DTU Space
National Space Institute

Data Error Covariances for Satellite Magnetic Data

Chris Finlay, Nils Olsen
DTU Space, Technical University of Denmark

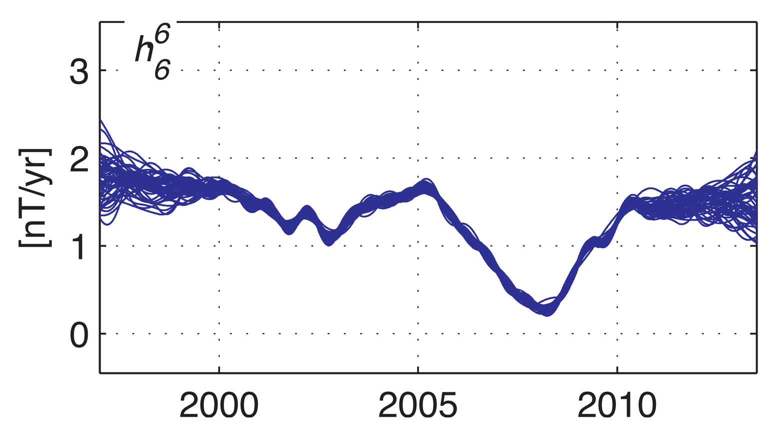
Summary

In geomagnetic field modelling, it is commonly assumed that satellite magnetic data errors are uncorrelated in both time and space. This results in a diagonal data error covariance matrix, simplifying the inversion procedure, but it ignores correlations due to any sources that have not been modelled. If full advantage is to be taken of Swarm data, better account should be taken of these spatially and temporally correlated errors, through an improved data error covariance matrix.

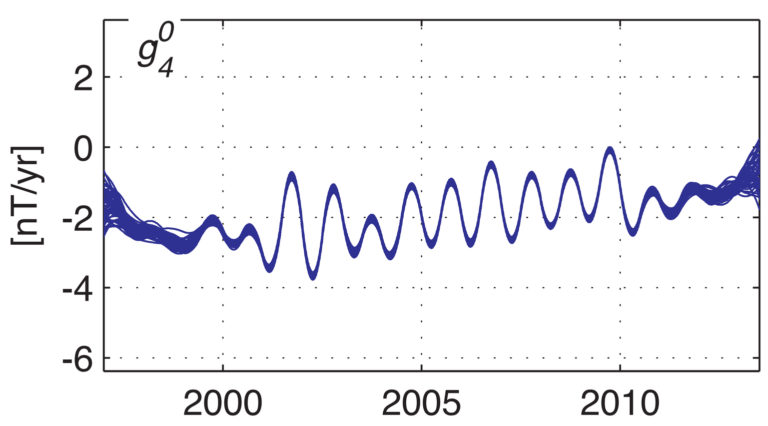
We show that attempts to build high time resolution field models with Ørsted, CHAMP and SAC-C data suffer from spurious oscillations, especially in the zonal components, once the standard temporal regularization is relaxed. Instability due to unmodelled fields also hamper attempts to build high resolution models of the lithospheric field. In an effort towards better handling this situation, we document the correlation of residuals (between CHAMP magnetic data and the CHAOS-4 field model) as a function of both time and of quasi-dipole latitude. We describe how these time-correlated errors can be included in practical inversion schemes. The success of future assimilation of Swarm magnetic data into numerical models of core dynamics may ultimately be reliant on access to suitable data error covariance matrices.

