THE INTERNATIONAL GEOMAGNETIC REFERENCE FIELD 1965.0

INTRODUCTION

Alfred J. Zmuda, Reporter
IAGA Working Group on Analysis of the Geomagnetic Field

The choice of an International Geomagnetic Reference Field (IGRF) for the field of internal origin and its secular variation is primarily the concern of two groups with overlapping objectives and personnel: (1) The International Association of Geomagnetism and Aeronomy (IAGA) World Magnetic Survey (WMS) Board and (2) The IAGA Working Group on Analysis of the Geomagnetic Field; this unit was Group No. 8 in Commission 3 (Magnetism of the Earth's Interior) from August 1963 through September 1967 and subsequently became Working Group No. 4 in Commission 2 (Representation of the Main Field). There was general recognition of the need for a reference field to form an agreed basis for main-field calculations and to unify results in studies on, for example, removal of trend to yield surface anomalies, field residuals potentially applicable to the calculation of ionospheric and magnetospheric currents, the shape of a field line, locations of conjugate points, and field values used in the B-L space of trapped particles.

An agreed reference field answering a wide spectrum of such needs was seen as attainable and desirable. There was less prospect of concurrence in higher terms such as might be needed for meeting the critical requirements of surface and air navigation. Regarding secular change, the benefits of adopting a definite model would include not only standardization in adjusting the main-field model to epoch but also convenience in, for example, testing models of dynamo action in the earth's core.

The following is a chronological development of the major discussions and studies on an IGRF. At the XII General Assembly of the International Union of Geodesy and Geophysics (IUGG) in Helsinki, Finland, July 25 to August 6, 1960, the IAGA Committee No. 5 on World Magnetic Survey and Magnetic Charts, with Chairman E. H. Vestine, made a recommendation that was subsequently adopted by the Assembly that as part of the World Magnetic Survey a potential analysis be made providing spherical harmonic terms up to and including a degree and order useful for adequate representation of the data [*Vestine*, 1961]. In a Working Group memorandum of May 1, 1964, the Group's Reporter, A. J. Zmuda, made a proposal, later accepted, that the Group undertake the evaluation of harmonic coefficients. At an informal open meeting of the WMS Board in Pittsburgh on November 18, 1964, the IGRF was discussed by L. R. Alldredge, J. C. Cain, P. F. Fougere, E. H. Vestine, and A. J. Zmuda [*WMS Notes No. 3*, 1966]. At the Herstmonceux Colloquium of October 4 - 6, 1966, *Leaton* [1967] stressed the need for the international adoption of a geomagnetic reference field.

In a major step, *B. R. Leaton and S. R. C. Malin* [1966] and *B. R. Leaton* [1967] examined relatively recent sets, extracted five [*Adam et al.*, 1962, 1963; *Nagata and Oguti*, 1962; *Leaton et al.*, 1965; *Hurwitz et al.*, 1966; *Cain et al.*, 1967], and then consolidated for 1965.0 a set with 48 terms ($n = m = 6$) for the main as well as secular change field as one case and a set with an additional 32 terms in the main field ($n = m = 8$) as another. These seven sets formed the initial group to be evaluated for an IGRF. In a Working Group communication dated July 7, 1967, J. C. Cain noted that the GSFC 12/66-1 field [*Cain et al.*, 1967] gave a better fit to the observations than either of the consolidated sets and suggested a truncation of this GSFC field for the IGRF.

The subsequent deliberations included the following considerations.

Form. The possibilities treated were a single series of spherical harmonics for all points, those on the earth's surface as well as those above and below the surface but external to sources of magnetization; one harmonic series for the earth's surface and one for the other points; and for the surface data, a group of values of the field components at specified grid points. In the spherical harmonic series, whose coefficients should be used? Is there one set clearly superior, or are there several good ones sufficiently close to warrant consolidating into a single group?

Number of terms. Harmonic coefficients contain errors stemming from the uneven distribution of, and errors in, the raw data; from the difficulty of removing transient fluctuations; from the influence of local anomalies that cannot be depicted by any practicable method of analytic representation; and from the mutual dependency of the coefficients [*Chapman and Bartels*, 1940a; *Zmuda and Neuman*, 1961]. With these fundamental limitations, what coefficients are reliable, and, relatedly, how many are to be used in the series for the main field and in the series for the secular variation? *Nagata* [1965] discusses the convergence of the harmonic series, and *Roberts and Scott* [1963] discuss the truncation errors.

Scalar Intensity. Satellite observations of the scalar intensity are numerous and made with a high precision. However, incorporating this element into calculations for the harmonic coefficients is not as direct as that with any of the vector components. *Cain et al.* [1967] and *Tyurmina and Cherevko* [1967] showed ways of using this scalar quantity in harmonic analysis.

Epoch and Duration. The epoch is 1965.0, but what should be the total period of applicability? The period must be long enough to preserve the utility of a reference field, but any reference chosen will undoubtedly contain imperfections that must be ultimately removed.

Coordinate System. Part of the problem concerning an IGRF is to specify the coordinate system to be used and to specify the reference shape of the earth. The question of how to use the reference field, particularly with respect to surface data, cannot be avoided, and a recommendation for a set of coefficients has limited value if its usage is not prescribed. The earth's surface and the geoid resemble an oblate spheroid more closely than they do a sphere, and surface component measurements are made with respect to the local vertical due to gravity and to directions in a plane normal to this vertical. As an expedient procedure compatible with the precision of the available data, some of the pioneer analyses treated the earth as a sphere, and the measured components are along the unit vectors of spherical polar coordinates relative to a spherical earth. The observational data were then analyzed with spherical harmonics to yield the coefficients. From the work of *Kahle et al.* [1964, 1966], *Cain* [1966], and *Cain et al.* [1967], to better the values of the coefficients and to take advantage of the improved observations, it is necessary to take into account the oblateness of the earth and the difference between the measured surface vector components and those referenced to a true sphere, and to make the harmonic analysis with respect to a true sphere.

The following analyses relate to at least one phase of the IGRF: wavelengths of field patterns and the marked decrease after the sixth order of the size of the distinguishing parameters [*Alldredge et al.*, 1963; *Alldredge*, 1965]; removal of trend in magnetic surveys [*Bullard*, 1967]; the separation of anomalies from the main field [*Heirtzler*, 1965]; statistical analysis of magnetic profiles [*Serson and Hannaford*, 1957] and of the main field [*Kautzleben*, 1965a, b]; evaluations and limitations of harmonic series [*Heuring*, 1964; *Cain et al.*, 1965; *Heuring*, 1965; *Cain*, 1966; *Adam et al.*, 1967; *Cain et al.*, 1967; *Heuring et al.*, 1968; *Malin and Pocock*, 1969; *Yukutake*, 1968]; magnetic data for trapped particle evaluations [*Hendricks and Cain*, 1966] and for locating conjugate points [*Leonard*, 1963; *Cain* 1968a]; and spheroidal harmonic functions [*Winch*, 1967].

At the XIV General Assembly of IUGG at St. Gall, Switzerland, September 25 to October 7, 1967, evaluations of harmonic descriptions were given at Assembly sessions and Working Group meetings. With some dissension, the consensus at St. Gall was for truncating the main-field series at the 80th term, $n = m = 8$, and for a secular change series with first derivatives only and containing at most 80 terms, up to $n = m = 8$, and possibly terminating at $n = m = 6$ with 48 terms. Arguments for this arrangement were the following: the predominant part of the geomagnetic field is accounted for; core and crustal features are approximately separated since nearly all the effects of currents in the core are included in the IGRF and the effects of the magnetization of crustal rocks are excluded by the smoothing produced by truncation of series; and additional terms do not contribute significantly to the field description for the purpose of the IGRF. Those opposed to this view believed that more terms were needed to describe the field adequately. No IGRF agreement was reached [*Transactions*, 1967], and it was then decided to allow submission of new harmonic sets until March 15, 1968, to continue the assessments, and to try choosing an IGRF at the IAGA Symposium on the Description of the Earth's Magnetic Field, in Washington, D. C., October 22 - 25, 1968.

In the post-St. Gall period, the Working Group considered eight harmonic sets for the main field and eight for the secular change. Table 1 shows some general characteristics of the sets. The coefficients are available on request to the editor or the individual contributors. Some of these coefficients were also considered in the pre-St. Gall evaluation: the main and secular variation fields of *Cain et al.* [1967] and *Leaton et al.* [1965], the latter resubmitted only as a potential surface reference if the Working Group were to recommend two IGRF's; and the main-field coefficients of *Hurwitz et al.* [1966]. The remaining candidates were new harmonic sets: *Cain* [1968b]; *Fougere* [1967, 1968a]; *Hurwitz* [1968]; *IZMIRAN* [1967a, b]; and *Malin* [1968].

TABLE 1

HARMONIC DESCRIPTIONS SUGGESTED FOR IGRF 1965.0

Authors	*Number of Coefficients* *Main Field*	*Secular Variation*
Cain, Hendricks, Langel, and Hudson [1967]—GSFC 12/66 - 1 field	120, n = m = 10	120 in first and 120 in second derivative
Cain, Hendricks, and Langel [1968b]—POGO	99, n = m = 9	99, n = m = 9
Fougere [1967]	120, n = m = 10	120 in first and 120 in second derivative
Fougere [1968]	120, n = m = 10	120 in first and 120 in second derivative
Hurwitz, Knapp, Nelson, and Watson [1966] for main field and *Hurwitz* [1968] for secular change field	168, n = m = 12 for field of internal origin 168, n = m = 12 for field of external origin	80, n = m = 8
IZMIRAN [1967a] for main field and *IZMIRAN* [1967b] for secular change field	99, n = m = 9	48, n = m = 6
Leaton, Malin, and Evans [1965]	80, n = m = 8	48, n = m = 6
Malin [1968]	80, n = m = 8	48, n = m = 6

The following are some characteristics related to the individual models:

Cain et al. [1967]: The GSFC 12/66-1 model is derived from a sample of all magnetic data available from the interval 1900 - 1964 plus preliminary total field observations from the OGO 2 (1965-81A) spacecraft for epoch 1965.8.

Cain et al. [1968b]: Based on a fit to a selection of scalar intensity values observed by the near-earth POGO spacecraft OGO 2 (1965-81A) and OGO 4 (1967-73A). The root mean square (rms) deviation between observed and computed satellite values equals 11γ.

Fougere [1967]: Derived using 1000 observations from the following sources: OGO 2 [1965-81A] and Vanguard [1959 η] satellites, the Project MAGNET (airplane) data, and surface observatory data.

Fougere [1968]: Here 3000 observations were applied using the sources cited in the earlier model augmented by additional survey data supplied by the U.S. Coast and Geodetic Survey. The method of analysis here and in the earlier model was a variation of one applied by *Jensen and Cain* [1962] but guaranteeing convergence

Hurwitz et al. [1966] and *Hurwitz* [1968a]: The main-field model was derived from approximately 425,000 airborne and surface measurements made since 1900 and reduced to 1965. Secular variation coefficients are for the interval 1960 - 1965 and are based on observatory annual means of X, Y, and Z since 1960.

IZMIRAN: COSMOS-49 [1964-69A] data for altitudes 200-500 km are used to correct a preexisting set of main-field coefficients. Secular coefficients are derived from charts of secular variation.

Leaton et al. [1965]: If two reference fields are considered, this model is submitted for the surface field. All available surface and airborne observations of magnetic declination from 1955.0 and of vector force components from 1945.0 up to mid-June 1964 are brought up to epoch 1965.0 and subjected to harmonic analysis.

Malin [1968]: Input data for the main field are those used by *Leaton et al.* [1965] and those from satellites OGO 2 (1965-81A) and 1964-83C. The secular variation coefficients are based on the observatory annual means published in the Royal Observatory Bulletin No. 134.

The differences in the raw data and methods used in the analyses are reflected in differences of values of the harmonic coefficients of the main field as well as of the secular variation. Figure 1 shows the range of values for each of 80 main-field coefficients from the eight harmonic sets, a range which extends from 7 to 115γ. The following are some examples obtained with the aid of the numbers on the graph and the associated algebraic signs preceding and following the numbers: the g_1^0 coefficient ranges from -30282 to -30388γ, a spread of 106γ; and the h_1^1 coefficient, from 5707 to 5782γ, a spread of 75γ. Figure 2 shows the range of values for each of 80 secular-variation first-derivative terms. Here the coefficients have small values but the spread about a mean is relatively large. For a specific coefficient the range of values lies within 2 to 21γ. There are some coefficients in specific harmonic sets considerably outside the range of values for the comparable coefficients in the remaining sets. For example, the $\dot{g}_1^0$ coefficient in the POGO series has the value 26γ/year while the values for the other sets lie within the range 12 to 17γ/year.

