Separation of core and lithospheric magnetic fields by co-estimation of equivalent source models from Swarm data

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Overview

The geomagnetic field of internal origin, as observed by modern satellite missions, contains the signatures of both the core dynamo and the magnetized lithosphere. It is common practise to estimate the core field either by truncating models at spherical harmonic degree 13, or by minimizing a norm at the core surface and subtracting an estimate of the high degree lithospheric field. Here, we instead explicitly co-estimate the core and lithospheric fields, seeking models that minimize the unsigned radial flux on the core surface and on the Earth's surface respectively.

We have applied this procedure to vector field data from the *Swarm* satellite constellation mission. The obtained field models possess reasonable spherical harmonic spectra. The inferred core surface field shows surprisingly weak radial field in many regions (particularly in the South Atlantic), along with some intense field concentrations. The lithospheric field derived in these preliminary tests is of comparatively poor quality and is clearly polluted by unmodelled external fields.

Motivation

- Core and lithospheric fields are both internal origin, and have overlapping wavelengths
- But the fields originate at different depths. Can this information be used to aid their separation?

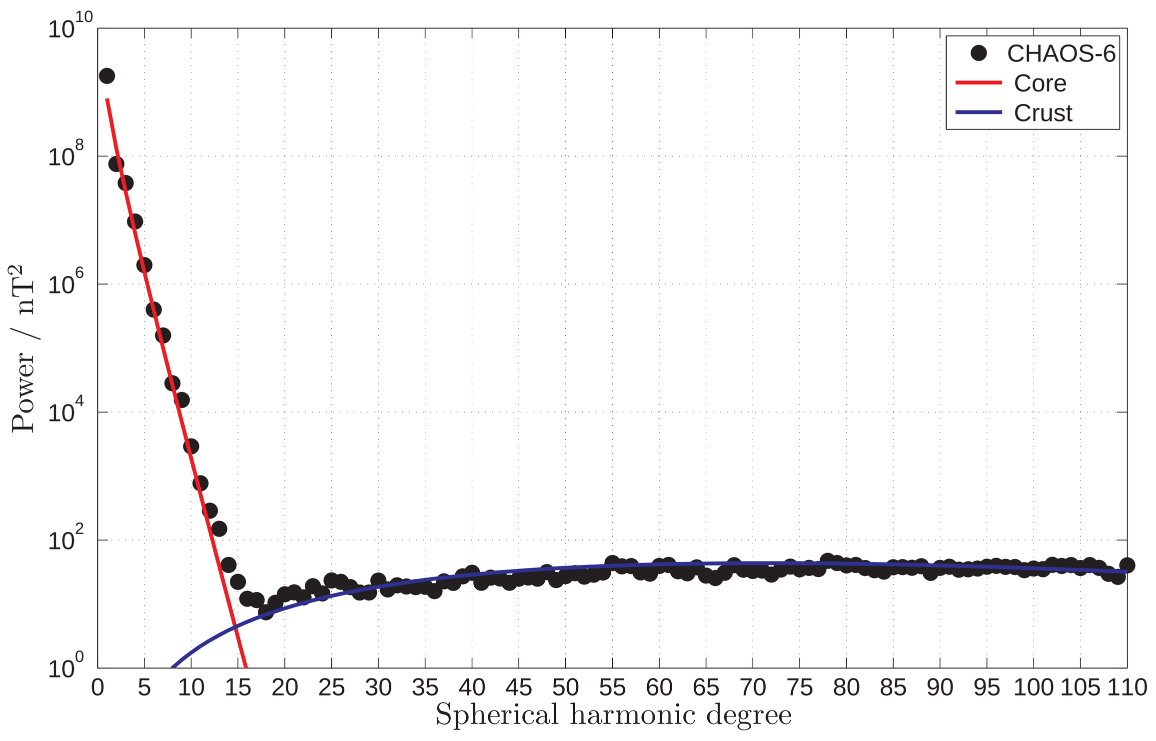

Fig. 1: Spherical harmonic power spectra of internal field from CHAOS-6 [Finlay et al., 2016] **(black)** with predictions from simple theoretical models [Voorhies et al., 2002] for the core field **(red)** and lithospheric field **(blue)**.

