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Towards a cm-geoid for Malaysia

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Abstract

This paper summarizes the underlying airborne gravity surveys, and the computation of the new Malaysian geoid to be used for height determination by GPS. The computed Malaysian geoid models are fitted to GPS on leveling benchmarks, and therefore improvements of the geoid will primarily be related to improvements in the vertical datum and GPS data; the intrinsic accuracy of the underlying gravimetric geoid is few cm only. Improvements of this underlying model are possible due to recent developments in satellite gravity field data, and when (or if) gravity data could be made available in border areas (Thailand and Indonesia).

Introduction

The Malaysian geoid project is unique, and first in the world to cover a complete major country with dense airborne gravity, with the aim to make the best possible national geoid model. The basic underlying survey and computation work of the Malaysian geoid project was done by GlobalTrak (Malaysia) and the Geodynamics Dept. of the Danish National Survey and Cadastre (KMS; since Jan 1 part of the Danish National Space Center) in cooperation with JUPEM. With the new data the geoid models are expected to be much improved over earlier models (Kadir et al. 1998).

The primary aim of the new Malaysian geoid model is to be able to compute orthometric (sea-level) heights H in the national height system by

$$H = h^{GPS} - N \qquad (1)$$

where h^{GPS} is the GPS height above the ellipsoid (e.g. from RTK-GPS) and N the geoid. In the above equation it is important to realize that H refers to a local vertical datum (typically defined by a tide gauge, with separate systems in Peninsular Malaysia, Sarawak and Sabah), h^{GPS} refers to a geocentric system (ITRF/WGS84), to which the computed (gravimetric) geoid also usually refers.

The gravimetric geoid height N is in principle determined by Stokes' equation of physical geodesy, which gives the expression of the geoid height N as an integral of gravity anomalies around the earth (σ)

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g \, S(\psi) d\sigma \qquad (2)$$

where Δg is the gravity anomaly, R earth radius, γ normal gravity, and S a complicated function of spherical distance ψ (Heiskanen and Moritz, 1967). In practice global models of the geopotential from analysis of satellite data and global mean gravity anomalies are used, e.g. for the current global model EGM96 (Lemoine et al., 1996)

$$N_{EGM96} = \frac{GM}{R\gamma} \sum_{n=2}^{N} \left(\frac{R}{r}\right)^n \sum_{m=0}^{n} \left(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda\right) P_{nm}(\sin\phi) \qquad (3)$$

Here the spherical harmonic coefficients C_{nm} and S_{nm}, for EGM96 complete to degree and order 360 define the long-wavelength gravity field (degree 360 corresponds to a resolution of 55 km). In practice, however, EGM96 is no better than the underlying regional gravity and satellite data, and large errors were e.g. found in Borneo during the Malaysian geoid project. Background and online information on EGM96 can be found at www.nga.mil. Currently a new global model EGM05 is being prepared in cooperation with the International Association of Geodesy; this new spherical harmonic model will have a maximal degree of 2160 (5' resolution).

For the Malaysian project new GRACE satellite data combination models was used (GGM01C). This model is a combination model to degree 180 based on 1° mean anomalies, essentially derived from the same terrestrial data as EGM96, but having superior new satellite information (GGM01S) at the lower harmonic degrees.

A third data source for the geoid determination is digital terrain models (DEM's), which provide details of the gravity field variations in mountainous areas (the mass of the mountains can change the geoid by several 10's of cm locally). The handling of digital terrain models is done by analytical prism integration assuming known rock density (Forsberg, 1984). The new satellite data SRTM was used together with JUPEM DEM's for this purpose.

With the data from spherical harmonic models, local or airborne gravity, and DEM's, the (gravimetric) geoid is constructed by remove-restore techniques as a sum

$$N = N_{EGM} + N_{gravity} + N_{DEM} \qquad (4)$$

However, to be consistent with GPS and local leveling systems, a correction between the *global* and *local* vertical datums must be made:

$$N^{GPS} = N + \varepsilon \qquad (5)$$

where ε is a *GPS-corrector surface* taking into account datum differences and possible errors in GPS, leveling and the gravimetric geoid N.