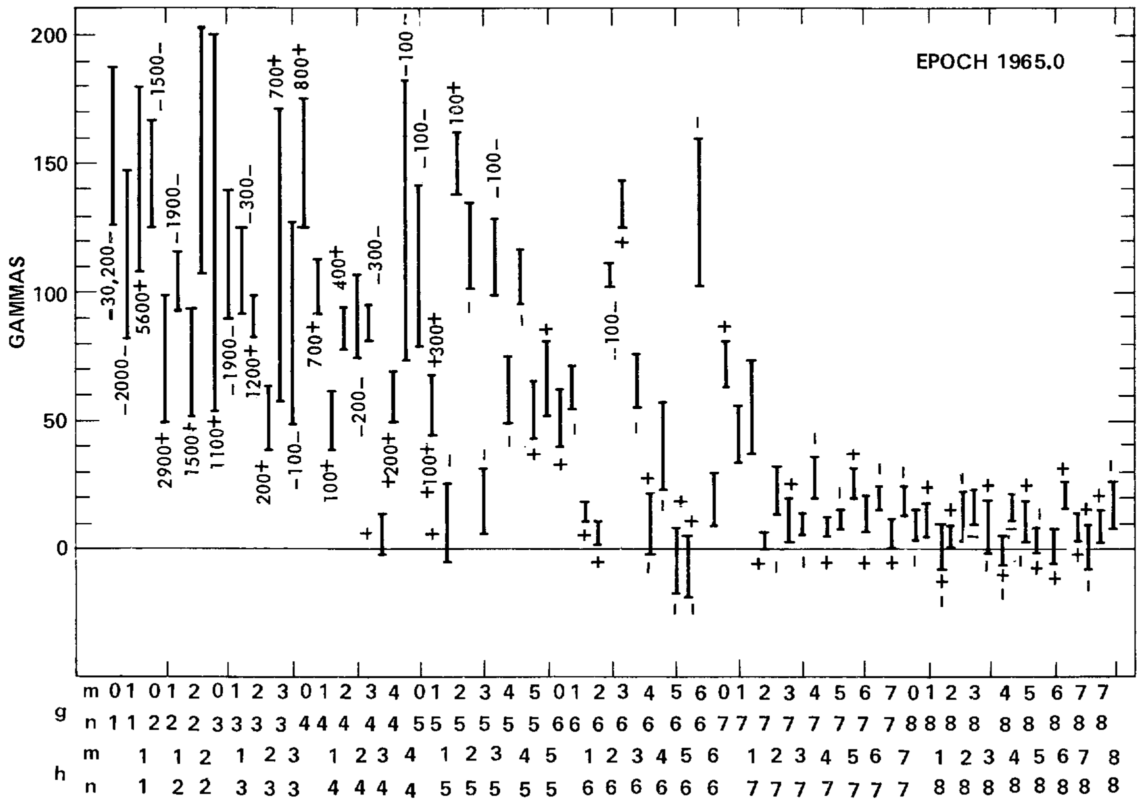


Fig. 1. *Range in Harmonic Coefficients Submitted for the Main Field of IGRF.*

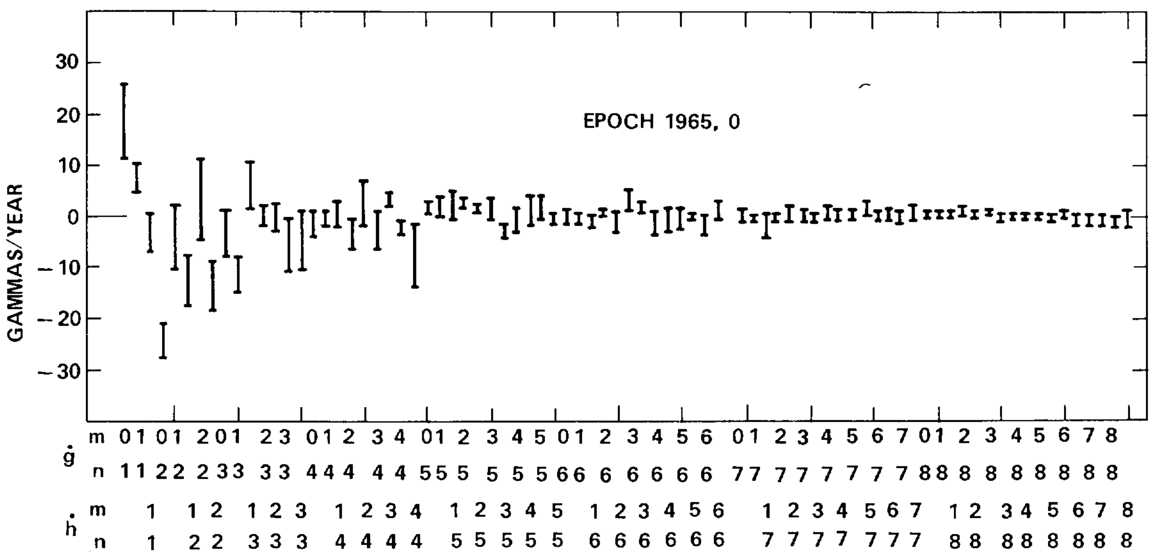


Fig. 2. *Range in Harmonic Coefficients Submitted for Secular Change of IGRF.*

The following persons served at some time on the Working Group:

IAGA WORKING GROUP ON ANALYSIS OF THE GEOMAGNETIC FIELD

Members	*Consultants*
N. P. Benkova, U.S.S.R.	L. R. Alldredge, U.S.A.
J. C. Cain, U.S.A.	E. C. Bullard, U.K.
P. F. Fougere, U.S.A.	E. Dawson, Canada
N. Fukushima, Japan	D. G. Knapp, U.S.A.
F. T. Heuring, U.S.A.	T. Nagata, Japan
L. Hurwitz, U.S.A.	J. H. Nelson, U.S.A.
A. B. Kahle, U.S.A.	V. P. Orlov, U.S.S.R.
Yu. D. Kalinin, U.S.S.R.	N. V. Pushkov, U.S.S.R.
H. Kautzleben, Dem. Rep. of Germany	P. H. Serson, Canada
B. R. Leaton, U.K.	E. H. Vestine, U.S.A.
S. R. C. Malin, U.K.	T. Yukutake, Japan
J. F. McClay, U.S.A.	
T. Nagata, Japan	
D. E. Winch, Australia	
A. J. Zmuda, U.S.A., Reporter of Working Group	

References included in this and the other papers on the IGRF are listed at the end of the IGRF section.

EVALUATIONS BY VARIOUS AUTHORS

Natalia P. Benkova, Shmyea Sh. Dolginov, Lia O. Tyurmina, and Tamara N. Cherevko

Academy of Sciences of the U.S.S.R.

Institute of Terrestrial Magnetism, Ionosphere, and Radiowave Propagation (IZMIRAN)

Moscow, U.S.S.R.

The use of an analytic model as a reference for the main geomagnetic field has applicability to at least three problems associated with the main field: the secular variation, the outward extrapolation, and the analytic representation. In this connection we investigated seven of the models proposed for an IGRF. Representations, which are discussed in a preceding article by Zmuda, are often labeled as follows: F_1 for *Fougere* [1967], G for GSFC 12/66-1 from *Cain et al.* [1967], H for *Hurwitz et al.*, [1966], I for IZMIRAN, L for *Leaton et al.*, [1965] and M for a median field with n = m = 8, and contained in the letter from B. R. Leaton dated March 3, 1967, P or POGO for the field from *Cain et al.* [1968b].

It is reasonable to divide the results of this work into three categories: comparisons between the computed intensities and those measured with the COSMOS-49 satellite; comparisons between the world magnetic charts and computed fields; and comparisons between models derived using satellite data and the observations used in their derivations—the COSMOS-49 data and the IZMIRAN model, the POGO data (from the OGO 2 and OGO 4 satellites) and the POGO model.

COMPARISON WITH COSMOS-49 MEASUREMENTS

As a basis for comparison, the measured values of the scalar intensity F were taken. Due to the comparatively low altitudes (260-490 km) and the near-circular orbit of inclination 50° as well as the quality of the measurements on magnetically quiet days, these data are very suitable to verify which of the analytical models offers the better outward extrapolation. Undoubtedly, such a comparison will be correct only within the latitudinal band ±50°. The scalar values were computed using all the coefficients in each model; the COSMOS-49 data used totaled 4000 values. The magnitudes and distribution of ΔF = F(COSMOS) − F(model) are shown in Fig. 1 in a series of histograms; root mean square (rms) and arithmetical means are also given for each of the models. Bearing in mind the accuracy of the measurements (positional errors being included) to be about ±30γ, one could initially consider deviations higher than 100γ as a result of some random errors or, if they are of a gross character, as a result of a low accuracy of the given model. But it was revealed that a rejection of all values with $|\Delta F| \geqq 100\gamma$ would lead to a rejection of a great amount of ΔF's for the models derived using mainly surface data. Therefore, the width of ΔF filter was increased up to 225γ. That had but little effect on the IZMIRAN, GSFC, and POGO models, since their ΔF's rarely exceeded ±100γ. As illustrated in the figure, the satellite models (or models with a great percentage of satellite data) give a much better fit to the measured values than the models based on surface or near-surface data. The IZMIRAN model gives

IGRF 1965.0

IAGA Commission 2 Working Group 4 on Analysis of the Geomagnetic Field

At an open meeting in Washington, D.C., on October 24, 1968, the Working Group Members and Consultants chose the International Geomagnetic Reference Field (IGRF) 1965.0. The reference field was endorsed by the International Association of Geomagnetism and Aeronomy (IAGA) World Magnetic Survey Board on October 28, 1968, and by the IAGA Executive Committee in February 1969.

The reference is a series of solid spherical harmonics and their derivatives in geocentric spherical coordinates, describing the geomagnetic potential V and the field components through

$$V = a \sum_{n=1}^{n=8} \sum_{m=0}^{m=n} \left(\frac{a}{r}\right)^{n+1} \cdot [g_n^m \cos m\lambda + h_n^m \sin m\lambda] P_n^m(\cos\theta)$$

$$X = \frac{1}{r}\frac{\partial V}{\partial \theta} = \sum_{n=1}^{n=8} \sum_{m=0}^{m=n} \left(\frac{a}{r}\right)^{n+2} \cdot [g_n^m \cos m\lambda + h_n^m \sin m\lambda] \frac{d}{d\theta} P_n^m(\cos\theta)$$

$$Y = \frac{-1}{r\sin\theta}\frac{\partial V}{\partial \lambda} = \sum_{n=1}^{n=8} \sum_{m=0}^{m=n} \left(\frac{a}{r}\right)^{n+2} \frac{-m}{\sin\theta} \cdot [-g_n^m \sin m\lambda + h_n^m \cos m\lambda] P_n^m(\cos\theta)$$

$$Z = \frac{\partial V}{\partial r} = \sum_{n=1}^{n=8} \sum_{m=0}^{m=n} -(n+1) \left(\frac{a}{r}\right)^{n+2} \cdot [g_n^m \cos m\lambda + h_n^m \sin m\lambda] P_n^m(\cos\theta),$$

where X, Y, and Z represent, respectively, the northward, eastward, and downward components of the intensity in geocentric coordinates; a, the radius (6371.2 km) of the reference sphere; r, the radial distance from the center of the reference sphere; θ, the colatitude, or $90° - \beta$ where β is the latitude; λ the east longitude measured from Greenwich; $P_n^m(\cos\theta)$, an associated Legendre function of degree n and order m and of the Schmidt quasi-normalized type [*Chapman and Bartels*, 1940b]; and g_n^m and h_n^m, spherical harmonic coefficients. Each series has 80 terms, up to n = m = 8. The scalar intensity $F = [X^2 + Y^2 + Z^2]^{1/2}$; the horizontal intensity $H = [X^2 + Y^2]^{1/2}$; the declination $D = \tan^{-1}\left[\frac{Y}{X}\right]$; the inclination $I = \tan^{-1}\left[\frac{Z}{H}\right]$. The function $P_n^m(\cos\theta)$ may be written as

$$P_n^m(\mu) = \frac{1}{2^n n!}\left[\frac{\epsilon_m (n-m)!(1-\mu^2)^m}{(n+m)!}\right]^{\frac{1}{2}} \frac{d^{m+n}(\mu^2-1)^n}{d\mu^{m+n}}$$

where $\mu = \cos\theta$; $\epsilon_m = 1$ for $m = 0$ and $\epsilon_m = 2$ for $m \geqq 1$.

The epoch is $t_0 = 1965.0$ and the value of a harmonic coefficient for another time t is obtained from

$$C_n^m(t) = C_n^m(t_0) + \dot{C}_n^m(t - t_0)$$

where $\dot{C}_n^m$ equals the secular change of the coefficient in gammas/year ($1\gamma = 10^{-5}$ gauss).