Impact of correlated errors on field models

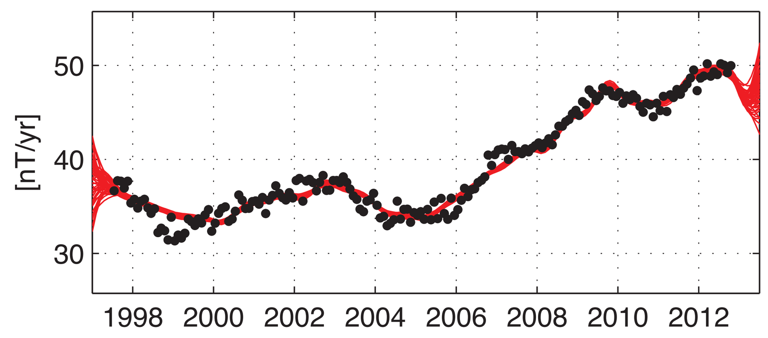
- Standard practice in time-dependent field modelling is to regularize the temporal evolution by penalizing second/third time derivatives at the CMB.
- This unfortunately damps rapid field variations at small length scales, and is incompatible with jerk events.
- Gillet et al. (2013) instead propose use of a less restrictive time-correlation function compatible with the observed PSD for annual to decadal field changes ($P(f) \propto f^{-4}$).
- To test this method on satellite magnetic data, we built an ensemble of CHAOS-4 type field models (Olsen et al,. 2014) compatible with the Gillet et al. correlation function. Results are shown below.



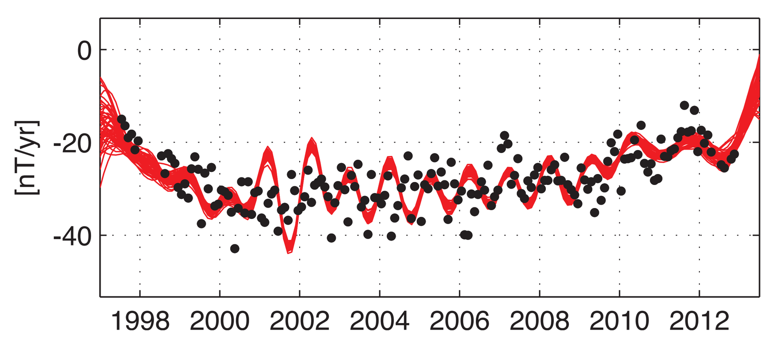
Above: Evolution of $d/dt(h_6^6)$, 50 models shown.



Above: Zonal coefficients e.g. $d/dt(g_4^0)$ show spurious approx annual oscillations.



Above: Fit of 50 models to annual diffs of monthly means: dB_ϕ/dt from NGK, Germany.