In practice ε is determined by *fitting* the gravimetric geoid at points with coincident GPS and leveling; at these points ε can be directly determined by

$$\varepsilon = N_{GPS} - N = h_{GPS} - H_{levelling} - N \tag{6}$$

and ε then interpolated to other points by e.g. least-squares collocation; in practice, however, the N^{GPS} will not be a classical geoid (an equipotential surface) because any error in H or h will be "inherited" in the final GPS geoid N^{GPS}. This is presently the major source of error in the Malaysian geoid.

The 2002-3 Malaysia airborne geoid project

The Malaysian airborne gravity survey was done on a 5 km line spacing, covering mostly West Malaysia fall 2002 and East Malaysia spring 2003. The airborne gravity data system used was based on the DNSC/University of Bergen system, used extensively for Arctic gravity field mapping. The system is based on differential GPS for positioning, velocity and vertical accelerations, with gravity sensed by a modified marine Lacoste and Romberg gravimeter. The system has a general accuracy better than 2 mgal at 5 km resolution (Olesen et al., 1997).

Fig. 1. An-38 aircraft used for aerogravity

For the Malaysia airborne survey, the system was installed in a An-38 aircraft (Fig. 1), operated by Layang-Layang aerospace, Kota Kinabalu. The An-38 aircraft turned out to be very suitable for the airborne survey, with accuracies estimated from cross-overs well below 2 mgal r.m.s., cf. Table 1.

Table 1. *Cross-over analysis of airborne gravity sets.*

East Malaysia

Unit: mgal	Mean x-over	R.m.s. x-over	R.m.s. error
Original free-air data at altitude	0.18	3.16	2.23
Bouguer anomalies at 2700 m	0.12	2.78	1.96
Do, after bias adjustment	-0.05	2.26	1.60

West Malaysia

Unit: mgal	Mean x-over	R.m.s. x-over	R.m.s. error
Original free-air data at altitude	-.09	2.37	1.68
Bouguer anomalies at 3400 m	-.06	2.36	1.67
Do, after bias adjustment	-.06	1.81	1.28

The airborne gravity survey was flown at different elevations, as topographic conditions permitted, cf. Figs. 2a and 2b. The data were therefore required to be downward continued to the surface, before applying the Stokes formula gravity to geoid transformation (2). The downward continuation was done by least-squares collocation using the planar logarithmic covariance model (Forsberg, 1987), using all available gravity data in the process (airborne, surface, marine and satellite altimetry gravity data). The Stokes' integration was implemented by spherical FFT methods (Forsberg and Sideris, 1993).

The existing surface gravity data coverage was only of significance in East Malaysia (Fig. 3). Here the relatively dense surface gravity data coverage in the lowlands will strengthen the geoid compared to the situation in Sarawak and Sabah, where essentially no existing gravity data was available. The computed geoids are shown in Fig. 4a and 4b.

