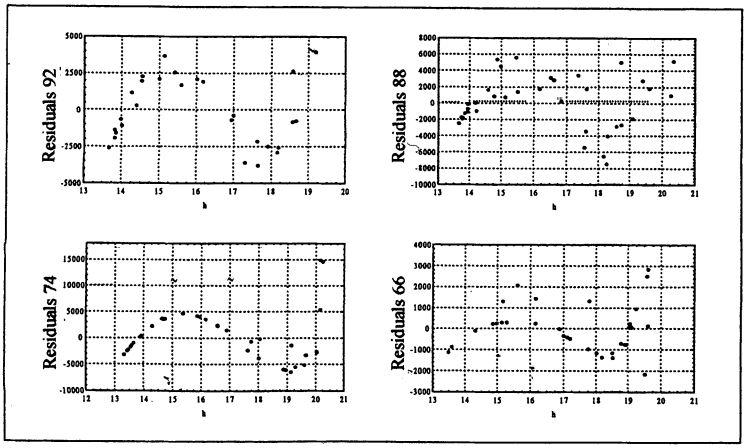


(9a)

(9b)

(9c)

Fig.9. Residuals of the (a) segmented logarithmic, (b) semi-logarithmic, and (c) polynomial curves for the Brahmaputra River.



(9d)

Fig. 9 cont. Residuals of the (d) quadratic curves for the Brahmaputra River.

removed by fitting third order polynomials.

Results of Student's t-test show that the residuals of the four types of rating curves are independent for the years 1966, 1974, 1988 and 1992.

The residuals of the segmented logarithmic and quadratic rating curves were found to be normally distributed for the years 1966, 1988 and 1992. Semi-logarithmic and third order polynomial residuals were normally distributed for only 1988. However, for the other three years, polynomial residuals were found to be very close to the critical values of the Shapiro-Wilk test. For all types of rating curves, residuals of 1974 were found non-normal. This was probably caused by the presence of two high outliers (≥ 8,000 m^3/sec).

As discussed in the section on methods, four goodness-of-fit criteria were chosen in order to select the best rating curve for the Ganges and Brahmaputra rivers. These criteria are: R^2, constant variance, independence, normality of residuals and RMSE. The properties of the four methods were ranked using a set of arbitrary points (Table 5). Constant variance, independence and normality of residuals have been discussed

Table 5 – Ranking criteria for the rating curves.

R^2	Independence	Residuals Variance	Normality	RMS
Highest (4)	Independent (1)	Heteroscedastic (0)	Absolutely normal (3)	Highest (1)
Second highest (3)	Non-independent (0)	Homosceda) stic (1	Near Normal (2)	Second highest (2)
Third highest (2)			Non-normal (1)	Third highest (3)
Lowest (1)				Lowest (4)

Table 6 – The R^2 and RMSE for various stage-discharge models for the Ganges River.

Year	Segmented Logarithmic		Semi-logarithmic		Cubic Polynomial		Quadratic	
	R^2 (%)	RMSE	R^2 (%)	RMSE	R^2(%)	RMSE	R^2 (%)	RMSE
1966	99.90	410.0	99.10	455.0	99.90	603.8	99.26	537.0
1974	99.86	697.0	99.91	553.0	99.94	596.4	99.80	1149.0
1988	99.76	1353.0	99.82	1193.0	99.76	1872.0	99.15	3414.2
1992	99.70	907.0	99.74	1041.0	99.67	1274.0	98.55	2643.0

Table 7 – The R^2 and RMSE for various stage-discharge models for the Brahmaputra River.

Year	Segmented Logarithmic		Semi-logarithmic		Cubic Polynomial		Quadratic	
	R^2 (%)	RMSE	R^2 (%)	RMSE	R^2(%)	RMSE	R^2 (%)	RMSE
1966	99.80	1220.6	99.83	936.6	99.91	948.5	99.85	1160.0
1974	99.50	2938.6	99.00	2757.0	99.30	2666.0	98.33	4970.0
1988	93.60	2656.7	98.90	2645.0	99.50	2563.0	99.00	3336.1
1992	96.60	1068.2	99.80	943.0	99.74	980.2	99.27	2442.0

above. The R^2 and RMSE values are given in Tables 6 and 7.

The total score was calculated as the sum of points for each method and each year. For the Ganges River, segmented logarithmic and semi-logarithmic rating curves scored the highest, 39 points. The polynomial and Quadratic rating curves scored 32 and 13 points, respectively. For the Brahmaputra River, the third order polynomials scored the highest, 41 points and the semi-logarithmic curve the second highest at 35 points. Quadratic and segmented logarithmic rating curves scored 30 and 29 points, respectively. Based on rankings, segmented logarithmic and semi-logarithmic rating curves were found to be suitable for the Ganges River and third order polynomials for the Brahmaputra River.

Conclusion

This study has a limited scope due to resource constraints. Despite these limitations, based on four years of data, the suitability of logarithmic, semi-logarithmic, polynomial and quadratic stage discharge relationships or rating curves were investigated for the Ganges and Brahmaputra rivers in Bangladesh. So far, in Bangladesh, biased least squares estimators have been used in order to inter- and extrapolate river discharges from the logarithmic rating curves. In this study, unbiased least squares estimators were determined which would fill the gap in knowledge. If applied, this will also increase the efficiency of logarithmic and semi-logarithmic rating curves in inter- and extrapolation.

Based on detailed analysis and goodness-of-fit criteria, segmented logarithmic and third order polynomial stage-discharge relationships were found to be the best for the Ganges and Brahmaputra rivers, respectively. The physical cause(s) for fitting two different types of rating curves for the Ganges and Brahmaputra rivers are unknown and need to be investigated. Year to year variation in river stage for any common threshold discharge was found to be negligible. This supports the previous finding that the Ganges and Brahmaputra rivers "are in dynamic equilibrium".

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