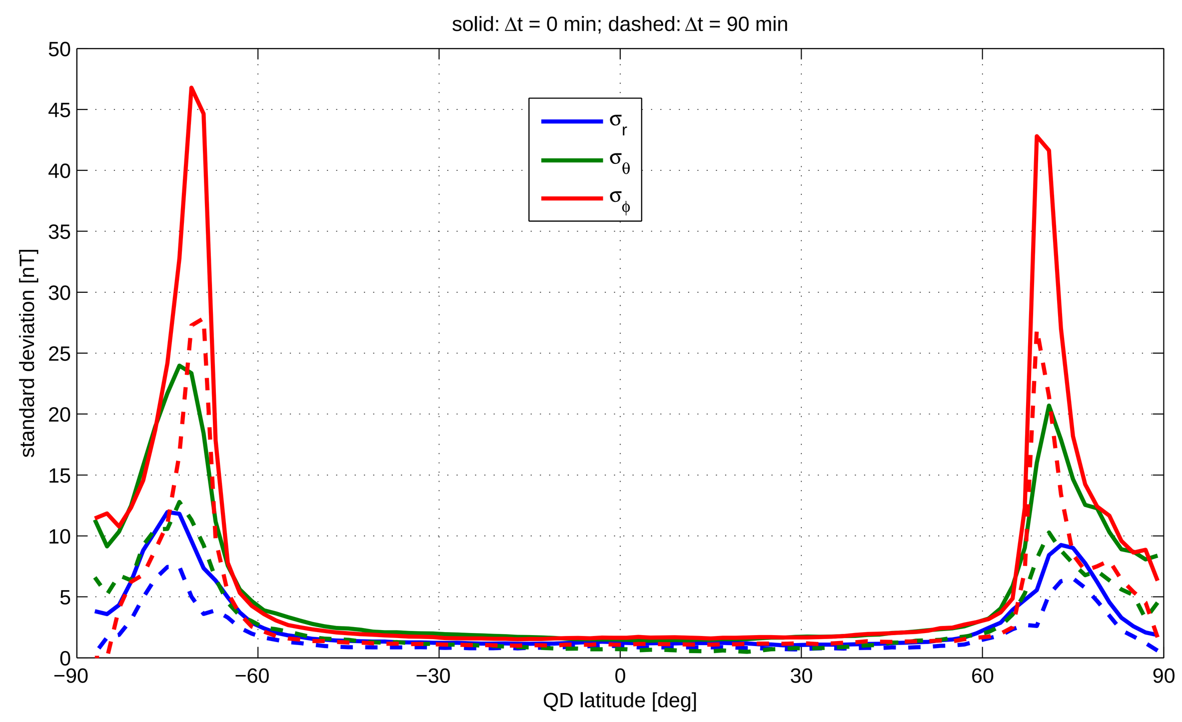


Above: Annual oscillations of all ensemble members as auroral latitudes approached. e.g.dB_r/dt at Macquarie Island, Australia.

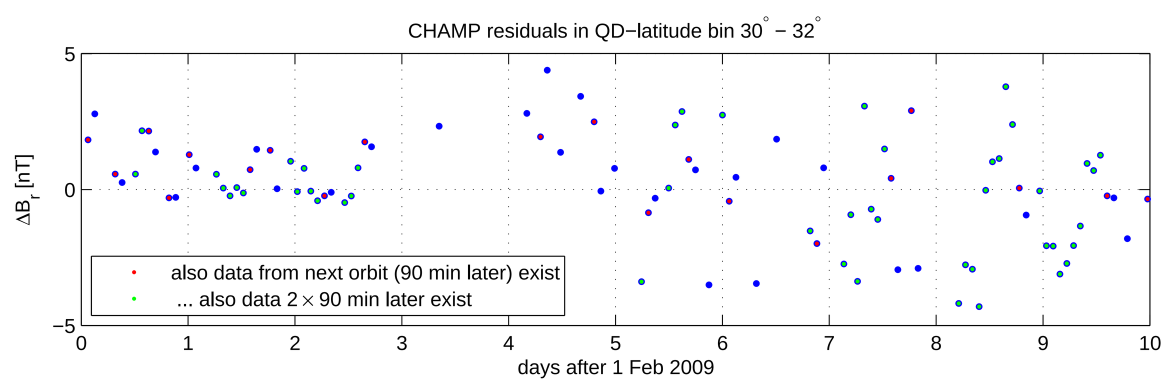
- **Conclusion: Correlated errors, due to unmodelled signals, destroy our ability to extract the rapid core field changes of interest.**