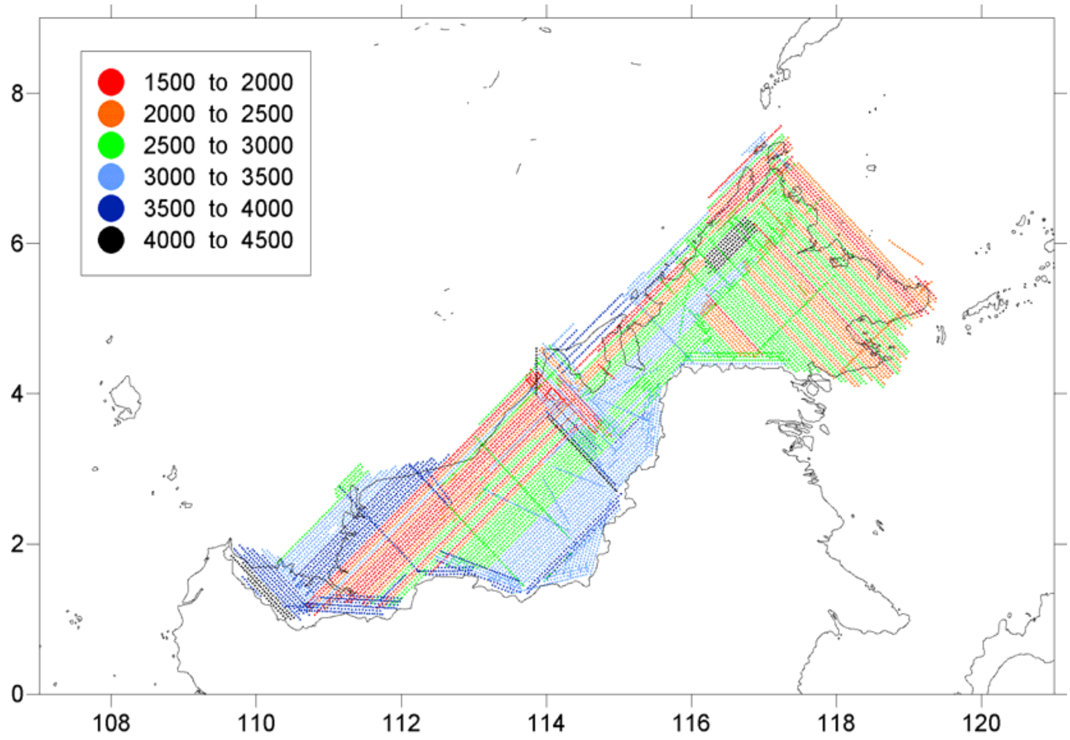

Fig. 2a. Flight lines in West Malaysia. Colour coding represents flight elevation.

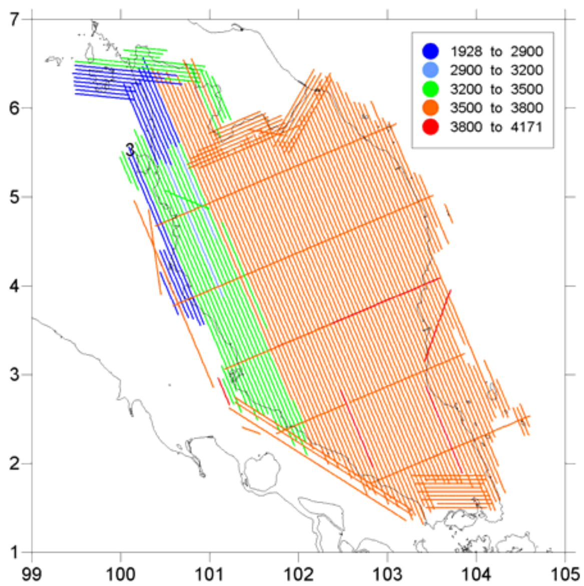

Fig. 2b. Flight lines in East Malaysia. High elevation mainly due to airspace restrictions.

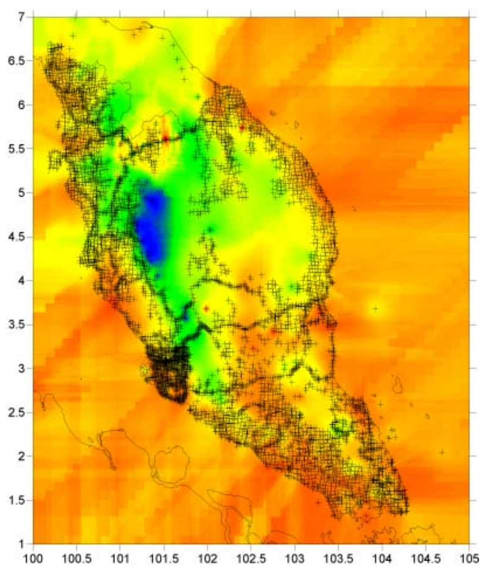

Fig. 3. Surface gravity coverage in East Malaysia (colours indicate anomalies)

