

SUMMARY

The CHAOS series of geomagnetic field models describe the internal geomagnetic field and its time variations. Here, we investigate how such models can be improved using data from ESA’s Swarm mission. We find unvalidated Swarm VFM data are well fit by a new model, CHAOS-4plus_V2, with rms total field residuals for the three satellites being 2.18 ± 0.02 nT at non-polar latitudes. Previous secular variation models have already been improved on, using only the first five months of Swarm data.

Field Modelling Methodology

- Potential field approach: $\mathbf{B} = -\nabla V$ where $V = V^{\text{int}} + V^{\text{ext}}$.
- The internal part of the potential takes the form
$$V^{\text{int}} = a \sum_{n=1}^{N_{\text{int}}} \sum_{m=0}^n (g_n^m \cos m\phi + h_n^m \sin m\phi) \left(\frac{a}{r}\right)^{n+1} P_n^m(\cos \theta)$$
- And internal coefficients are further expanded as
$$g_n^m(t) = \sum_l g_{n,l}^m \cdot M_l(t) \quad \text{for } n = 1 - 20$$
$$= g_n^m(t_0) \quad (\text{const.}) \quad \text{for } n = 21 - 80$$
where $M_l(t)$ are order 6 B-splines with 0.5 yr spacing.
- The external part is defined in the SM and GSM co-ordinate systems, with θ_d and T_d being dipole co-lat. and dipole local time
$$V^{\text{ext}} = a \sum_{n=1}^2 \sum_{m=0}^n (q_n^m \cos mT_d + s_n^m \sin mT_d) \left(\frac{r}{a}\right)^n P_n^m(\cos \theta_d) + a \sum_{n=1}^2 q_n^{0,\text{GSM}} R_n^0(r, \theta, \phi).$$
- Degree-1 coefficients in SM coordinates also depend explicitly on $\epsilon(t)$ and $\iota(t)$, the external and induced parts of the RC index.
- The field in magnetometer frame is related to that in the North-East-Center (NEC) frame by
$$\mathbf{B}_{\text{NEC}} = -\nabla V = \underline{\mathbf{R}}_q \cdot \underline{\mathbf{R}}_3 \cdot \mathbf{B}_{\text{VFM}}$$