Observations: the Swarm satellite constellation

Fig. 2: Artist's view of the 3 Swarm satellites (credit: ESA).

- Successfully launched on 22nd November, 2013
- Two satellites flying close-by (approx. 150 km apart) at a relatively low altitude $\sim$ 460 km and a third higher $\sim$ 500 km
- Satellites slowly descending with the higher satellite drifting to a different local time (now 3hrs apart)
- Use L1b 1Hz data product, release 0408/09, from June 2014 to June 2015
- Vector field data from *Swarm* A, B and C (NEC frame) decimated to 1minute sampling
- Select 'quiet-times'; criteria for K_p, $|dRC/dt|$, E_m and IMF $B_z > 0$ as for CHAOS-6, data from dark regions only
- Latitude-dependent error estimates [Kother et al., 2015] estimated using CHAOS-6 residual provide diagonal entries for the data error covariance matrix $\mathbf{C_e}^{-1}$
- Huber weights are derived using residuals from CHAOS-6; these form the diagonal entries of the weighting matrix $\mathbf{W}_h$

Equivalent source modelling

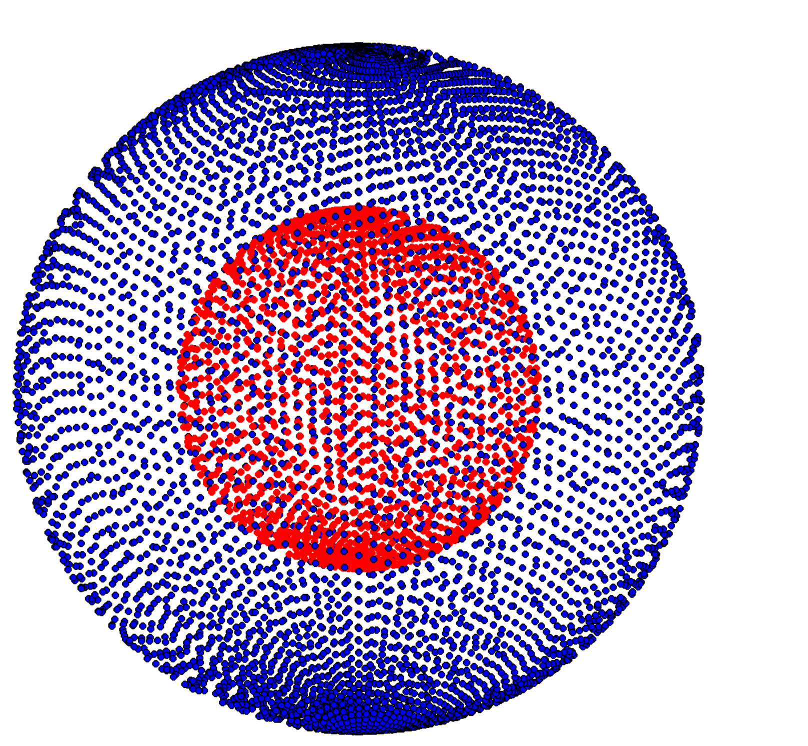

Fig. 3: Locations of equivalent point sources (monopoles) used to represent the core **(red)** and lithospheric **(blue)** fields.

Assuming we can write $\mathbf{B} = -\nabla\Phi$, the potential Φ at position $\mathbf{r}_i$ may be expressed as a sum over point sources (monopoles) of magnitude $\hat{m}_k$ (defined to be in units nT) arranged at positions $\mathbf{r}_k$ [O'Brien and Parker, 1994; Kother et al., 2015]

$$\Phi(\mathbf{r}_i) = \sum_{k=1}^{K} \frac{(\hat{m}_k r_k^2)}{r_{ik}} \quad \text{where} \quad r_{ik} = \sqrt{r_i^2 + r_k^2 - 2 r_i r_k \cos(\mu_{ik})}$$

$$= \sum_{k=1}^{K} r_k \hat{m}_k \sum_{n=0}^{\infty} \left(\frac{r_k}{r_i}\right)^{n+1} \sum_{m=0}^{n} P_n^m(\cos\theta_i) P_n^m(\cos\theta_k) \cos m(\phi_i - \phi_k)$$