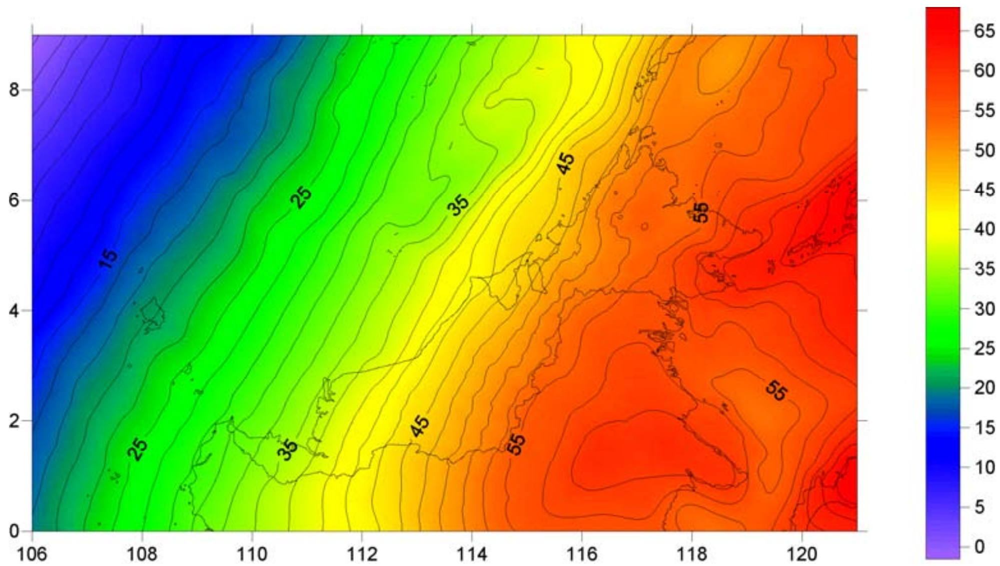

Fig. 4a. Final gravimetric geoid for East Malaysia (EMG03A). Contour interval 2 m.

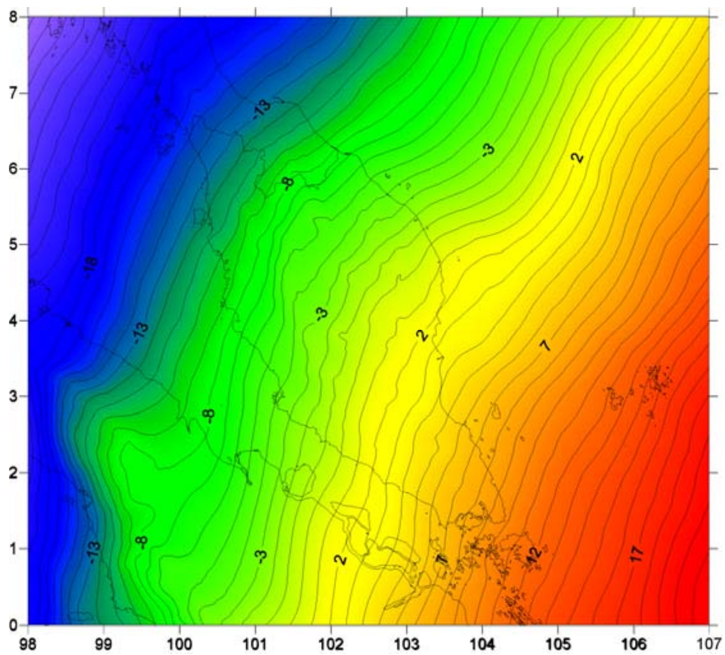

Fig. 4b. Final gravimetric geoid for peninsular Malaysia (WMG03C). Contour interval 1 m.

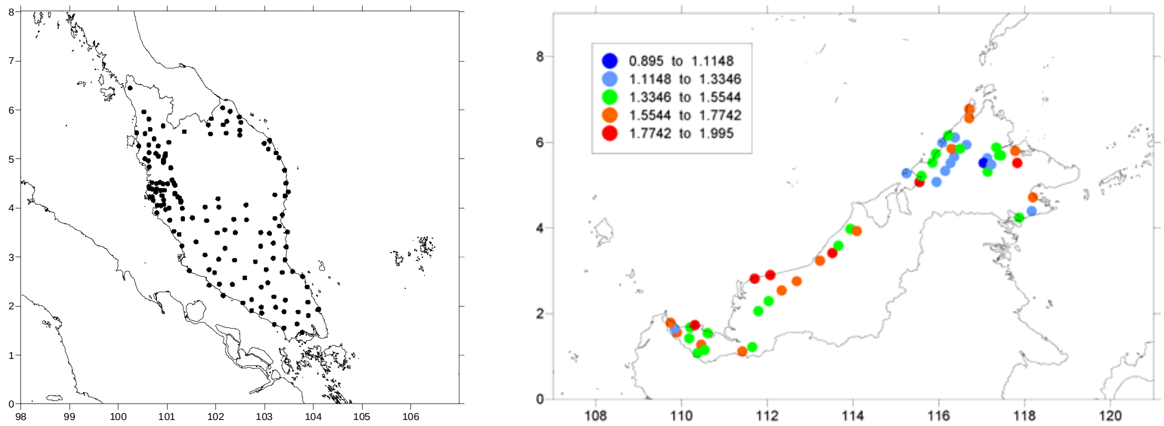

Fig 5. GPS/levelling data in West (left) and East Malaysia (east, with offset N^{GPS} – N in colours).