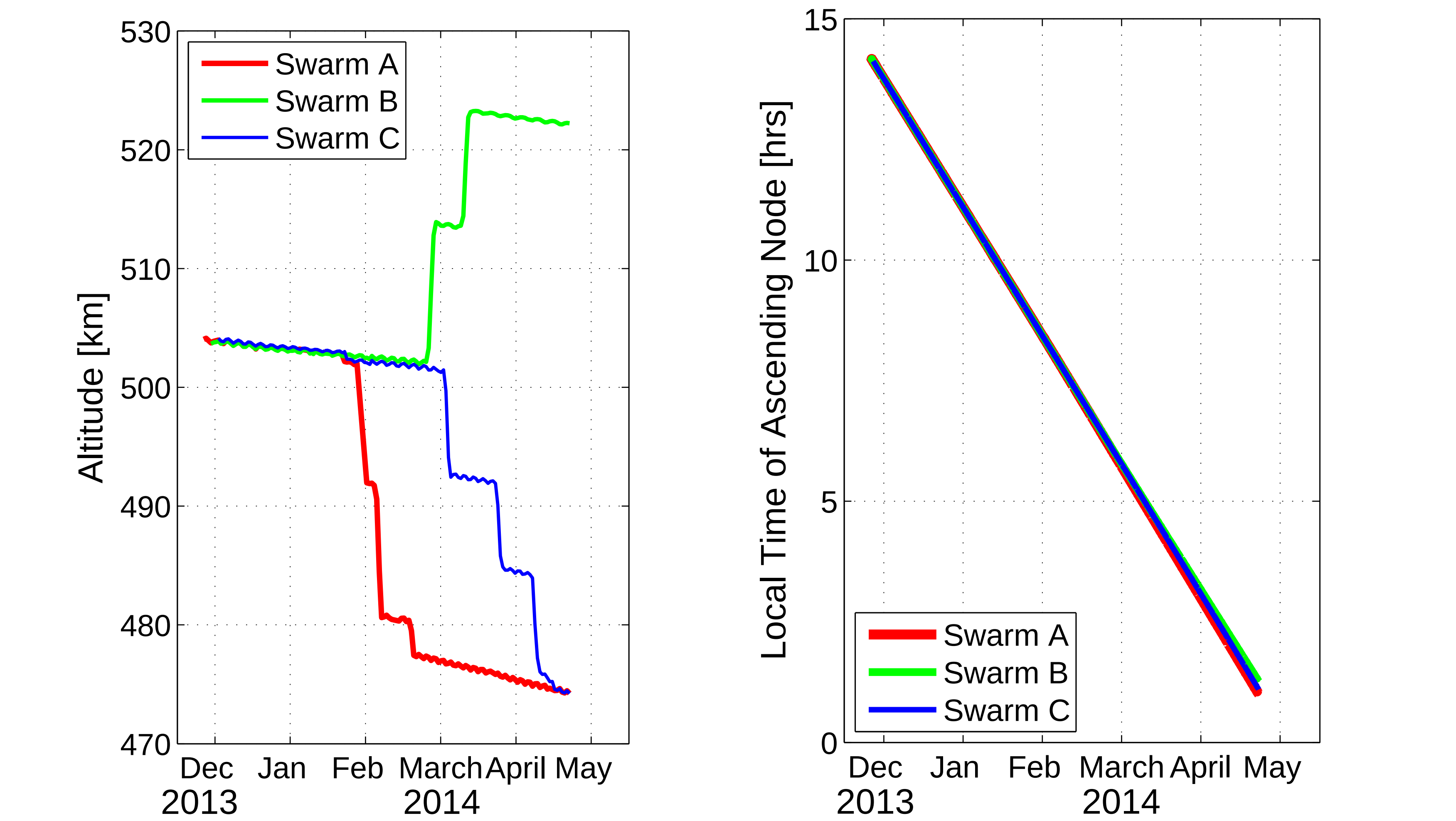
$\underline{\mathbf{R}}_3$ rotates from the magnetometer VFM system to the Common Reference Frame (CRF) of the star tracker ,while $\underline{\mathbf{R}}_q$ rotates from the CRF to the NEC frame. $\underline{\mathbf{R}}_3$ involves **co-estimated Euler angles**.

- Model estimation is via **regularized, robust, non-linear least squares** minimizing
$$\mathbf{e}^T \underline{\mathbf{C}}^{-1} \mathbf{e} + \lambda_3 \mathbf{m}^T \underline{\mathbf{\Lambda}}_3 \mathbf{m} + \lambda_2 \mathbf{m}^T \underline{\mathbf{\Lambda}}_2 \mathbf{m}$$

This is implemented using *Iteratively Reweighted Least-Squares* with Huber weights. $\underline{\mathbf{\Lambda}}_3$ and $\underline{\mathbf{\Lambda}}_2$ are regularization matrices constraining the third, respectively second, time derivatives at the core surface.