Spatial structure of variance and time correlation

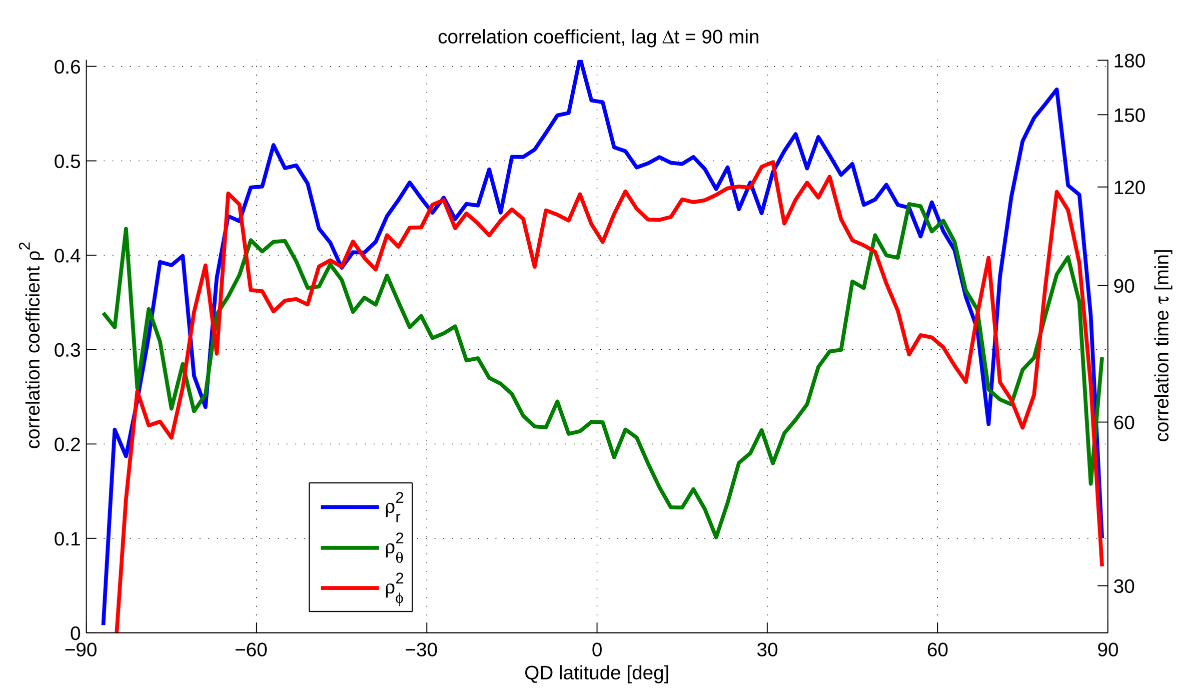
- We have analyzed residuals between the CHAOS-4 model and low altitude, solar minimum CHAMP data, looking for evidence of trends in their spatial structure.
- Plotting the (robust) standard deviation of the residuals as a function of Quasi-Dipole (QD) latitude (see solid lines in figure below) we find evidence for three distinct regimes:
  - non polar latitudes ($< 60°$): $\sigma_{B_r} \approx 1.2\mathrm{nT} < \sigma_{B_\theta} \approx \sigma_{B_\phi}$
  - auroral oval ($65° - 80°$): $\sigma_{B_r} \ll \sigma_{B_\theta} \ll \sigma_{B_\phi}$
  - polar cap ($> 80°$): $\sigma_{B_r} < \sigma_{B_\theta} \approx \sigma_{B_\phi}$



- Model residuals seem better organized in (B_r, B_θ, B_ϕ) compared to (B_B, B_2, B_3) provided attitude from > 1 star imager is available (e.g. CHAMP, Swarm).
- Given trend is primarily in QD latitude, we next test whether residuals from the same QD latitudes on consecutive orbits (90 minutes later) co-vary (i.e. are correlated in time). The square-root of this co-variance is shown by the dashed lines in the figure above.
- **The square-root of the co-variance is comparable to the standard deviation, particularly at low/mid QD latitudes, indicating strong temporal correlation between times 90mins apart that has previously been ignored.**
- How often are data available at the same QD latitude, in consecutive orbits? As an example we mark below those data points in Feb 2009 that have data available in the next and newt but one orbits, in the same QD latitude bin - there are many such examples.