The computed gravimetric geoids were subsequently fitted to the GPS control by least squares collocation. Generally there was a better agreement between the geoid model (N) from gravity and the GPS-levelling derived values (N^{GPS}) in West than in East Malaysia (cf. Table 2), expressing the use of separate reference tide gauges in Sarawak and Sabah, and possibly larger errors in leveling. The fitted geoid standard deviation is of course just expressing the a priori sigma values applied in the geoid fitting process.

Table 2. *Geoid comparison to GPS-levelling of geoids, West Malaysia (145 points)*

Unit: m	mean	standard dev.
Gravimetric geoid ("geoid12.gri")	0.72	0.12
GPS geoid, fitted to 145 contrained points	0.00	0.05
GRACE model GGM01C alone	0.62	0.26
EGM96	0.47	0.35

Error estimates for the final geoid, fitted to GPS, were derived by least-squares collocation. Because the data sets were too large for running full collocation error estimations, a thinned subset of data was used, and error estimates are therefore slightly pessimistic. The error estimates was obtained by assigning apriori errors to surface gravity of 1 mgal, airborne data of 2 mgal, and GPS-levelling data (N^{GPS}) of 5 cm. The results, shown in Fig. 6a and 6b, clearly illustrate the decay in accuracy of the geoid away from GPS control and gravity data coverage, but also indicates a good coherent accuracy of 2-3 cm r.m.s. in over much of peninsular Malaysia.

It should be stressed, however, that the error estimates do not take into account possible errors, especially in N^{GPS}. The fitted GPS-geoids (during the project denoted WMG03D and EMG03B) in the Malaysian vertical datum system should therefore always be treated with some care as outliers are possible close to such erroneous points, all of which could not be edited away during the fitting process, carried out in close dialogue with JUPEM staff.

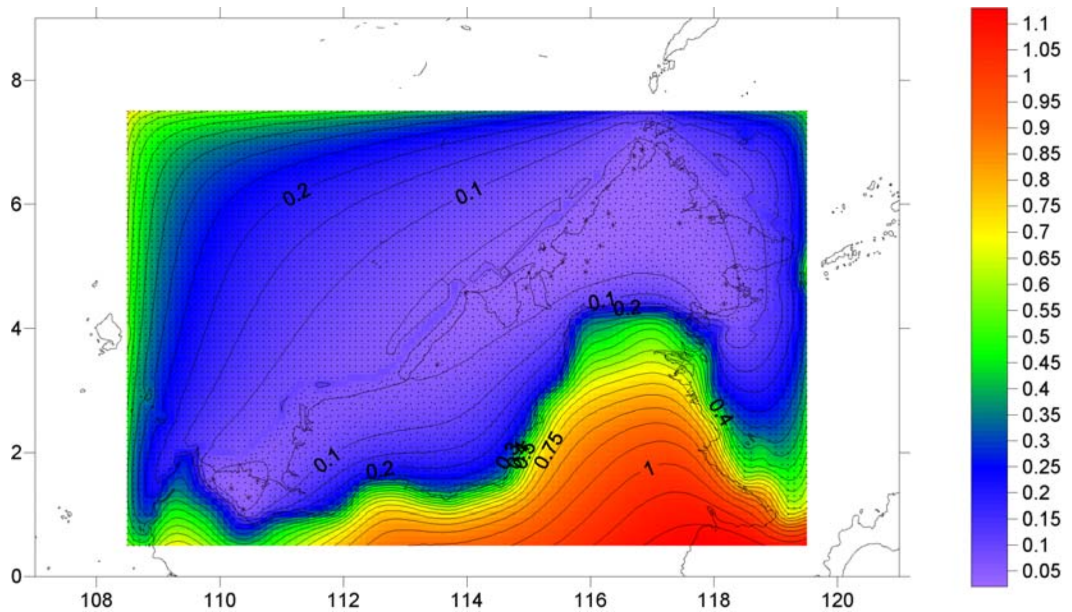

Fig. 6a. Collocation geoid error estimates in East Malaysia. Unit meter.