Comparison with the conventional spherical harmonic expansion, reveals the Gauss coefficients are simply related to $\hat{m}_k$ via

$$g_n^m = \sum_{k=1}^{K} \left(\frac{r_k}{a}\right)^{n+2} \hat{m}_k\, P_n^m(\cos\theta_k) \cos(m\phi_k)$$

$$h_n^m = \sum_{k=1}^{K} \left(\frac{r_k}{a}\right)^{n+2} \hat{m}_k\, P_n^m(\cos\theta_k) \sin(m\phi_k)$$

where a=6371.2 km is Earth's spherical reference radius.

Here, we consider a joint equivalent source model (see Fig.3)

$$\mathbf{m} = \begin{pmatrix} \mathbf{m}^{core} \\ \mathbf{m}^{lith} \end{pmatrix}$$

$\mathbf{m}^{core} = \{\hat{m}_k^{core}\}$ is a set of 2000 sources, 300km below the CMB
$\mathbf{m}^{lith} = \{\hat{m}_k^{lith}\}$ is 6000 sources, 300km below Earth's surface
Each is approx equal area distributed on sphere [Leopardi, 2006]

Solving the inverse problem

- Estimate $\mathbf{m}$ from vector magnetic field data from *Swarm* $\mathbf{d} = \{(B_r)_i; (B_\theta)_i; (B_\phi)_i\}$
- Seek $\mathbf{m}^{core}$, $\mathbf{m}^{lith}$ minimizing $|B_r|$ at CMB, $S(c)$, and Earth's surface, $S(a)$, respectively
- This requires minimization of an objective function

$$\Theta(\mathbf{m}) = (\mathbf{d} - \mathbf{Gm})^T \mathbf{C}_e^{-1/2} \mathbf{W}_h \mathbf{C}_e^{-1/2} (\mathbf{d} - \mathbf{Gm}) + \lambda_{core} \int_{S(c)} |B_r^{core}| d\Omega + \lambda_{lith} \int_{S(a)} |B_r^{lith}| d\Omega$$

i.e. l_1 norm regularized weighted least squares, that promotes sparsity of B_r on $S(c)$ and $S(a)$

- Use an iteratively-reweighted-least squares (IRLS) algorithm [Farquharson and Oldenburg (1998)]. At the jth iteration,

$$\boxed{\mathbf{m}_{j+1} = (\mathbf{G}^T\mathbf{W}\mathbf{G} + \lambda_{core}\mathbf{R}_j^{core} + \lambda_{lith}\mathbf{R}_j^{core})^{-1}\mathbf{G}^T\mathbf{W}\mathbf{d}}$$

where $\mathbf{W} = \mathbf{C}_e^{-1/2}\mathbf{W}_h\mathbf{C}_e^{-1/2}$ and $\mathbf{R}_j^x = (\mathbf{G}_{B_r}^x)^T \mathbf{L}_j^x \mathbf{G}_{B_r}^x$

and $\mathbf{L}_j^x = diag\left[(\mathbf{B_r}^x)_j^2 + \epsilon^2\right]^{-1/2}$ for x=*core* or *lith*,

where $(\mathbf{B_r}^x)_j = \mathbf{G}_{B_r}^x \mathbf{m}_j^x$ and $\epsilon << |\mathbf{B}_{rj}^x|$

- λ_x chosen using misfit norm vs l_1 model norm L-curves, first considering the core field, then the lithospheric field

References

Farquharson and Oldenburg (1998) Non-linear inversion using general measures of data misfit and model structure *Geophys. J. Int*, 134, 213-227.

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Leopardi (2006) A partition of the unit sphere into regions of equal area and small diameter, *Electronic Transactions on Numerical Analysis*, 25, 309-327.