Table 1 shows the IGRF coefficients, which apply to the period 1955.0 - 1972.0. For dates after the

TABLE 1

IGRF 1965.0 COEFFICIENTS

		Main Field (γ)		*Secular Change* (γ/yr)	
n	*m*	g_n^m	h_n^m	$\dot{g}_n^m$	$\dot{h}_n^m$
1	0	−30339		15.3	
1	1	−2123	5758	8.7	−2.3
2	0	−1654		−24.4	
2	1	2994	−2006	0.3	−11.8
2	2	1567	130	−1.6	−16.7
3	0	1297		0.2	
3	1	−2036	−403	−10.8	4.2
3	2	1289	242	0.7	0.7
3	3	843	−176	−3.8	−7.7
4	0	958		−0.7	
4	1	805	149	0.2	−0.1
4	2	492	−280	−3.0	1.6
4	3	−392	8	−0.1	2.9
4	4	256	−265	−2.1	−4.2
5	0	−223		1.9	
5	1	357	16	1.1	2.3
5	2	246	125	2.9	1.7
5	3	−26	−123	0.6	−2.4
5	4	−161	−107	0.0	0.8
5	5	−51	77	1.3	−0.3
6	0	47		−0.1	
6	1	60	−14	−0.3	−0.9
6	2	4	106	1.1	−0.4
6	3	−229	68	1.9	2.0
6	4	3	−32	−0.4	−1.1
6	5	−4	−10	−0.4	0.1
6	6	−112	−13	−0.2	0.9
7	0	71		−0.5	
7	1	−54	−57	−0.3	−1.1
7	2	0	−27	−0.7	0.3
7	3	12	−8	−0.5	0.4
7	4	−25	9	0.3	0.2
7	5	−9	23	0.0	0.4
7	6	13	−19	−0.2	0.2
7	7	−2	−17	−0.6	0.3
8	0	10		0.1	
8	1	9	3	0.4	0.1
8	2	−3	−13	0.6	−0.2
8	3	−12	5	0.0	−0.3
8	4	−4	−17	0.0	−0.2
8	5	7	4	−0.1	−0.3
8	6	−5	22	0.3	−0.4
8	7	12	−3	−0.3	−0.3
8	8	6	−16	−0.5	−0.3

epoch 1972.0, recommendations will be made at the XV General Assembly of the International Union of Geodesy and Geophysics (IUGG) in 1971; future modifications of the IGRF are likely to apply only to the secular change coefficients. For the period near but preceding 1955.0, the 1945.0 field of *Vestine et al.* [1947] is suggested. A Fortran program to compute field values from the IGRF 1965.0 is obtainable from the U.S. National Space Science Data Center, NASA Goddard Space Flight Center, Greenbelt, Maryland, U.S.A., 20771; the Institute of Geological Sciences, Royal Greenwich Observatory, Herstmonceux Castle, Hailsham, Sussex, England; or the World Data Center A for Geomagnetism, U. S. Coast and Geodetic Survey—ESSA, Rockville, Maryland, U.S.A., 20852.

The centered dipole of the IGRF has a magnetic moment of 8.01×10^{25} gauss cm^3 for 1965.0, and its axis intercepts the earth's surface at the point with $\beta = 78.6°$N and $\lambda = 290.2°$E, and at that with $\beta = 78.6°$S and $\lambda = 110.2°$E.

The main field coefficients represent a composite from four sets listed in order of importance in Table 2 and weighted proportional to $1/\sigma^2$ with the σ values shown in the table. The secular change coefficients are the resultant from the five sets of Table 2, all with equal weights. The coefficients were published in three journals [*IAGA*, 1969].

TABLE 2
HARMONIC SETS INCLUDED IN IGRF 1965.0

Title and/or Authors	*Main Field, Weight* σ	*Secular Change*
GSFC 12/66 - 1, *Cain, Hendricks, Langel and Hudson* [1967]	40	All five sets had equal weights for the secular change coefficients
Fougere [1968]	70	
Malin [1968]	80	
IZMIRAN, main field coefficients from *IZMIRAN* [1967a]; secular change from *IZMIRAN* [1967b]	100	
Hurwitz [1968] (Secular change only)		

It will sometimes be desirable to transform from geodetic to geocentric coordinates and from the vector field in geodetic coordinates to that in geocentric. Fundamental considerations have been discussed in a number of publications: coordinate transformations for points on the earth's surface [*Bomford*, 1962]; and coordinate and field transformations and their importance at the earth's surface [*Kahle et al.*, 1964, 1966; *Kahle*, 1968] and at points above the earth's surface [*Cain*, 1966; *Cain et al.*, 1967; *Cain et al.*, 1968c; *Malin and Pocock*, 1969]. The following treatment incorporates various parts from these publications.

For use with the IGRF, the earth's surface has an equatorial radius A and flattening f from the International Ellipsoid recently adopted by the International Astronomical Union [*Transactions of the International Astronomical Union*, 1966]:

$$A = 6378.160 \text{ km} \quad (1)$$

$$f = \frac{A - B}{A} = \frac{1}{298.25} = 0.0033529 \quad (2)$$

where B is the polar radius (6356.775 km).

The mean radius $\frac{2A + B}{3}$ of this ellipsoid equals

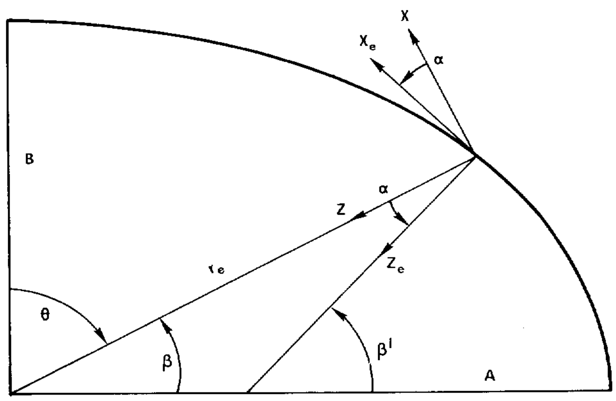


Fig. 1. Geocentric and Geodetic Coordinates and Vector Field Components Referred to a Sphere and to a Spheroid. See Text for Discussion.

6371.03 km and is less than the radius a (6371.2 km) of the IGRF reference sphere that is, however, retained because of its use in deriving the harmonic coefficients, though it represents also the mean radius of an earlier ellipsoid.

The latitude of a point on the surface of the earth as determined astronomically refers to the local vertical, normal to the geoid (here taken to be the spheroid).

This is the geodetic latitude, β' (Fig. 1) which relates to the geocentric latitude β through

$$\tan \beta = (1 - e^2) \tan \beta' \qquad (3)$$

where $e^2 = \frac{A^2 - B^2}{A^2}$ with e the eccentricity and

where, following the normal practice, each latitude is positive in the northern hemisphere and negative in the southern.

The geocentric radial distance, r_e, to the surface of the spheroid can be found from the equations

$$\begin{aligned} r_e^2 &= x^2 + y^2 + z^2 \\ x &= \nu \cos \beta' \cos \lambda \\ y &= \nu \cos \beta' \sin \lambda \\ z &= (1 - e^2)\nu \sin \beta' \qquad (4) \\ \nu &= \frac{A}{(1 - e^2 \sin^2 \beta')^{\frac{1}{2}}} \end{aligned}$$

with λ the east longitude.

For a point h above the earth's surface and along the normal to the ellipsoid, the geocentric colatitude and geodetic latitude connect through

$$\cos \theta = \frac{\sin \beta'}{[P \cos^2 \beta' + \sin^2 \beta']^{\frac{1}{2}}} \qquad (5)$$

where

$$P = \frac{h[A^2 - (A^2 - B^2) \sin^2 \beta']^{\frac{1}{2}} + A^2}{h[A^2 - (A^2 - B^2) \sin^2 \beta']^{\frac{1}{2}} + B^2}. \qquad (6)$$

The geocentric radial distance r relates to the geodetic latitude β' and height h through

$$r^2 = h^2 + 2h[A^2 - (A^2 - B^2) \sin^2 \beta']^{\frac{1}{2}} + \frac{A^4 - (A^4 - B^4) \sin^2 \beta'}{A^2 - (A^2 - B^2) \sin^2 \beta'}. \qquad (7)$$

Additional relationships of interest are

$$\cos \alpha = \frac{1}{r} [h + \sqrt{A^2 \cos^2 \beta' + B^2 \sin^2 \beta'}], \qquad (8)$$

$$\sin \alpha = \frac{1}{r} [A^2 - B^2][A^2 \cos^2 \beta' + B^2 \sin^2 \beta']^{-\frac{1}{2}} \cos \beta' \sin \beta', \qquad (9)$$

$$\cos \beta = \cos \beta' \cos \alpha + \sin \beta' \sin \alpha, \qquad (10)$$

$$\sin \beta = \sin \beta' \cos \alpha - \cos \beta' \sin \alpha, \text{ and} \qquad (11)$$

$$\alpha = \beta' - \beta. \qquad (12)$$

Transformation equations between field components (X, Y, and Z) derived in the geocentric coordinates of the IGRF and those (X_e, Y_e, and Z_e) referenced to a geodetic coordinate system are

$$\begin{aligned} X_e &= X \cos \alpha + Z \sin \alpha \\ Y_e &= Y \qquad (13) \\ Z_e &= -X \sin \alpha + Z \cos \alpha. \end{aligned}$$

Both sets of field components can be obtained from the Fortran programs mentioned earlier.

IGRF CHARTS

Brian R. Leaton
Chairman, IAGA Commission 2
Institute of Geological Sciences
Hailsham, Sussex, U.K.

Charts for all seven elements and annual secular change are given in Figs. 1-14 for the surface of the earth. A stereographic projection has been used to enable worldwide data to be shown on one sheet. These charts are meant to correspond to the IGRF.

Draft charts were produced using a computer contour program at NASA-Goddard Space Flight Center, U.S.A., under the supervision of Dr. J. C. Cain. The charts were completed manually by Mr. M. Fisher at the Institute of Geological Sciences, U.K. This work involved the removal of plotter waviness, supplying where necessary the missing lines (mostly in D and $\dot{D}$), labeling, and general tidying.

Grid values for the elements and their secular change are given in IAGA Bulletin no. 29 by myself and D. R. Barraclough.