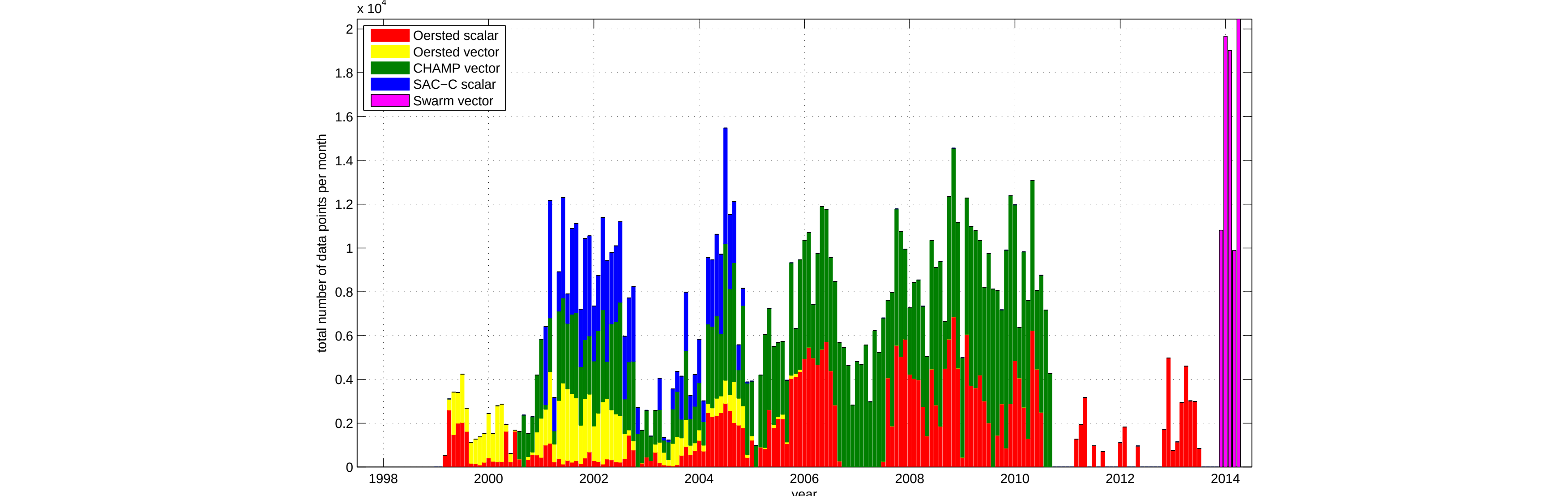
Swarm

Unvalidated Swarm VFM data, from the Level 1B prototype processor, until 23rd April 2014, have been used in this study. Below is the evolution of the Swarm satellites mean daily altitudes (left) and local times of the ascending nodes (right) for the available data.