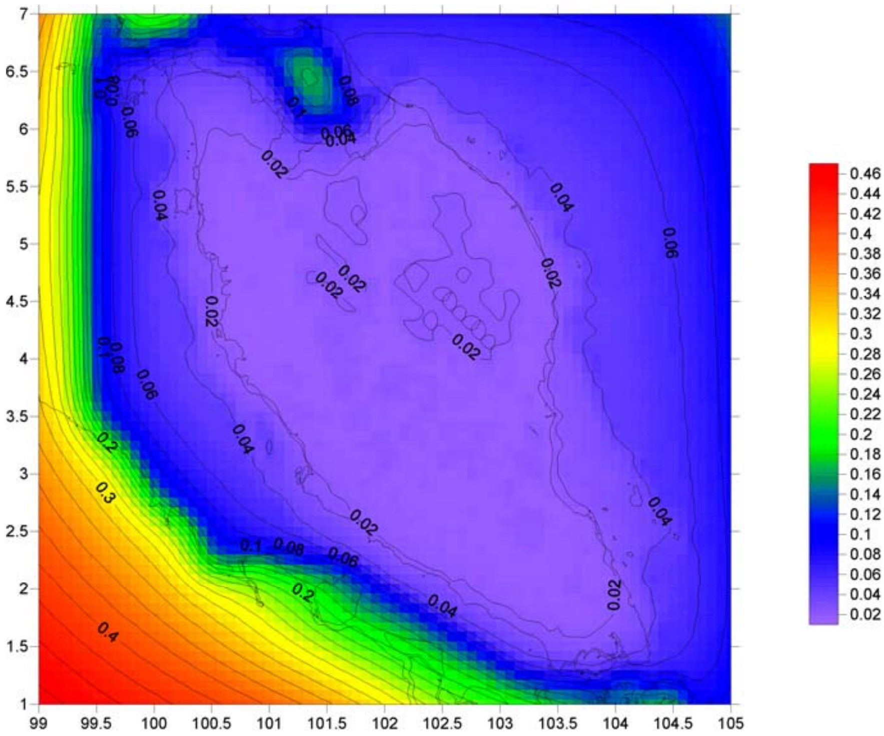

Fig. 7. Collocation error estimates in peninsular Malaysia. Unit: meter.

Possibilities for improving the Malaysian geoid towards the cm-level

With the airborne gravity data set being of high accuracy, and in good agreement with long-wavelength satellite gravity data, there should be no need for further airborne survey activities to improve the Malaysian geoid. It is clear, however, that the geoid would be improved if gravity data could be exchanged across the borders, first of all with Indonesia in Kalimantan, if a similar airborne survey was carried out on the Indonesian side of the border. But also the geoid of the northern provinces in peninsular Malaysia would be improved if more gravity data from Thailand were made available (some data were secured, a.o. thanks to permission to overfly the border region during the airborne survey). For Brunei and Singapore data sufficient data are available already now for the geoid determination.

For the global satellite data, several improvements since 2003 have taken place. Improved satellite altimetry solutions have yielded better satellite gravimetry close to the coasts (e.g., KMS03 global solution), and new improved global GRACE gravity fields are now available (GGM02S and GGM02C, courtesy University of Texas at Austin). The difference between the older and newer GRACE satellite gravity fields at the wavelength bands used in the Malaysian geoids is shown in Fig. 8. This kind of difference shows up in the gravimetric geoids, but not in the *fitted* geoids (where the long wavelength signals are fitted to GPS leveling). There is therefore only expected a marginal improvement in the Malaysian geoid by recomputation with the new satellite fields.