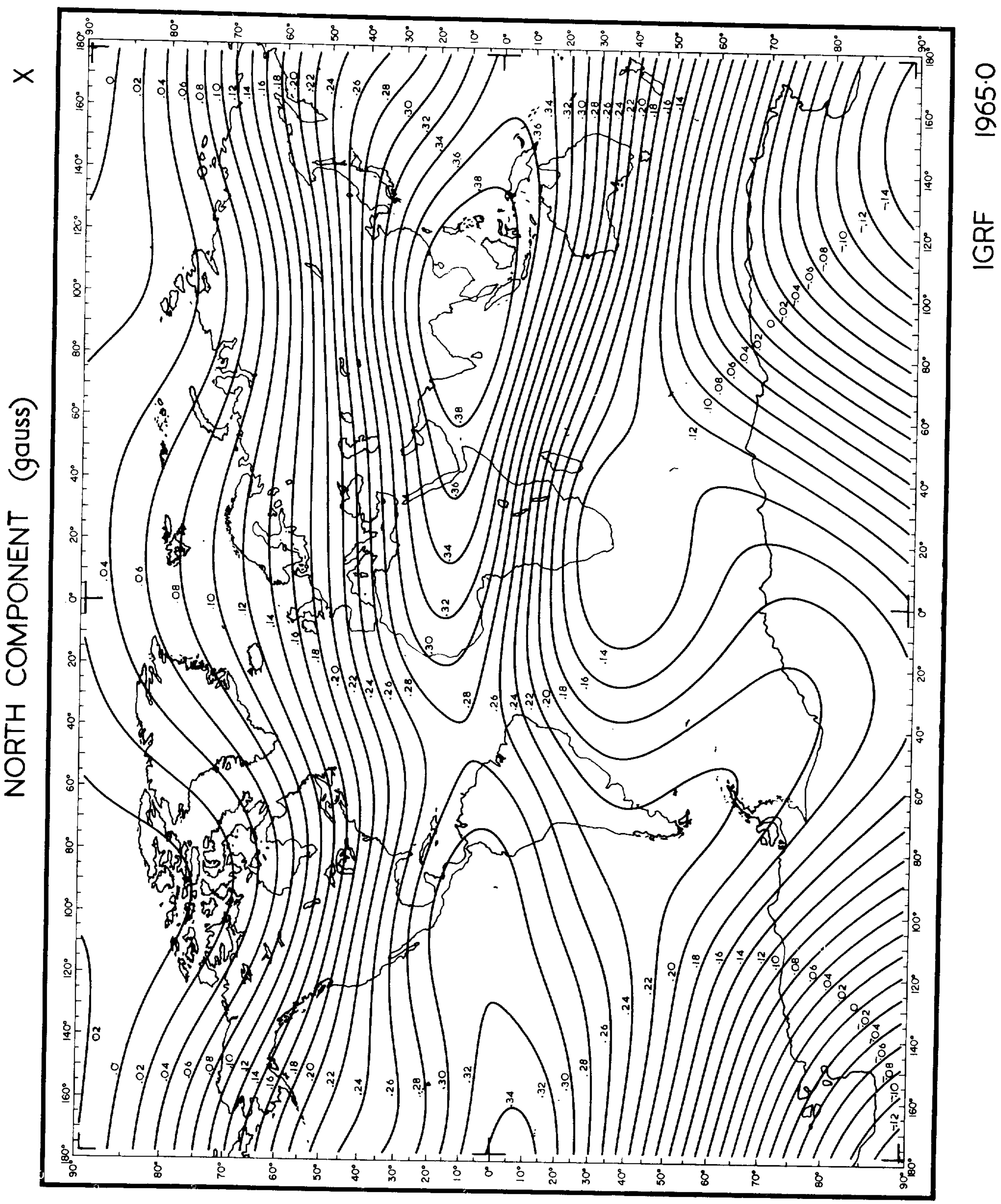
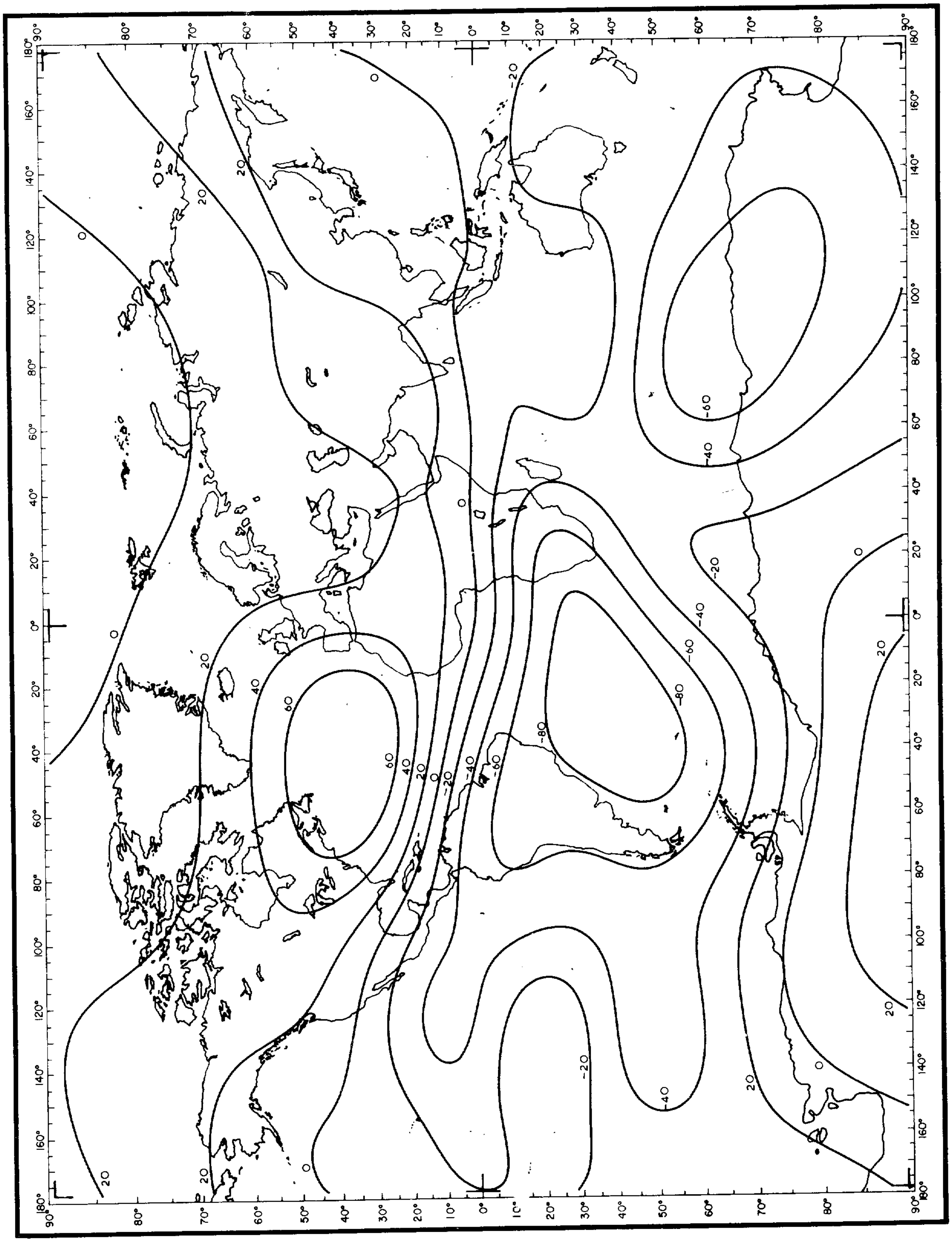