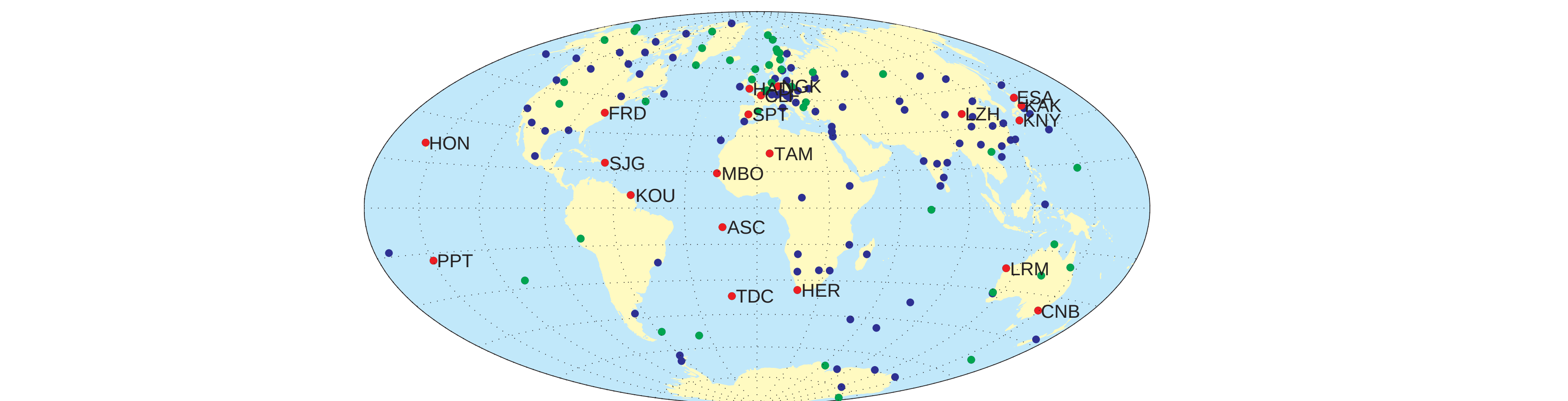


Other Data Sources

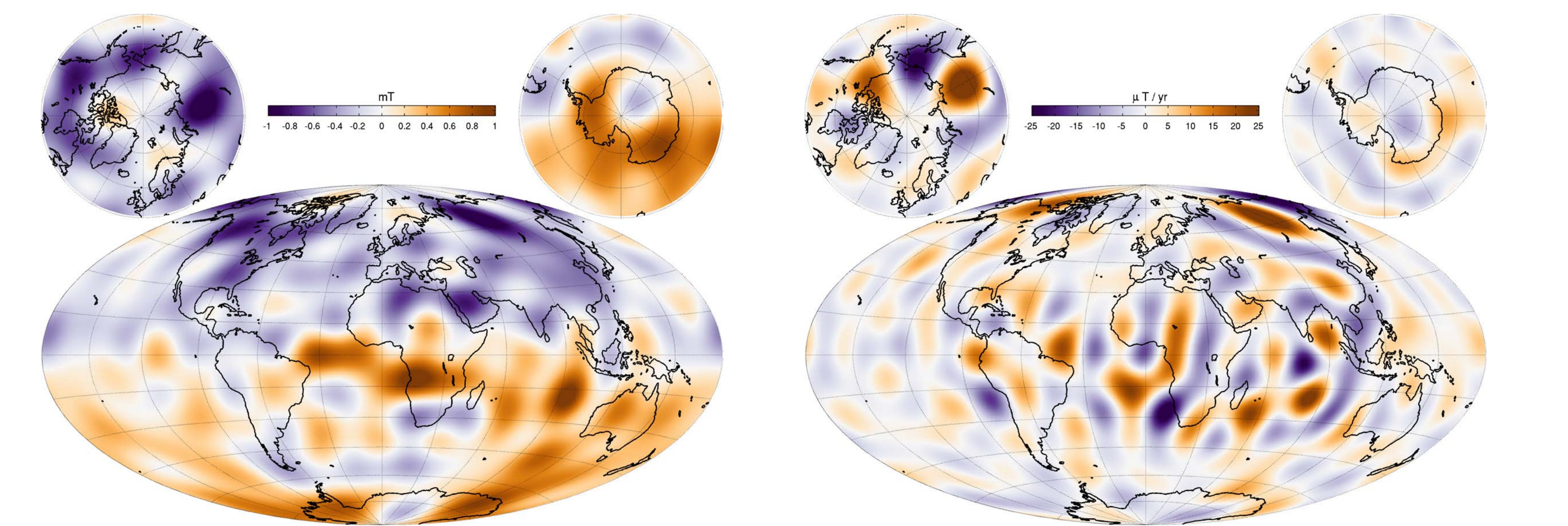
CHAMP, Ørsted and SAC-C satellite data, as previously used in the construction of CHAOS-4, were also employed. Below: Histogram showing all sources of vector and scalar satellite data each month.