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Voorhies, et al. (2002) On magnetic spectra of Earth and Mars. *J. Geophys. Res.*, 107, E6, 5034.

Power spectra of co-estimated models

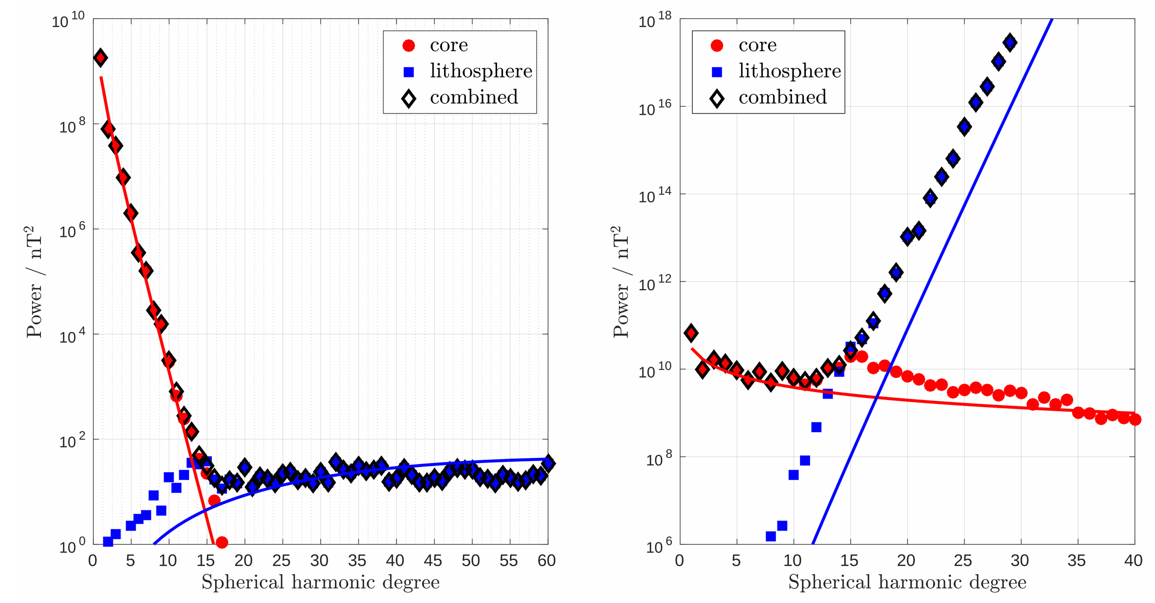

Fig. 5: Power spectra at Earth's surface (left) and at the core surface (right) of co-estimated core field **(red)**, lithospheric field **(blue)** and their combination **(black)**. Reference lines are the simple theoretical spectra predicted by Voorhies et al., 2002.