Fig. 1

SECULAR CHANGE OF NORTH COMPONENT (gammas/year) $\dot{X}$



IGRF 1965·0

Fig. 2

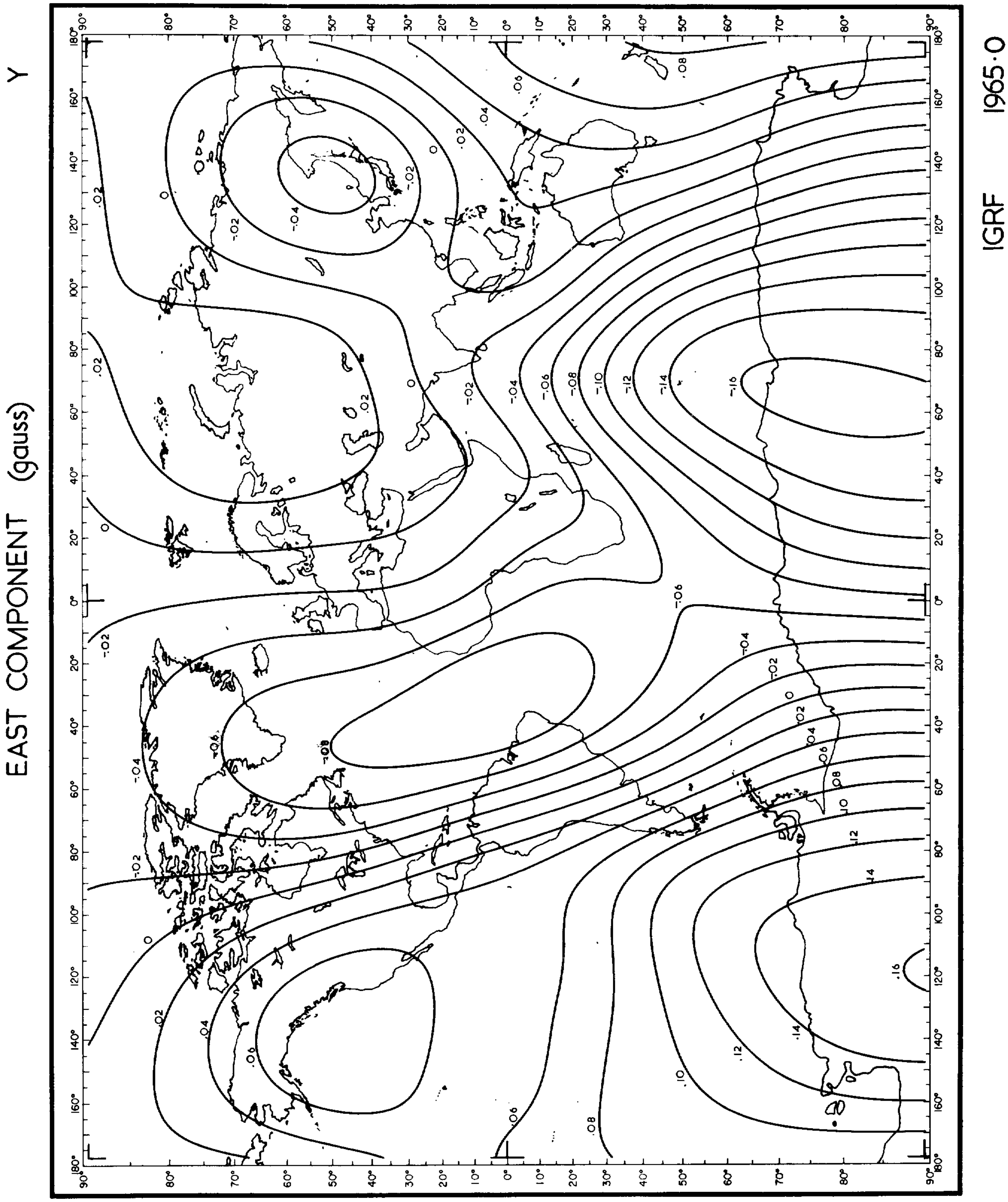


Fig. 3

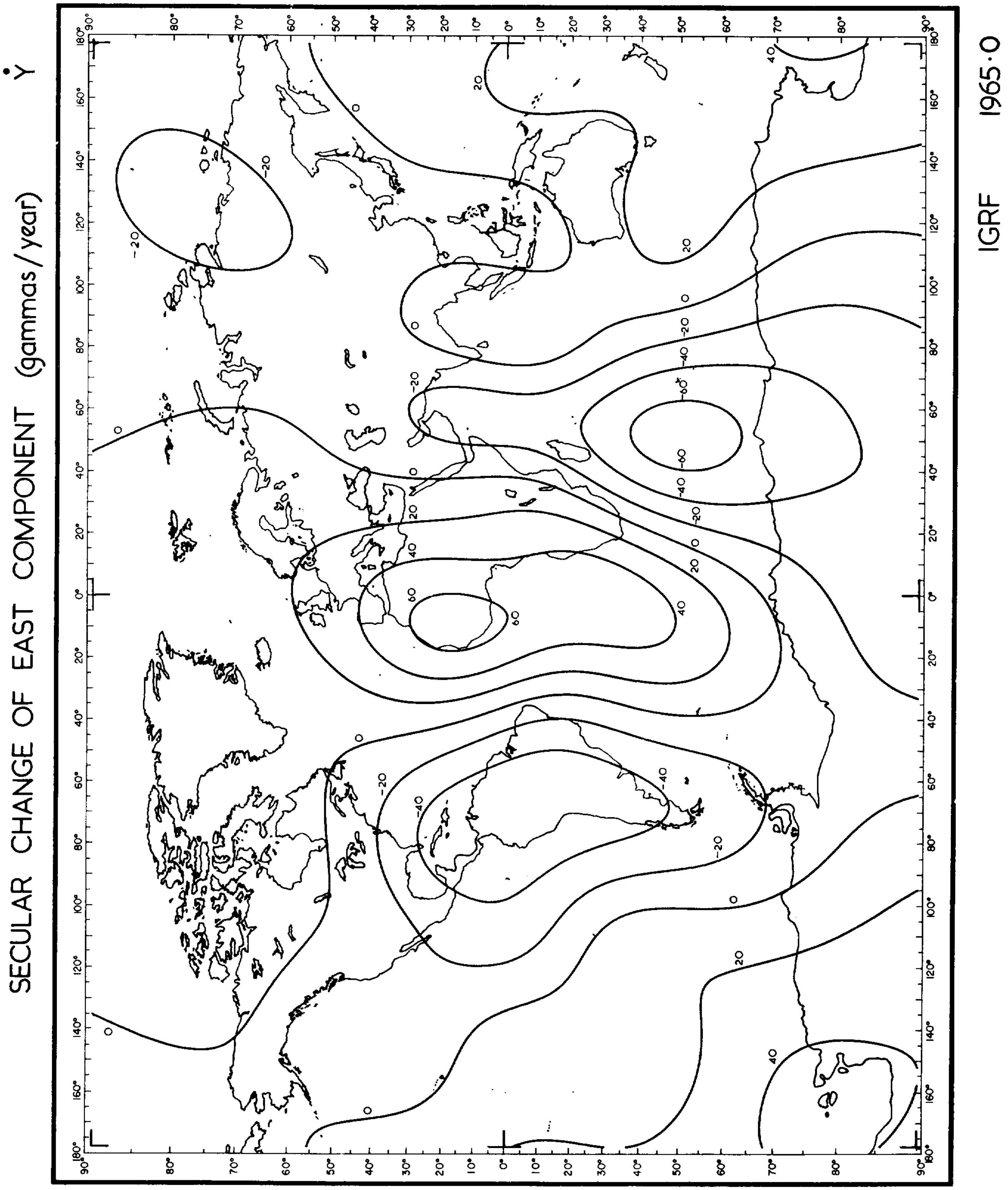


Fig. 4

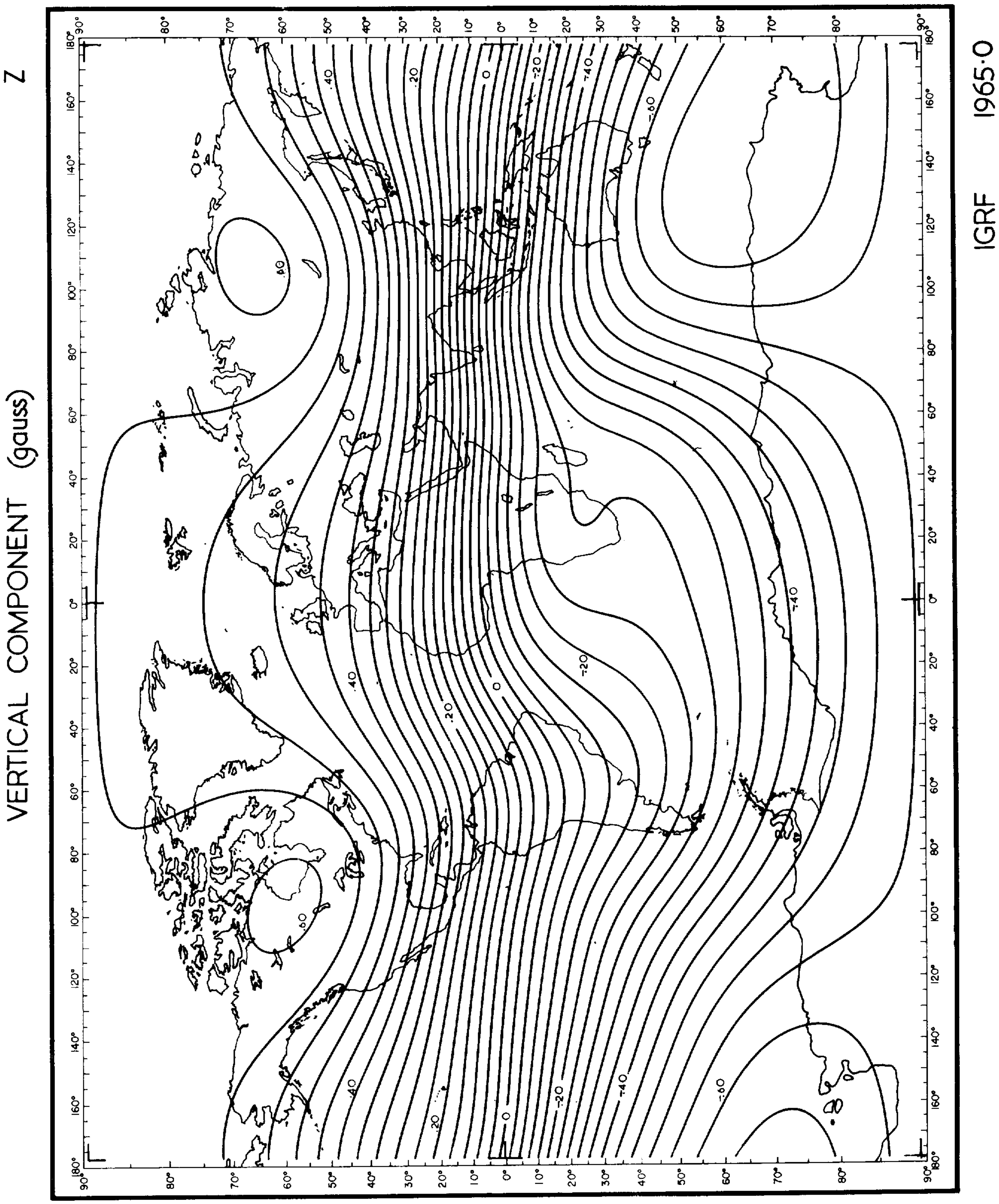


Fig. 5

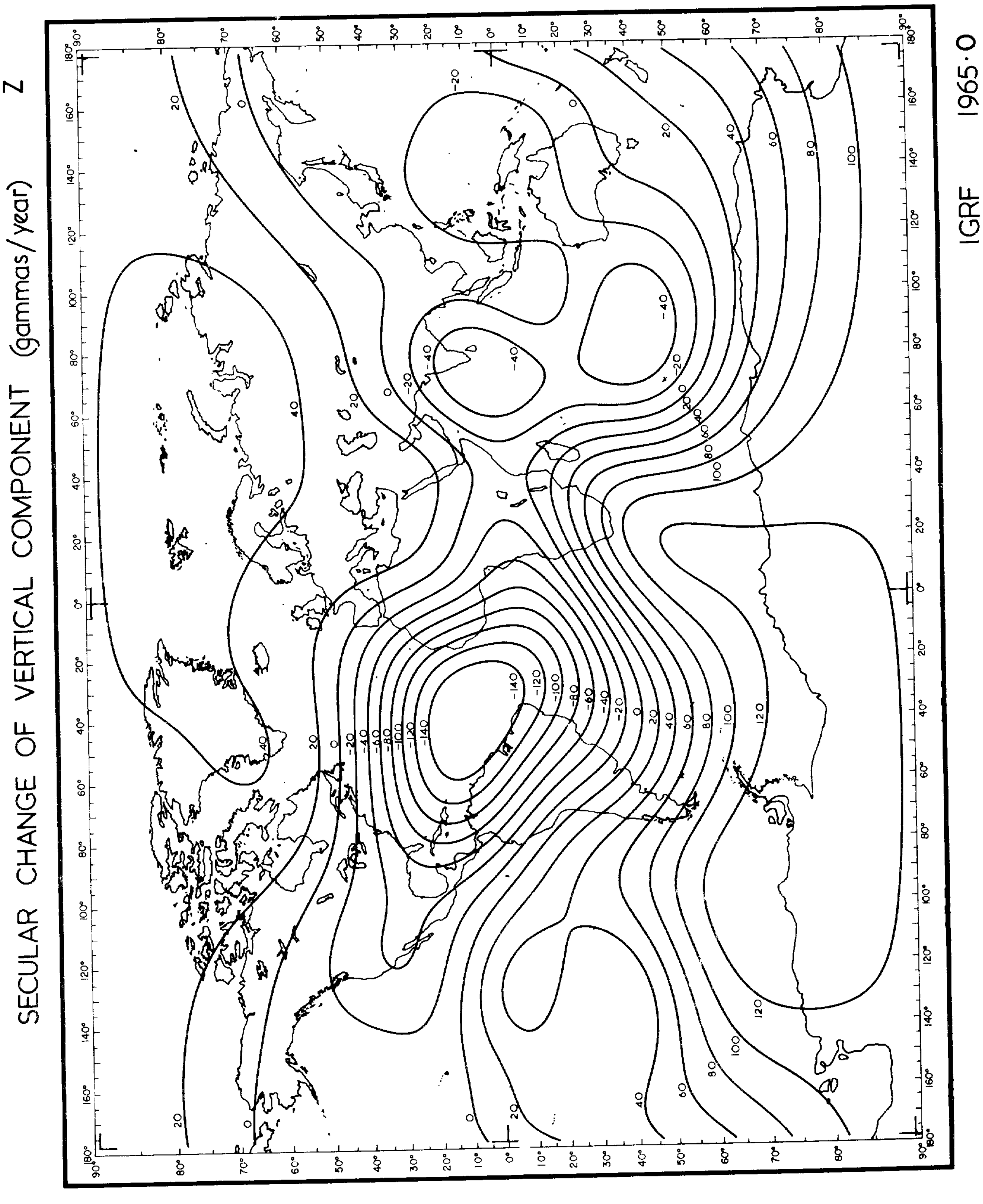

SECULAR CHANGE OF VERTICAL COMPONENT (gammas/year)
Ż
IGRF 1965·0

Fig. 6

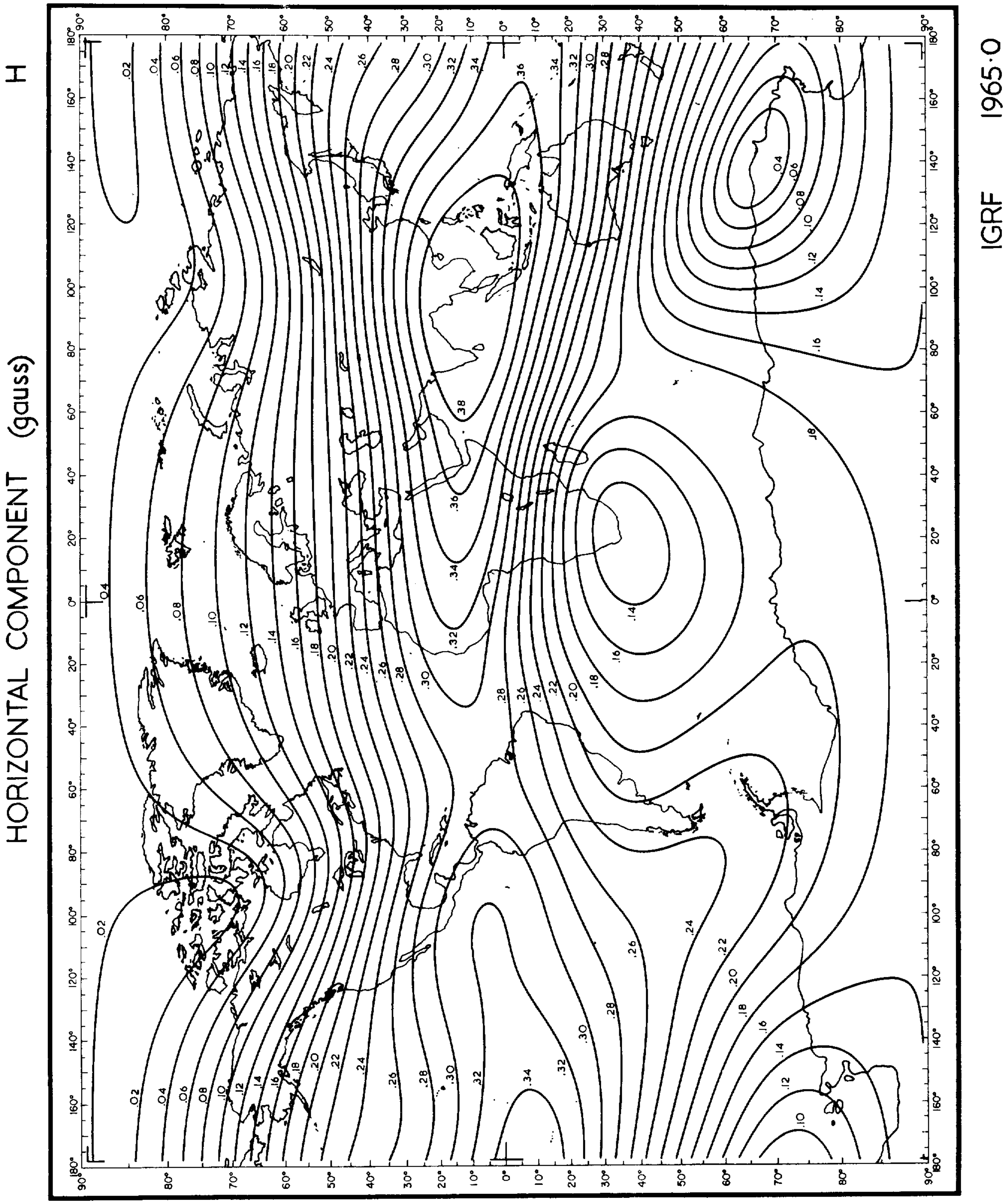