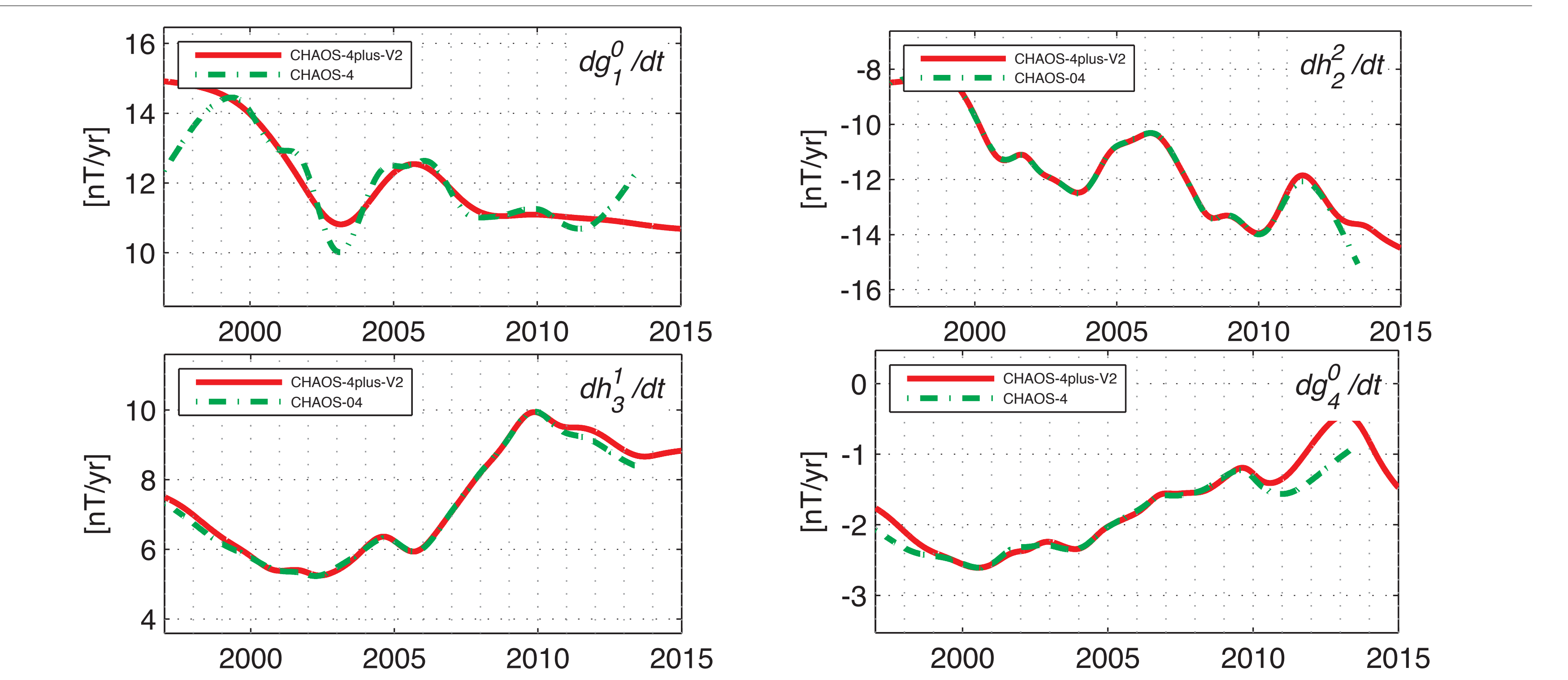
Below: Ground observatory data from the sites marked was also used. Emphasized (green or red symbols) are observatories providing data after May 2013; red were used to derive the RC index.