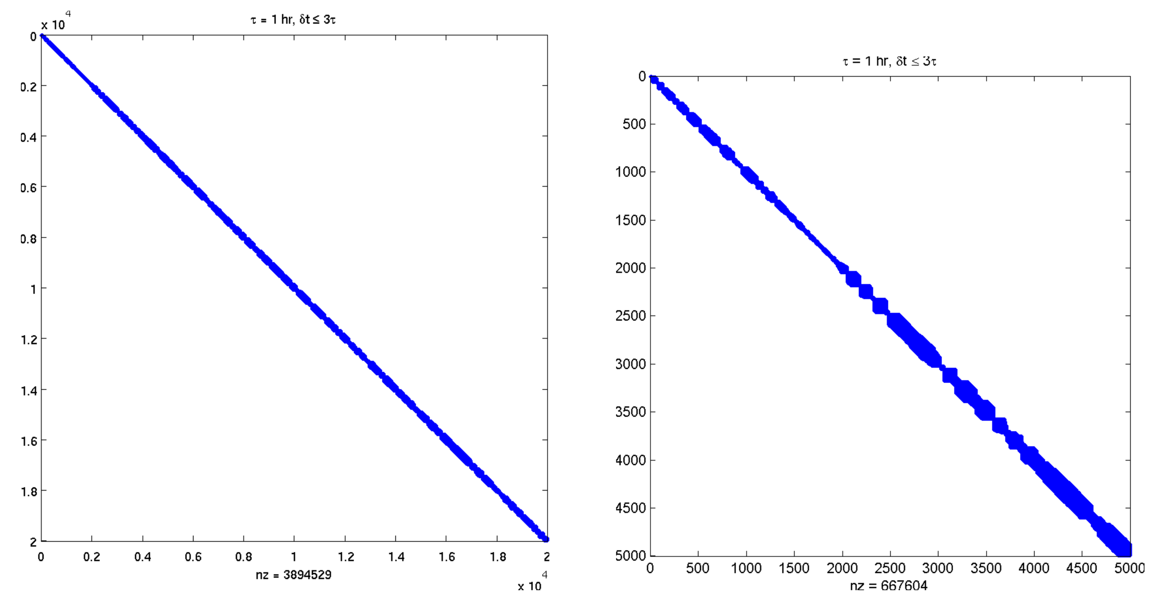
- Adopting the following simple exponential model for the co-variance in time,
$$\sigma^2(\delta t) = \sigma^2(0)\exp(-\delta t/\tau)$$
we can estimate the correlation time τ from $\sigma^2(0)$ and $\sigma^2(\delta t = 90\mathrm{min})$.
- Find typical correlation times of 1- 2 hours, relatively independent of QD latitude.
- Another way to illustrate this is by plotting the correlation $\rho^2(\delta t) = \sigma^2(\delta t)/\sigma^2(0) = \exp(\delta t/\tau)$



- **Conclusion: points on consecutive orbits, from the same QD latitude, have residuals that are highly correlated in time. This should be reflected in off-diagonal elements in the data error covariance matrix. Ignoring it (and hence modulations of unmodelled fields) may lead to model instabilities.**