Fig. 7

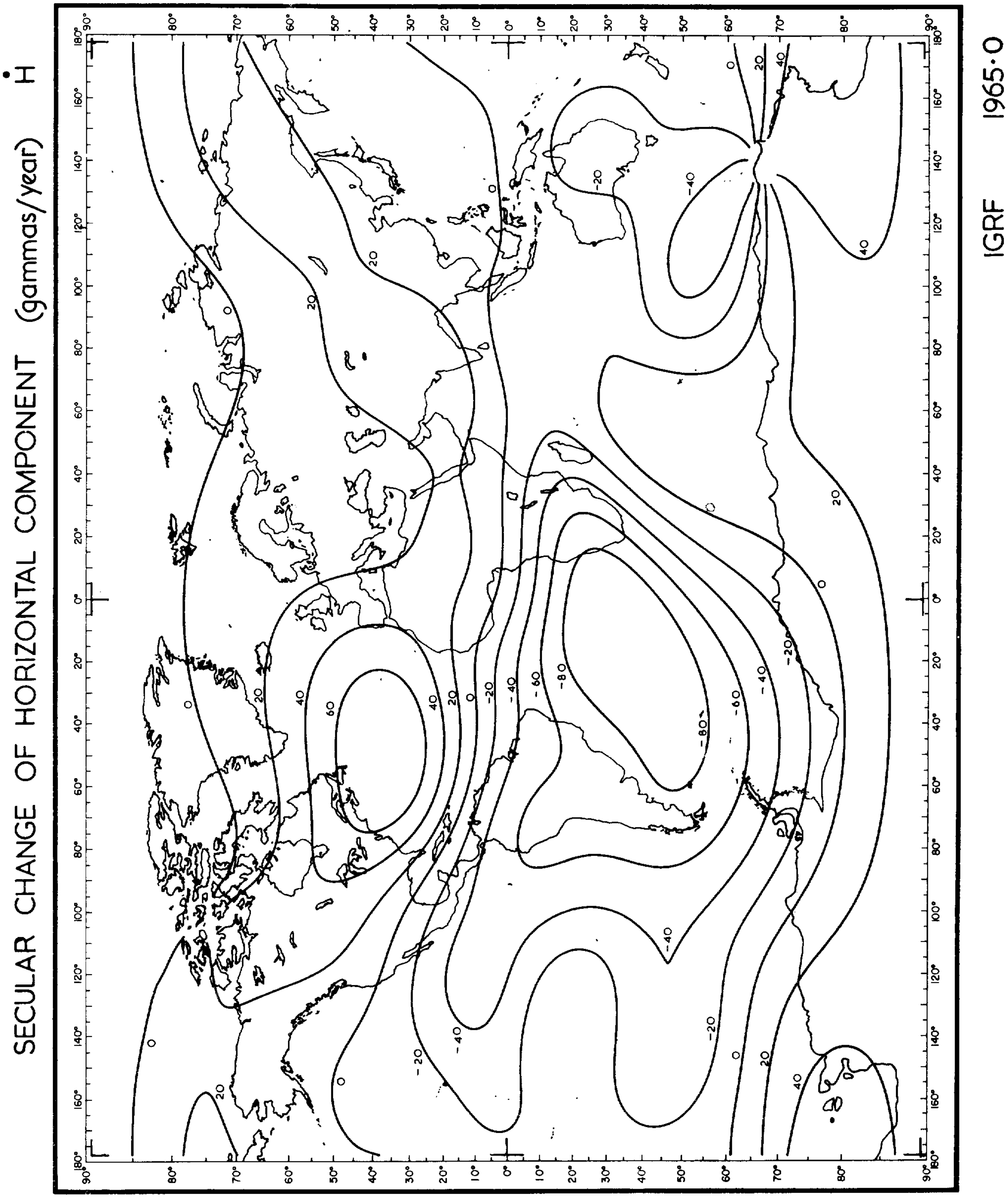

SECULAR CHANGE OF HORIZONTAL COMPONENT (gammas/year) Ḣ
IGRF 1965·0

Fig. 8

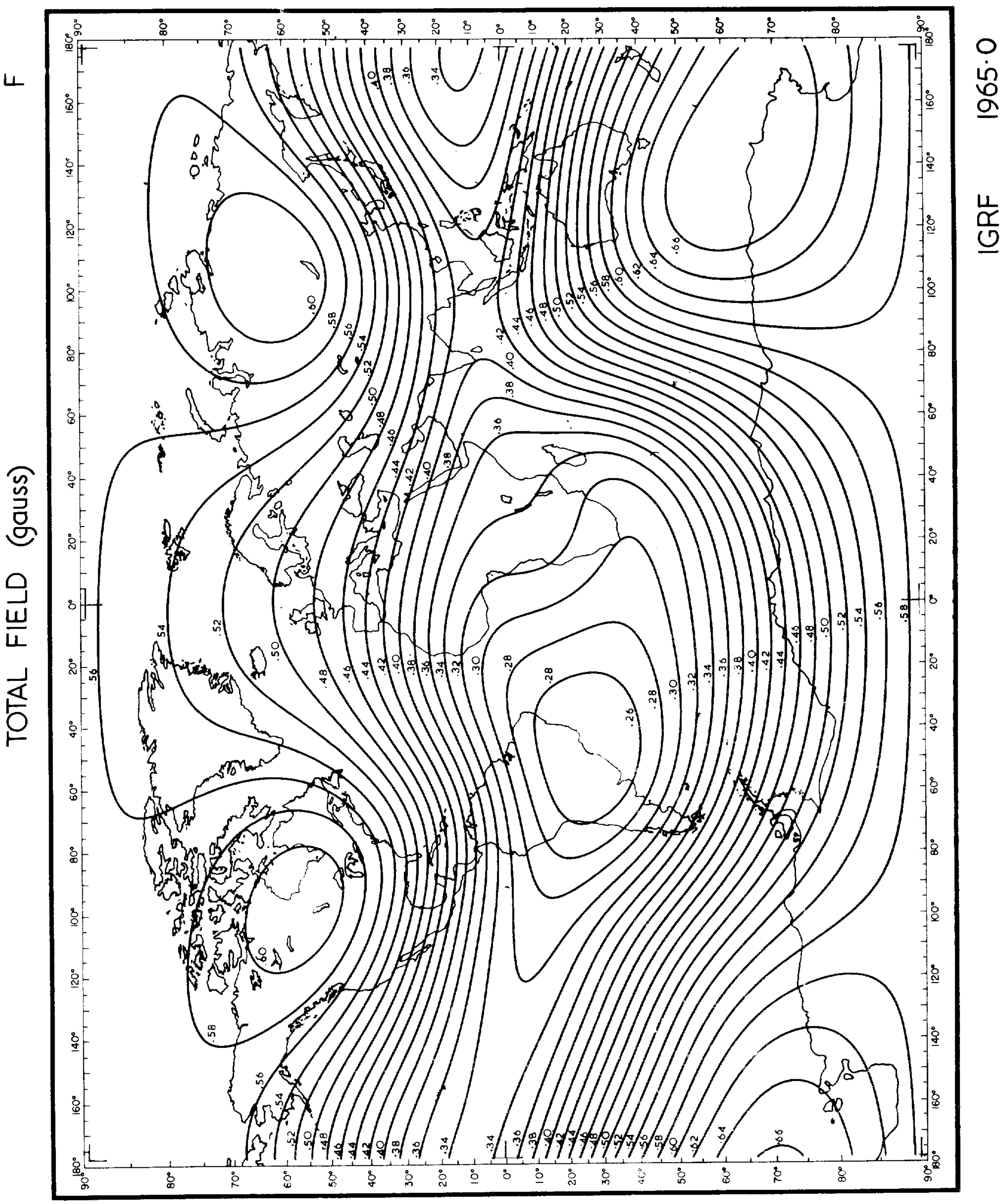


Fig. 9

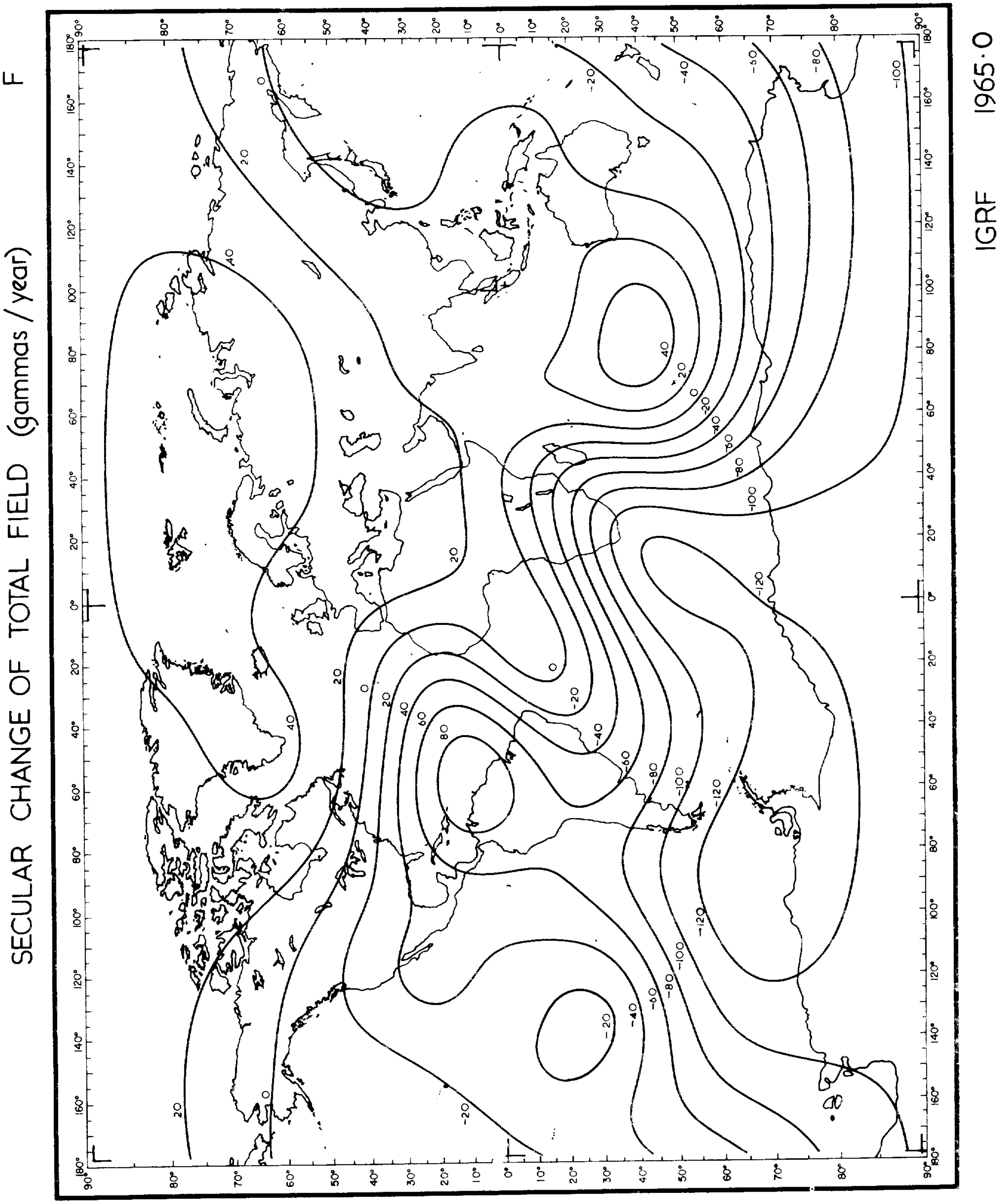


Fig. 10

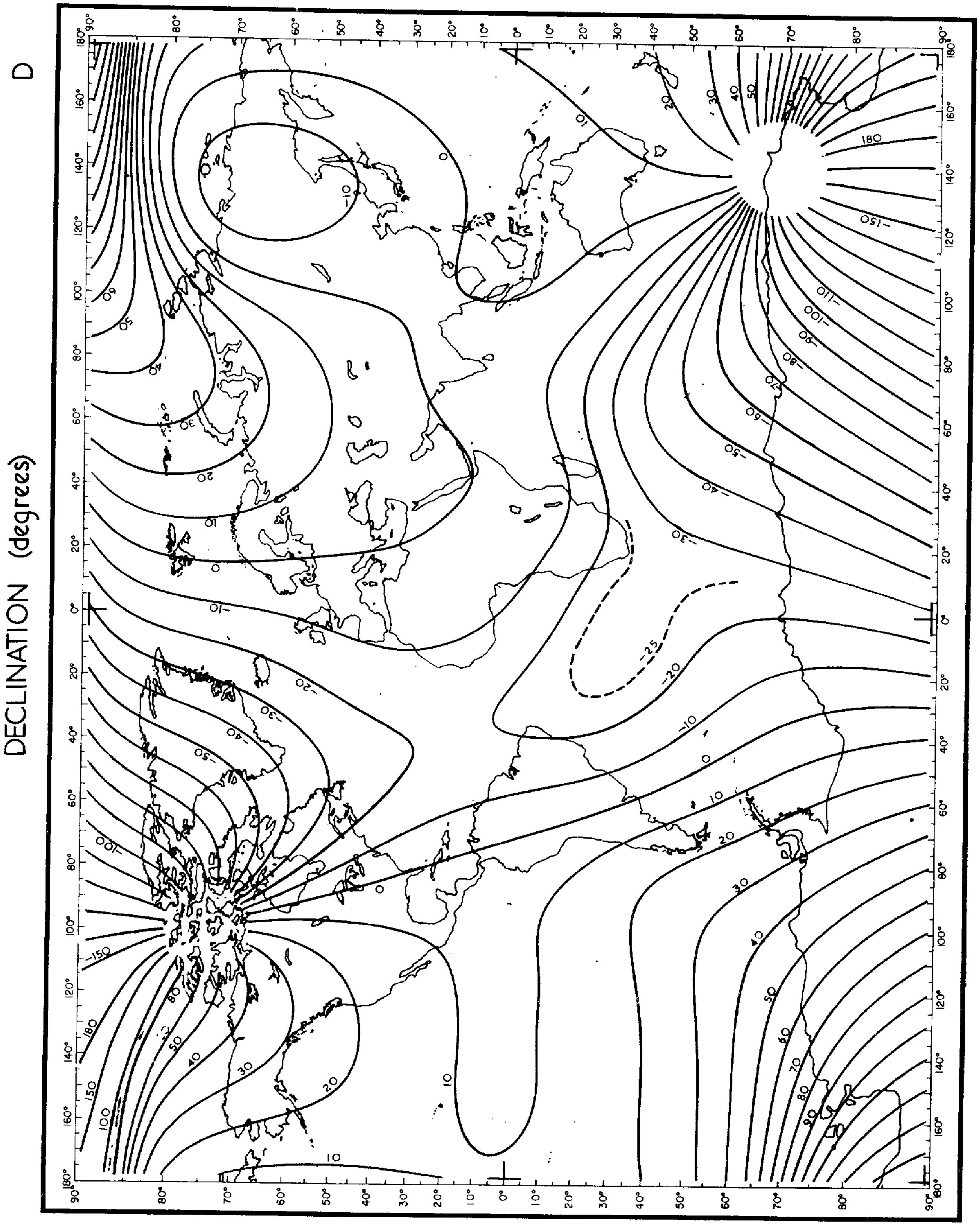
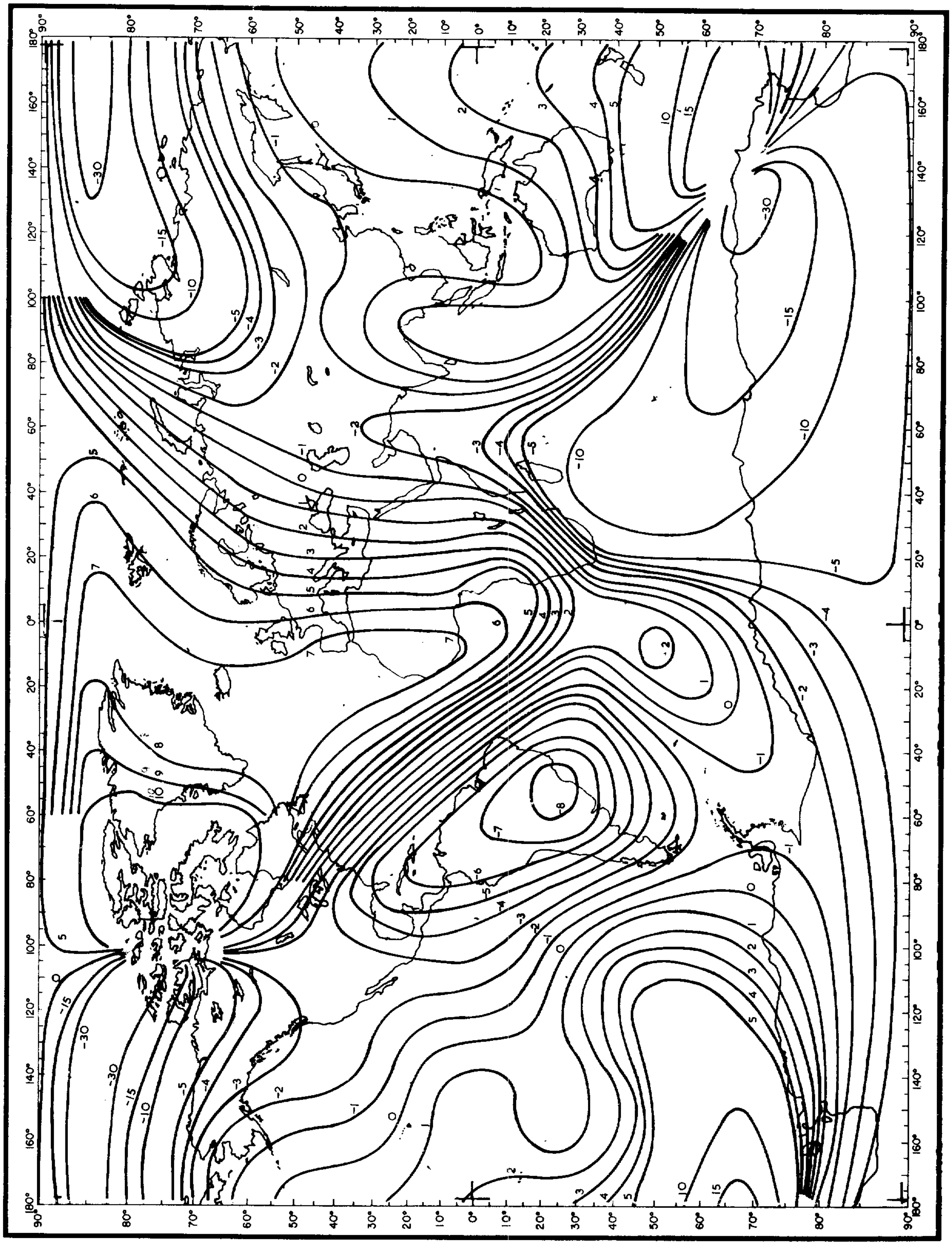


Fig. 11

SECULAR CHANGE OF DECLINATION (minutes/year) Ḋ