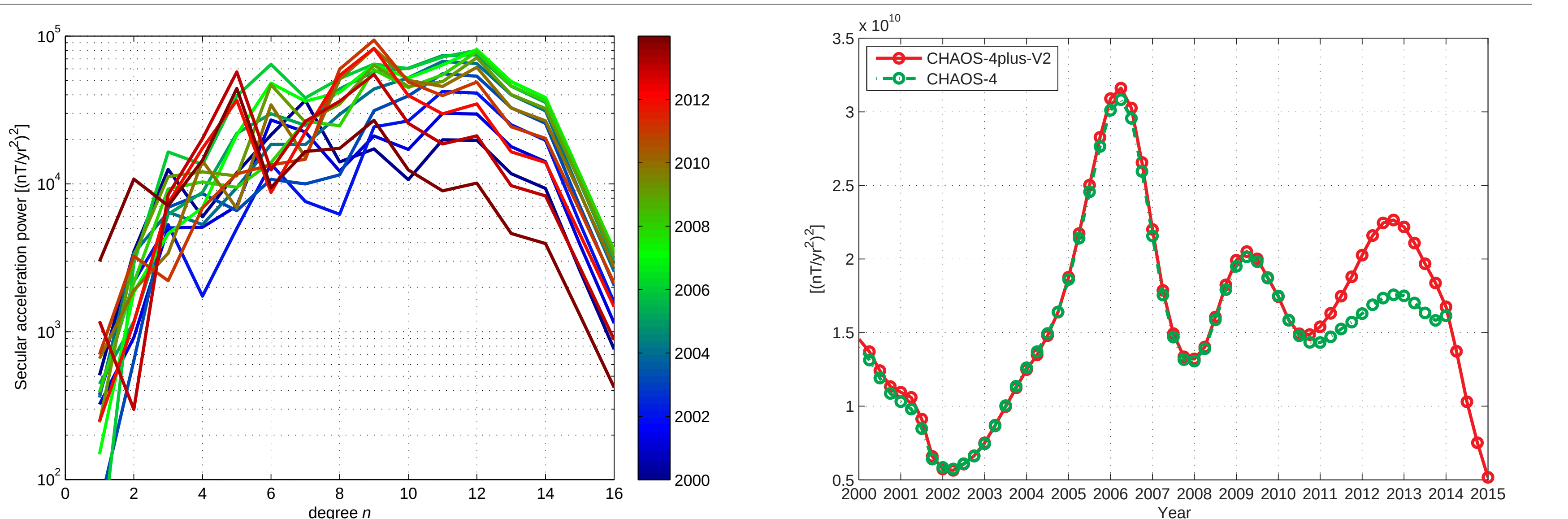
Core Field Variations 1999-2014



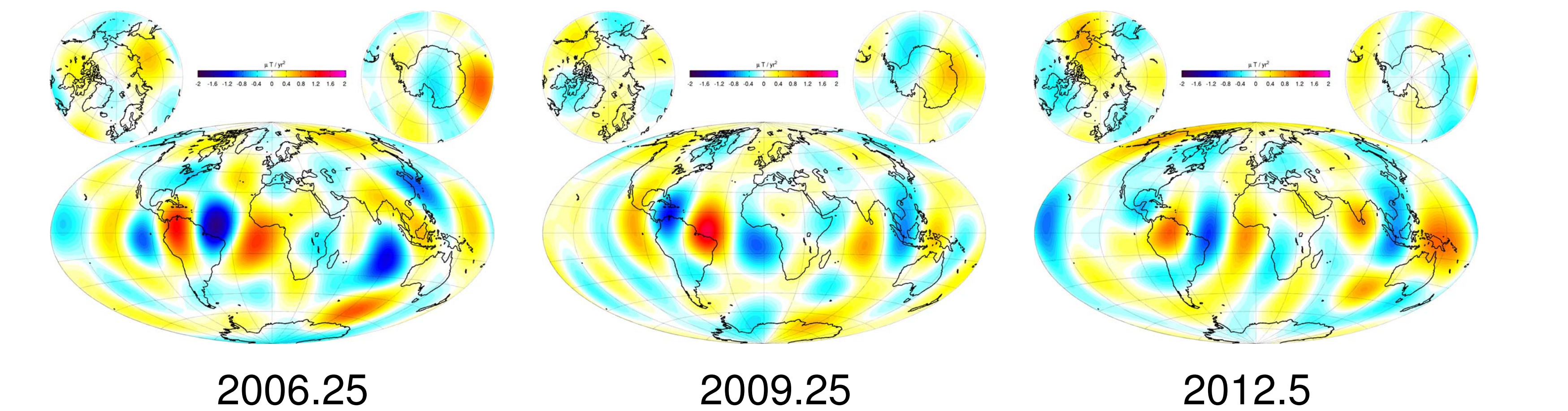
Above: **Radial field** (left) and its time rate of change or **secular variation (SV)** (right), to deg. 14 and 15, at the CMB in 2014.25.