Core field

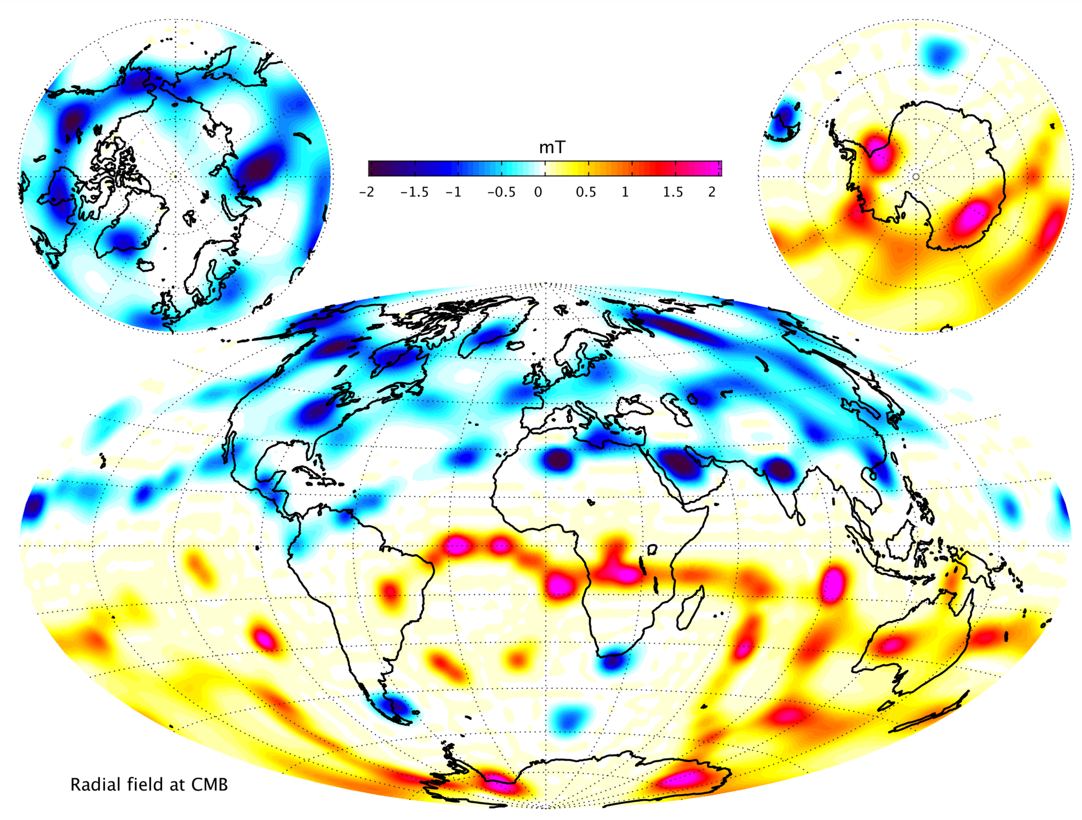

Fig. 6: Co-estimated core field, B_r at the core surface in 2015

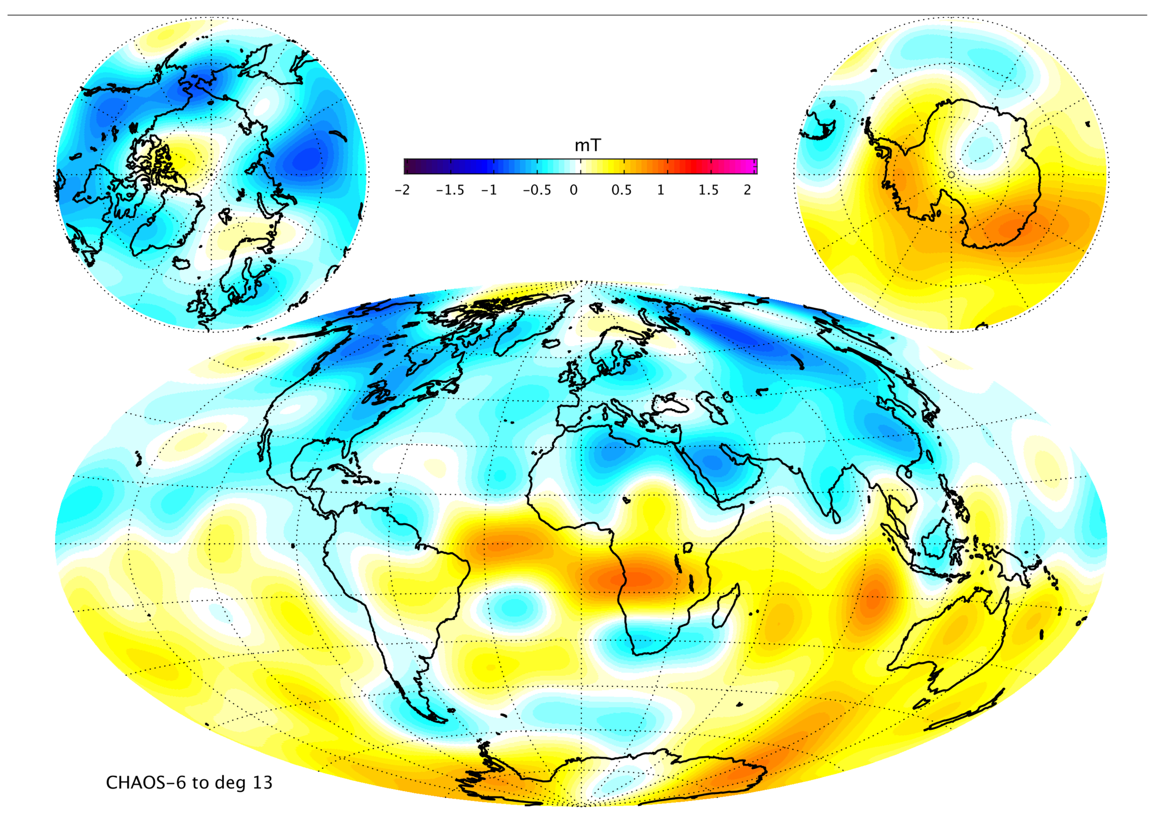

Fig. 7: For comparison: Internal field to degree 13, B_r at the core surface. From the CHAOS-6 field model [Finlay et al., 2016]

- Large regions of the co-estimated core surface field (e.g. South Atlantic) have surprisingly low amplitude
- Also some very high ampliude features, especially in equatorial region [reminiscent of the results of Jackson (2003)]