IGRF 1965·0

Fig. 12

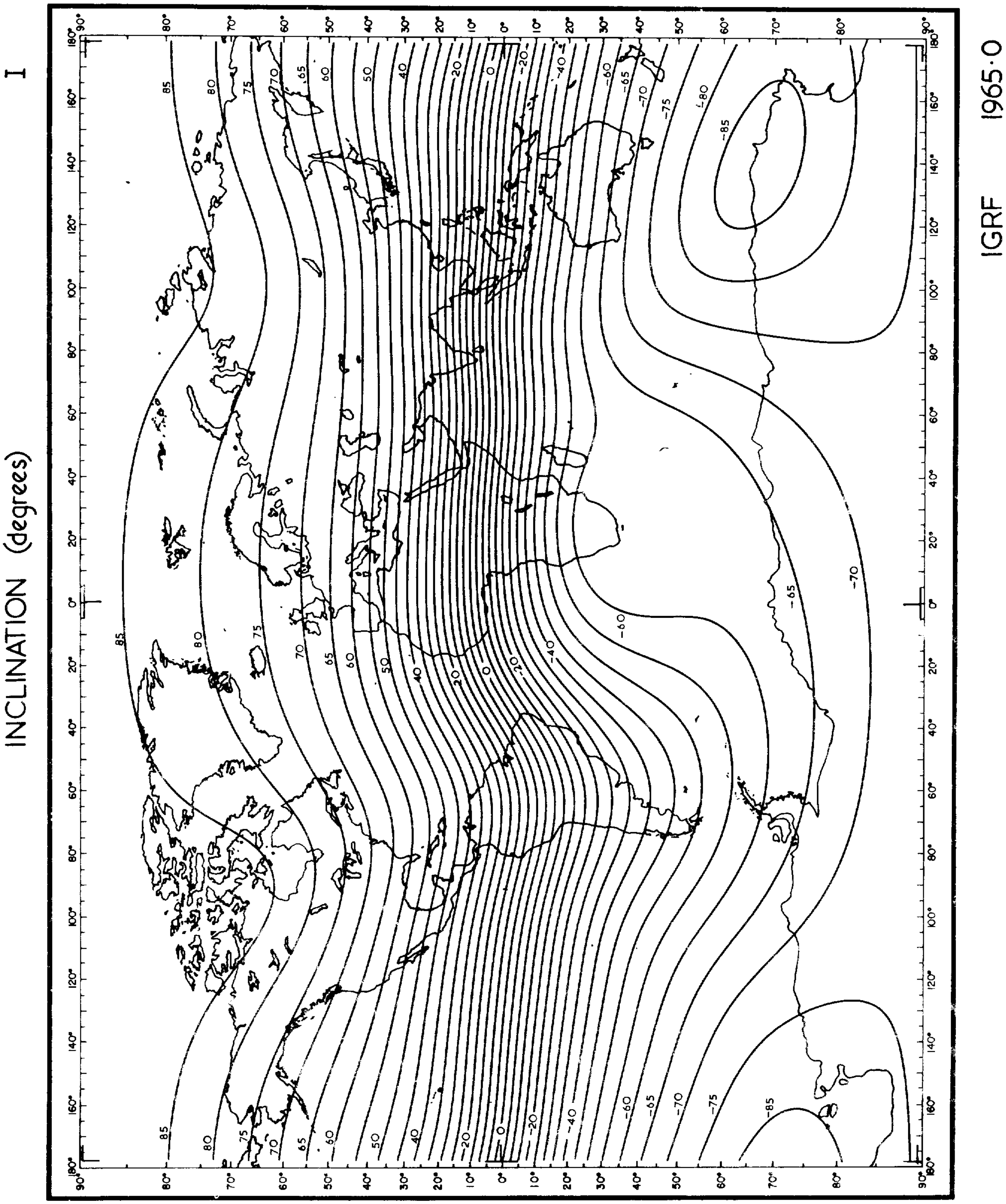


Fig. 13

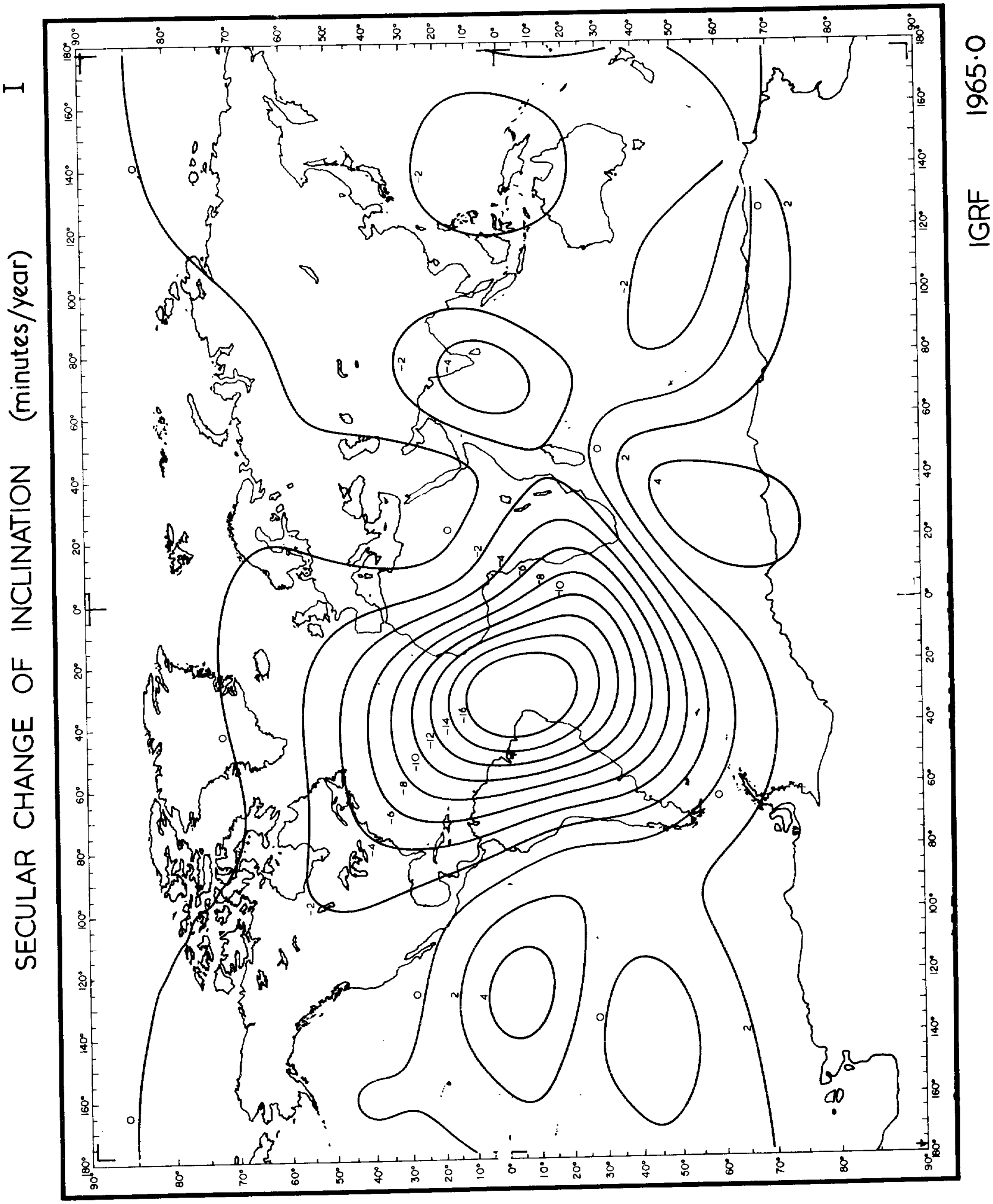


Fig. 14

REFERENCES FOR IGRF PAPERS

Adam, N. V., N. P. Benkova, V. P. Orlov, N. K. Osipov, and L. O. Tyurmina, Spherical analysis of the main geomagnetic field for epochs, 1955 and 1958, 1. *Geomagnetism and Aeronomy, U.S.S.R., 2*, 949 (English transl., 785), 1962.