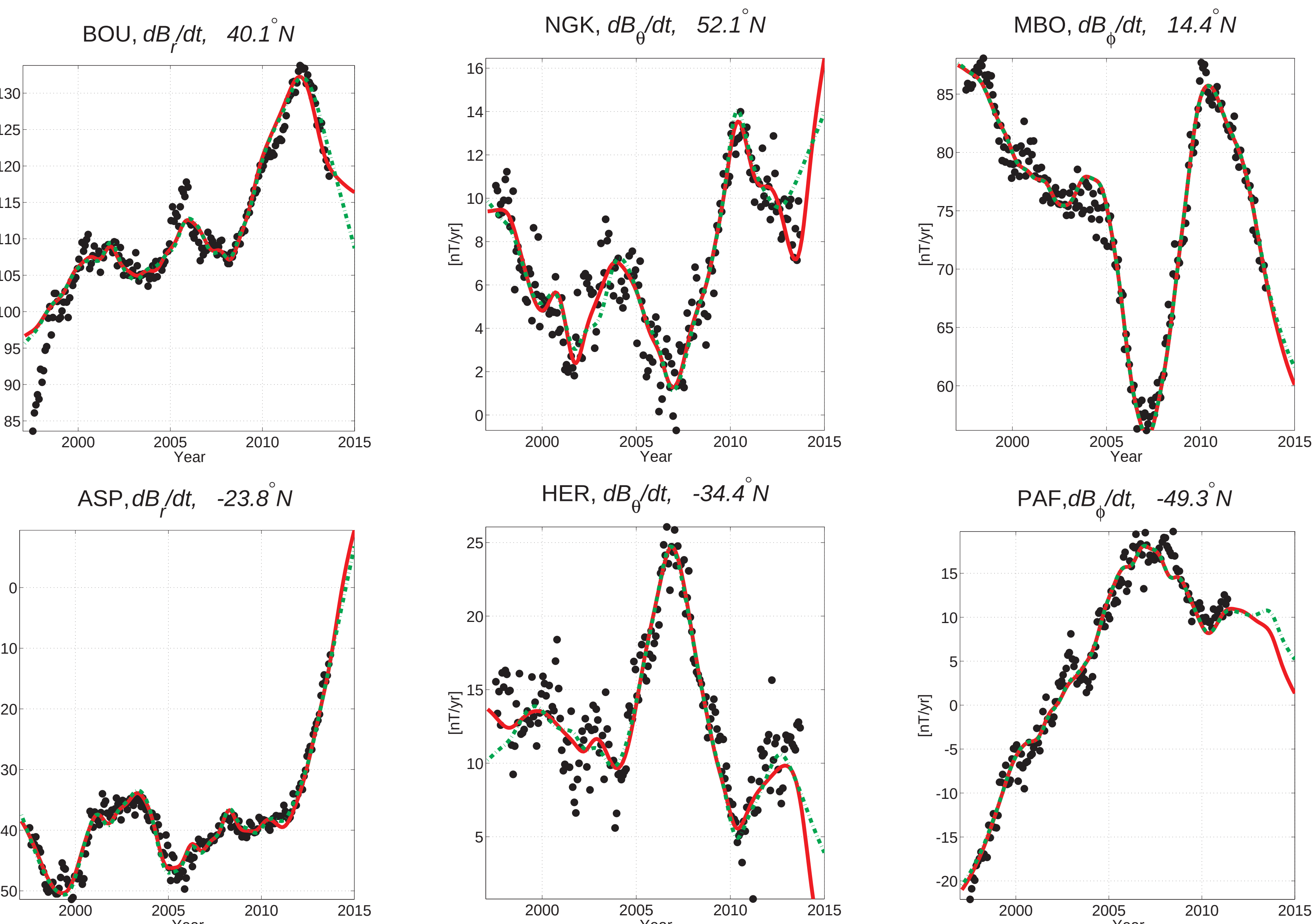
Above: **Time evolution of the secular variation** (at Earth’s surface) of selected spherical harmonic coefficients of the internal potential.



Secular acceleration (SA): Above left: SH spectra of SA at CMB ; Above right: Norm (squared, integrated value to deg. 8) of SA at CMB ; Below: Maps of radial SA (to deg. 8) at CMB during SA pulses.