Practical Implementation of Covariance Matrices

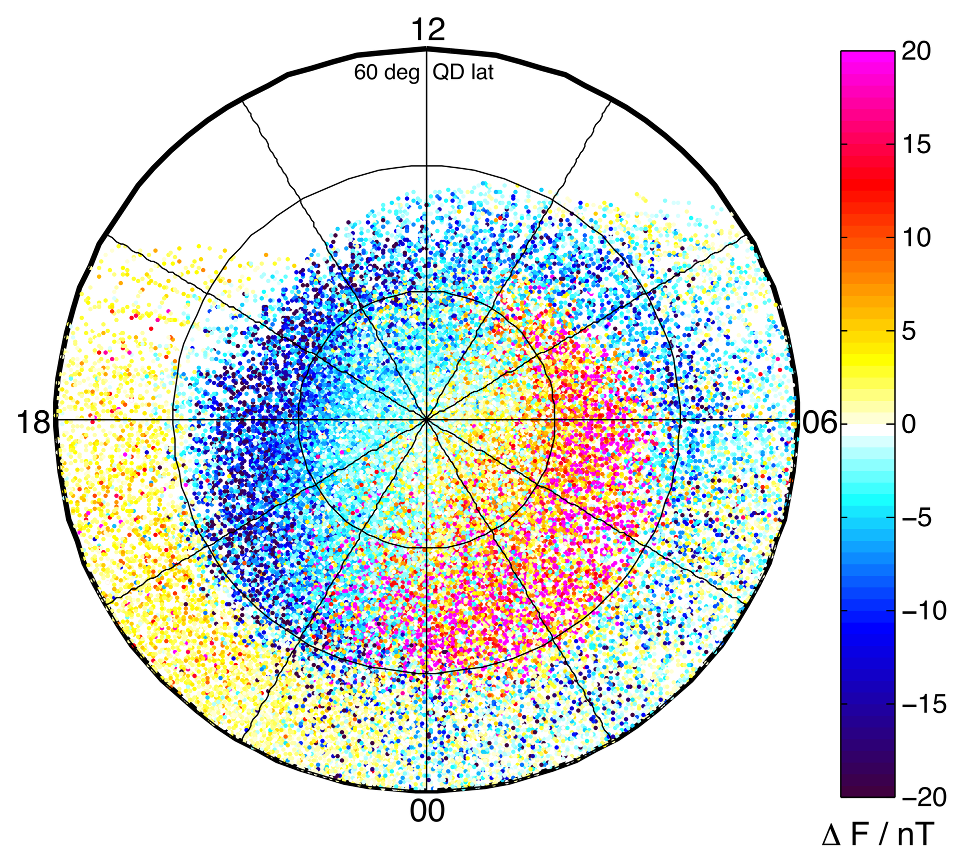
- It is often said that modelling the full data covariance matrix for satellite magnetic data is impossible, since this is of size N_{data} x N_{data} (Huge!).
- But in practice we can accumulate the normal equations matrix $\mathbf{G^T C_e^{-1} G}$ for chunks of data (e.g. split across processors). This are only of size $N_{model} x N_{model}$.
- With a correlation time τ much less than the data time span, if we only consider points up to 3τ apart as correlated (by then the correlation is only $\exp(-3) \approx 1/20$), then $\mathbf{C_e}$ **is sparse**.
- For example, for a chunk of 20,000 data and a correlation time $\tau = 1$ hr, only 3.9 mio out of 400 mio matrix elements are non-zero and C_e is of size 62 Mb. (see below, left for full matrix, right for a zoom.)



Conclusion: Processing data in chunks of up to 20,000 allows consideration of time-correlated errors.

Challenges at polar latitudes

- The **polar latitudes** are characterized by field aligned and polar electrojet current systems, rapidly varying in both space and time.
- Nonetheless we find a clear statistical structure to the residuals. The figure below shows scalar field residuals between CHAMP data (2008-2010) and the CHAOS-4 field model, at high latitudes in the northern hemisphere during winter, plotted in a QD latitude - Magnetic Local Time (MLT) frame.



- The structure above is well known to the space physics community from studies with ground and satellite data of the westward and eastward polar electrojet currents respectively at dawn and dusk (e.g. Friis-Christensen and Wilhjem, 1975).
- Correlated field residuals internal to satellites in winter hemisphere and little data selected in the summer could contribute to the spurious annual oscillation.
- **Conclusion: Handling this requires more than an improved data covariance matrix, since it acts a bias. Need to model MLT dependence of field disturbances at high latitudes.**

References

Friis-Christensen, E. and Wilhjelm, J. (1975) Polar cap currents for different directions of the interplanetary magnetic field in the Y-Z plane. *JGR*, Vol 80, Issue 10, pp 1248-1260.

Gillet, N., Jault, D., Finlay C. C., and Olsen, N. (2013) Stochastic modeling of the Earth's magnetic field: Inversion for covariances over the observatory era. *GGG*, Vol. 14, Issue 4, pp. 766-786.

Olsen, N., Luehr, H., Finlay, C.C., Sabaka, T. J., Michaelis, I., Rauberg, J. Toeffner-Clausen, L. (2014) The CHAOS-4 geomagnetic field model, *GJI*, Vol 197, pp. 815-827.

DTU Space, Technical University of Denmark, Electrovej 327, Kngs. Lyngby, Denmark
cfinlay@space.dtu.dk