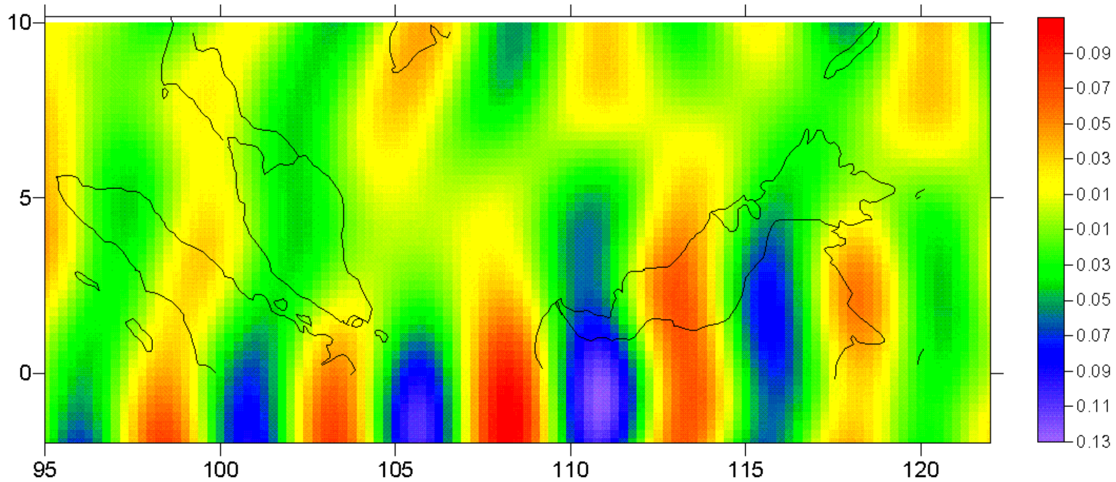

Fig. 8. Difference between GRACE models GGM01S and GGM02S to degree 80 in the Malaysian region. The N-S lineations are typical of GRACE data errors. The present errors will be inherited directly in East Malaysia (where degree 80 was effective cut-off for influence of local gravity data), but not in West Malaysia (where a cut-off of 40 was used in the kernels).

In the southern-central parts of Western Malaysia (e.g., the KL region) the gravimetric geoid should already now have reached the few cm-level r.m.s., as the gravity spacing is relatively dense, and the topography relatively benign. The fit to GPS is, however, not at the expected accuracy level, which is probably due to occasional errors in leveling and/or GPS data (especially antenna offsets to leveling points are often a source of error). Crustal movements can also play a role if subsidence has occurred between the epochs of leveling and GPS observation.

To further improve the Malaysian geoid models we therefore would recommend the following actions:

- *Carefully analyze leveling networks, and possibly perform a new adjustment including analysis of subsidence and land uplift (where possible by repeated surveys).*
- *Reanalyze GPS connections and antenna heights at leveling benchmarks.*
- *Resurvey by leveling and GPS of selected, suspected erroneous points with large geoid outliers.*
- *Make a new GPS-fitted version of the gravimetric geoid as new batches of GPS-levelling data become available, and as RTK-GPS users report problem regions for heights.*

In the longer term gravimetric geoids should be recomputed with updated satellite gravity and satellite altimetry models. GRACE data will likely continue to improve, and with the launch of the GOCE satellite 2006 gravity field information could improve spherical harmonic information to degree 200.

If possible, new gravity data surveys around major city and development areas, e.g. at 2-3 km spacing, could improve the geoid details further, as the relatively large flight height over the Peninsula means that the airborne survey only has limited resolving power for very local gravity field variations. Such terrestrial regional gravity densification would be logistically rather simple, when taking advantage of RTK positioning techniques, and using gravimeters of type Lacoste and Romberg or Scintrex referenced to the recent Malaysian high-precision gravity network. A high production rate of 15-20 stations per day should be possible, meaning e.g. the whole capital region could be covered in few weeks by a single team.

Conclusions

The principles for the new Malaysian geoid model have been outlined. The geoid project is unique in that it is the first time a whole country is mapped completely by airborne gravity, subsequently used for geoid determination. The Malaysian experience will undoubtedly be a useful inspiration for many other countries with a similar mix of well-developed and less accessible (jungle) regions, especially in the Asia-Pacific region.

The gravimetric geoid is apparent accurate to few cm r.m.s., with larger errors closer to the borders. The geoid is fitted to GPS-levelling information, and it is clear that this fit may actually in some cases degrade the high quality of the gravimetric geoid; in other cases it will help control longer wavelength errors. The balance between fit of GPS, and errors in geoid and GPS, is delicate, and undoubtedly there will be many regions in the present geoid where RTK-GPS users can expect problems due to fitting of GPS-levelling data with errors. This situation is similar to all countries of the world implementing national GPS networks and a geoid model for height determination.

It is possible that at some future epoch, with further improvement in satellite gravity models(e.g., after the GOCE mission) that geoid models will actually be so accurate that GPS+geoid alone will be able to define a vertical datum, making the maintenance of leveling networks obsolete. Malaysia has with the airborne survey data an excellent base for such redefinitions in the future.

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