Adam, N. V., N. P. Benkova, V. P. Orlov, N. K. Osipov, and L. O. Tyurmina, Spherical analysis of the constant geomagnetic field for the epochs, 1955 and 1958, 2. *Geomagnetism and Aeronomy, U.S.S.R.*, 3, 121, (English transl., 96), 1963.

Adam, N. V., N. P. Benkova, V. P. Portnova, and L. O. Tyurmina, Comparison of the computed geomagnetic field with that measured on the COSMOS-49 satellite, *Geomagnetism and Aeronomy, U.S.S.R., 7*, (English transl., 166), 1967.

Alldredge, L. R., G. D. VanVoorhis, and T. M. Davis, A magnetic profile around the world, *J. Geophys. Res., 68*, 3679, 1963.

Alldredge, L. R., Analysis of long magnetic profiles, *J. Geomagnet. Geoelec., Kyoto, 17*, 173, 1965.

Bazarzhapov, A. D., and G. I. Kolomiitzeva, Improvement of the analytical representation of secular variations, *Geomagnetism and Aeronomy, 7*, 868 (English transl., 704), 1967.

Benkova, N. P., Sh. Sh. Dolginov, L. O. Tyurmina, N. V. Adam, and T. N. Cherevko, Some comments on the International Geomagnetic Reference Field, Communication to IAGA Working Group, April 1968.

Bomford, G., *Geodesy*, Second edition, 495–496, Oxford University Press, London, 1962.

Bullard, E. C., The removal of trend from magnetic surveys, *Earth and Planetary Science Letters, 2*, 293, 1967.

Cain, J. C., W. E. Daniels, and S. J. Hendricks, An evaluation of the main geomagnetic field, 1940–1962, *J. Geophys. Res., 70*, 3647, 1965.

Cain, J. C., Models of the earth's magnetic field, in *Radiation Trapped in the Earth's Magnetic Field*, edited by B. M. McCormac, p. 7, D. Reidel Publishing Company, Dordrecht, Holland, 1966.

Cain, J. C., S. J. Hendricks, R. A. Langel, and W. V. Hudson, A proposed model for the International Geomagnetic Reference Field—1965, *J. Geomagnet. Geoelec., Kyoto, 19*, 335, 1967.

Cain, J. C., The role of the main geomagnetic field in locating conjugate points, *Radio Science, 3*, 766, 1968a.

Cain, J. C., Coefficients for an IGRF and derived using only data from the POGO satellite, submitted to Working Group in letter dated March 15, 1968b.

Cain, J. C., S. Hendricks, W. E. Daniels, and D. C. Jensen, with revision by S. H. Cain, Computation of the main geomagnetic field from spherical harmonic expansions, *Data Users Note NSSDC 68–11*, NASA/GSFC, Greenbelt, Maryland, May 1968c.

Cain, J. C., and S. J. Hendricks, The geomagnetic secular variation 1900–1965, *NASA Technical Note D-4527*, April 1968.

Cain, J. C., and S. J. Cain, Derivation of the International Geomagnetic Reference Field [IGRF (10/68)], NASA/GSFC preprint X-612-68-501, Greenbelt, Maryland, December 1968. (Also published as NASA *Technical Note D-6237*, 1971.)

Chapman, S., and J. Bartels, *Geomagnetism, 2*, Oxford University Press, London, England, pp. 633–634, 1940a, pp. 611–612, 1940b.

Dawson, E., and L. C. Dalgetty, Magnetic charts of Canada for epoch 1965.0, *Publication Dominion Observatory 31*, No. 9, 1966.

Fabiano, E. B., and N. W. Peddie, Grid values of total magnetic intensity, IGRF-1965, *ESSA Tech. Report, U.S. Coast and Geodetic Survey, 38*, April 1969.

Fougere, P. F., First set of coefficients for an IGRF, submitted to Working Group in a letter dated October 18, 1967, with corrections in letter dated November 15, 1967; published in article, Spherical harmonic analysis 3, the earth's magnetic field 1900–1965, *J. Geomag. Geoelec., Kyoto, 21*, 1–11, 1969.

Fougere, P. F., Second set of coefficients for an IGRF, submitted to Working Group in a letter dated March 18, 1968; published also in article noted in preceding reference.

Heirtzler, J. R., Marine geomagnetic anomalies, *J. Geomagnet. Geoelec., Kyoto, 17*, 227, 1965.

Hendricks, S. J., and J. C. Cain, Magnetic field data for trapped particle evaluations, *J. Geophys. Res., 71*, 346, 1966.

Heuring, F. T., The analytic description of the geomagnetic field at satellite altitude, *J. Geophys. Res., 69*, 4959, 1964.

Heuring, F. T., Comparison of some recently defined geomagnetic field models to the Vanguard 3 (1959η) data, *J. Geophys. Res., 70*, 4968, 1965.

Heuring, F. T., A. J. Zmuda, W. E. Radford, and P. Verzariu, An evaluation of geomagnetic harmonic series for 1100-kilometer altitude, *J. Geophys. Res., 73*, 2505, 1968.

Hurwitz, L., D. G. Knapp, J. H. Nelson, and D. E. Watson, Mathematical model of the geomagnetic field for 1965, *J. Geophys. Res., 71*, 2373, 1966.

Hurwitz, L., Secular change coefficients for an IGRF, submitted to Working Group in a letter dated March 15, 1968, with modifications dated March 25, 1968.

IAGA Commission 2 Working Group No. 4., Analysis of the Geomagnetic Field, the International Geomagnetic Reference Field, *Geomagnetism and Aeronomy, U.S.S.R., 72*, 956 (English transl., 772) 1969; *J. Geomagnet. Geoelec., Kyoto, 21*, 569, 1969; *J. Geophys. Res., 74*, 4407, 1969.

IZMIRAN, Catalogue of measured and computed values of the geomagnetic field intensity along the orbit of the COSMOS-49 satellite, compiled by Sh. Sh. Dolginov, V. N. Nalivaiko, A. V. Tyurmin, M. M. Chinchevoi, R. E. Brodskaya, G. N. Zlotin, I. N. Kikenadze, and L. O.

Tyurmina, edited by V. P. Orlov. Part 1 of three parts, page 21, *Academy of Sciences U.S.S.R., IZMIRAN*, Moscow, 1967a.

IZMIRAN, Secular change coefficients for IGRF, submitted to Working Group as part of letter from N. P. Benkova and V. P. Orlov, October 18, 1967, 1967b.

Kahle, A. B., J. W. Kern, and E. H. Vestine, Spherical harmonic analyses for the spheroidal earth, 1. *J. Geomagnet. Geoelec., Kyoto, 16*, 229, 1964.

Kahle, A. B., J. W. Kern, and E. H. Vestine, Spherical harmonic analyses for the spheroidal earth, 2. *J. Geomagnet. Geoelec., Kyoto, 18*, 349, 1966.

Kahle, A. B., Three ways to treat the IGRF on the earth's surface, Paper distributed at the IAGA Symposium on Description of the Earth's Magnetic Field, held in Washington, D. C., October 22–25, 1968.

Kaula, W. M., Test and combination of satellite determinations of the gravity field with gravimetry, *J. Geophys. Res., 71*, 5303, 1966.

Kautzleben, H., Statistical analysis of the main part of geomagnetic field, 1. General theory. *Geomagnetism and Aeronomy, U.S.S.R., 5*, 502, (English transl., 387), 1965a.

Kautzleben, H., Statistical analysis of the main part of geomagnetic field, 2. Analysis of the world charts of the 1955.0 epoch. *Geomagnetism and Aeronomy, U.S.S.R., 5*, 511 (English transl., 394), 1965b.

Kolomiitseva, G. I., and L. O. Tyurmina, Experience in using satellite data for the study of secular variations, *Geomagnetism and Aeronomy, 8*, 1068, 1968.

Leonard, R. S., Selection of a model of earth's magnetic field, *J. Geophys. Res., 68*, 6437, 1963.

Leaton, B. R., S. R. C. Malin, and M. J. Evans, An analytical representation of the estimated geomagnetic field and its secular change for the epoch 1965.0., *J. Geomagnet. Geoelec., Kyoto, 17*, 187, 1965.

Leaton, B. R., in World Magnetic Survey, p. 527, Report of colloquium held at Herstmonceux, 1966, October 4, 5, and 6, *Geophys. J. R. Astr. Soc., 12*, 521, 1967.

Leaton, B. R., and S. R. C. Malin, personal communication to Working Group, November 21, 1966.

Leaton, B. R., personal communication to Working Group, March 3, 1967.

Malin, S. R. C., Coefficients for an IGRF, submitted to Working Group in a letter dated March 11, 1968.

Malin, S. R. C., The International Geomagnetic Reference Field, Paper presented at IAGA Symposium on Description of the Earth's Magnetic Field, held in Washington, D. C., October 22–25, 1968.

Malin, S. R. C., and S. B. Pocock, Geomagnetic spherical analysis, *Pure and Applied Geophysics, 75*, 117–132, 1969.

Malin, S. R. C., Geomagnetic secular variations and its changes 1942.5 to 1962.5, *Geophys. J. R. Astr. Soc., 17*, 415–441, 1969.

Nagata, T., and T. Oguti, Magnetic charts for the epoch of 1958.5 corrected for the Antarctic region and spherical harmonic coefficients of the revised geomagnetic field, *J. Geomagnet. Geoelec., Kyoto, 14*, 125, 1962.

Nagata, T., Convergency of the spherical harmonic coefficients of the geomagnetic field, *J. Geomagnet. Geoelec., Kyoto, 17*, 2, 153, 1965.

Nagata, T., *Report on aeromagnetic survey in Japan*, World Data Center C-2, Kyoto, Japan, 1966.

Orlov, V. P., M. P. Ivchenko, A. D. Bazarzhapov, and G. I. Kolomiitseva, World charts of isoporics for the period 1960–1965, *Geomagnetism and Aeronomy, 9*, 135 (English transl., 106), 1969.

Roberts, P. H., and S. Scott, Truncation errors in the spherical harmonic analysis of the geomagnetic field and the problem of downward extrapolation, *J. Geomagnet. Geoelec., Kyoto, 15*, 148, 1963.

Serson, P. H., and W. L. W. Hannaford, A statistical analysis of magnetic profiles, *J. Geophys. Res., 62*, 1, 1957.

Tyurmina, L. O., Analytical model of the geomagnetic field according to COSMOS-49 data, *Geomagnetism and Aeronomy, U.S.S.R. 8*, 977 (English transl., 786), 1968.

Tyurmina, L. O., and T. N. Cherevko, Analytic representation of the geomagnetic field according to COSMOS-49 data. 1. *Geomagnetism and Aeronomy, U.S.S.R., 7*, 264 (English transl., 207), 1967.

Transactions of the International Astronomical Union, *Proceedings of the Twelfth General Assembly, Hamburg, Germany*, XIIB *(1964)*, 594–595, 1966.

Transactions of the XIV General Assembly, St. Gall, Switzerland, 1967, *IAGA Bulletin No. 25*, IUGG/IAGA, ed. L. R. Alldredge, IUGG Publication Office, 39 ter Rue Gay-Lussoc, Paris (V) 1967.

U.S. Navy Oceanographic Office, Airborne geomagnetic data, special pub. 66, supplement no. 1, 1962–1963, *USNOO, Washington, D. C.*, 1965.

Vestine, E. H., L. Laporte, I. Lange, and W. E. Scott, *The Geomagnetic Field, Its Description and Analysis*, Carnegie Inst. Washington Publication 580, Washington, D.C., 1947.

Vestine, E. H., *Instruction Manual on World Magnetic Survey*, Monograph No. 11, International Union of Geodesy and Geophysics, International Association of Geomagnetism and Aeronomy, Imprime Par L'Institut Geographique National Paris, August 1961.

Winch, D. E., An application of oblate spheroidal harmonic functions to the determination of geomagnetic potential, *J. Geomagnet. Geoelec., Kyoto, 19*, 1, 1967.

WMS Notes No. 3, International Union of Geodesy and Geophysics, International Association of Geomagnetism and Aeronomy, pp. 6–7, January 1966.

Yukutake, T., Synthesis of the non-dipole component of the earth's harmonic coefficients, *Bulletin of the Earthquake Research Institute, 46*, 385–403, 1968.

Zmuda, A. J., and M. Neuman, The correction and mutual dependence of harmonic coefficients, *Space Res., 2*, 701, ed. by H. C. Van de Hulst, C. DeJager, and H. F. Moore, North-Holland Publishing Company, Amsterdam, 1961.

Zmuda, A. J., W. E. Radford, F. T. Heuring, and P. Verzariu, The scalar magnetic intensity at 1100 kilometers in middle and low latitudes, *J. Geophys. Res., 73*, 2495–2503, 1968.