Lithospheric field (+ external field contamination)

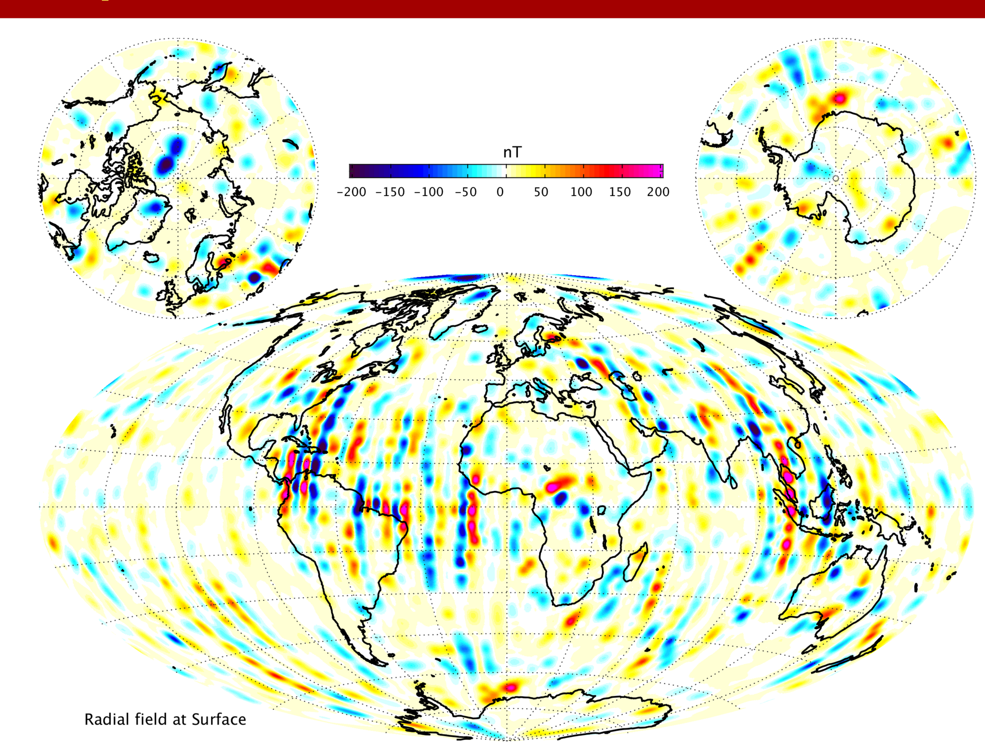

Fig. 8: Co-estimated lithospheric field, B_r at Earth's surface

Outlook

- Work in progress, still much to do
- Use gradient data to constrain the lithospheric field
- Include time-dependence of core field
- Include more detailed prior information on both sources
- Need to better quantify model uncertainties and trade-offs
- Hope to eventually contribute towards improved accounting for modelling errors in core flow inversions/data assimilation