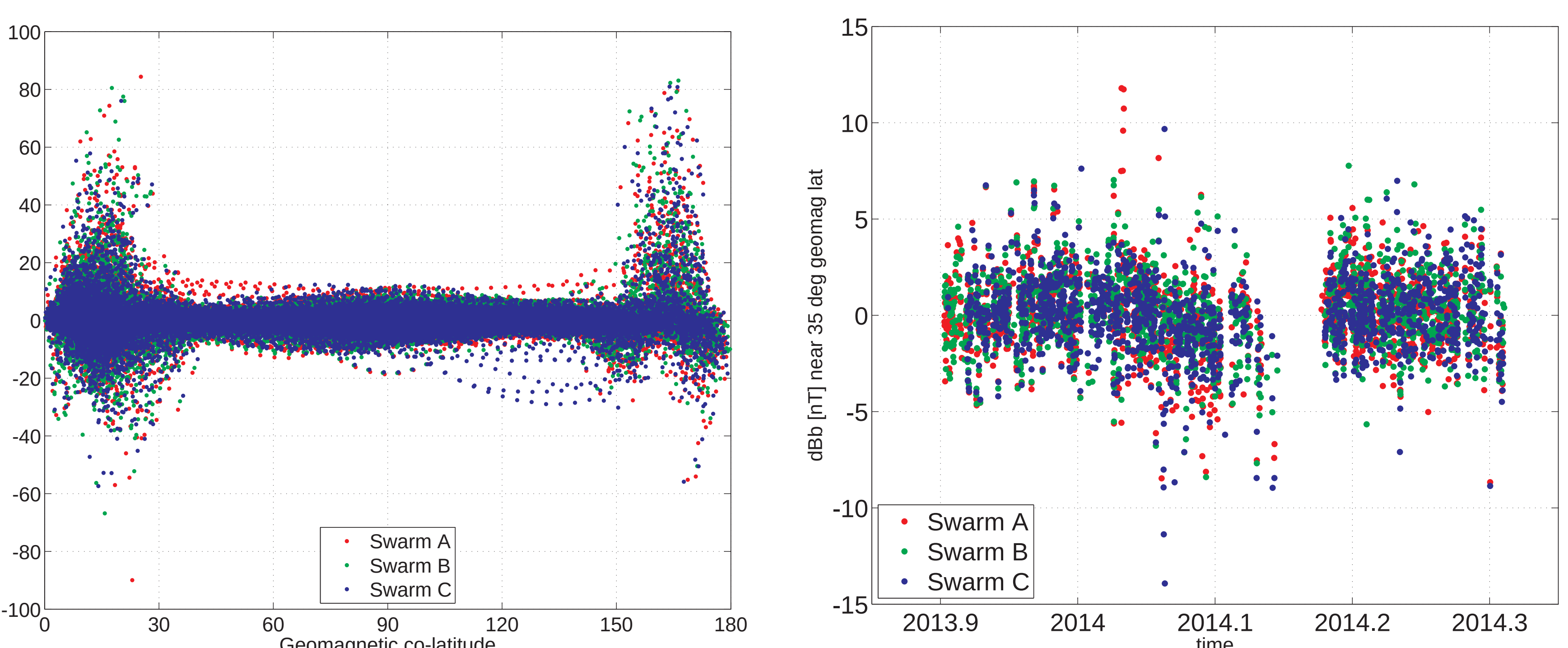


Fit to Ground Observatory Data



Above: Annual differences of revised observatory monthly means (**black dots**) compared to the SV from **CHAOS-4plus_V2** (red line) and the pre-Swarm model **CHAOS-4** (green dashes).

Swarm Residuals



Above: **Total field residuals dB_B** between Swarm A, B, C and the CHAOS-4plus V2 model. Above left: Versus geomag. colat.; Above right: Versus time, at 35° geomag. lat., where the ring current disturbance is $\perp$ to the main field. Right: Swarm residual statistics, Huber weighted.

Satellite	Component	CHAOS-4plus V2		
		N	mean [nT]	rms [nT]
Swarm A	F_{polar}	10,896	0.10	4.46
	$F_{\text{nonpolar}} + B_B$	26,107	0.06	2.18
	$B_{\perp}$	26,107	-0.07	2.96
	B_3	26,107	0.08	2.88
Swarm B	F_{polar}	11,206	-0.07	4.37
	$F_{\text{nonpolar}} + B_B$	27,184	0.12	2.16
	$B_{\perp}$	27,184	-0.09	3.05
	B_3	27,184	0.05	3.27
Swarm C	F_{polar}	10,903	0.01	4.41
	$F_{\text{nonpolar}} + B_B$	26,545	-0.07	2.20
	$B_{\perp}$	26,545	-0.07	3.02
	B_3	26,545	0.09	3.39