Proudman-Taylor constraint, increases the length scale of the magnetoconvection cell and, hence, makes the system convect more readily and efficiently. This characteristic of magnetoconvection has led to an important suggestion that the dynamo of the Earth's core operates in the regime $T_a^{1/2} \approx Q$ where the geodynamo as a thermal engine is most effective.

Magnetoconvection in spherical geometry

In rotating spherical geometry, magnetoconvection in the presence of an imposed azimuthal field whose strength is proportional to distance from the rotation axis has been extensively studied (for example, Fearn, 1979, 1998; Proctor, 1994). Spherical magnetoconvection exhibits similar features in a plane layer: the critical Rayleigh number R_c reaches an overall minimum as the magnetic field strength increases to $Q = O(T_a^{1/2})$ at which the Lorentz and Coriolis forces are of comparable size. For larger values of Q, the effects of the magnetic field inhibit convection and thus R_c increases with growing Q; for smaller values of Q, the small scale of the convection cells resulting from the rotational constraint leads to extremely large R_c.

It should be pointed out, however, that spherical magnetoconvection in the presence of a more realistic magnetic field that satisfies electrically insulating boundary conditions shows a quite different behavior (Fearn and Proctor, 1983; Zhang, 1995). It was found that the two-dimensionality of purely thermal convection survives under the influence of a strong Lorentz force and that there exist no optimum values of Q that can give rise to an overall minimum of the critical Rayleigh number (Zhang, 1995). The value of R_c is a monotonically, smoothly decreasing function of Q. This is because the more realistic magnetic field can become unstable when Q is sufficiently large (Zhang and Fearn, 1993).

Keke Zhang and Xinhao Liao

Bibliography

Aurnou, J.M., and Olson, P.L., 2001. Experiments on Rayleigh-Bénard convection, magnetoconvection and rotating magnetoconvection in liquid gallium. *Journal of Fluid Mechanics*, **430**: 283–307.

Chandrasekhar, S., 1961. *Hydrodynamic and Hydromagnetic Stability.* Oxford: Clarendon Press.

Fearn, D.R., 1979. Thermal and magnetic instabilities in a rapidly rotating sphere. *Geophysics, Astrophysics, and Fluid Dynamics*, **14**: 103–126.

Fearn, D.R., 1998. Hydromagnetic flow in planetary cores. *Reports on Progress in Physics*, **61**: 175–235.

Fearn, D.R., and Proctor, M.R.E., 1983 Hydromagnetic waves in a differentially rotating sphere. *Journal of Fluid Mechanics*, **128**: 1–20.

Proctor, M.R.E., 1994. Convection and magnetoconvection in a rapidly rotating sphere. In Proctor, M.R.E., and Gilbert, A.D. (eds.), *Lectures on Stellar and Planetary Dynamos.* Cambridge: Cambridge University Press, pp. 75–115.

Roberts, P.H., 1978. Magneto-convection in a rapidly rotating fluid. In Roberts, P.H., and Soward, A.M. (eds.), *Rotating Fluids in Geophysics.* London: Academic Press, pp. 421–435.

Soward, A.M., 1979. Thermal and magnetically driven convection in a rapidly rotating fluid layer. *Journal of Fluid Mechanics*, **90**: 669–684.

Zhang, K., 1995. Spherical shell rotating convection in the presence of toroidal magnetic field. *Proceedings of the Royal Society of London Service A*, **448**: 245–268.

Zhang, K., and Fearn, D., 1993. How strong is the invisible component of the magnetic field in the Earth's core? *Geophysical Research Letters*, **20**: 2083–2088.

Cross-references

Core Convection
Core Motions
Geodynamo
Magnetohydrodynamic Waves
Proudman-Taylor Theorem

MAGNETOHYDRODYNAMIC WAVES

Introduction

Magnetohydrodynamic waves are propagating disturbances found in electrically conducting fluids permeated by magnetic fields where magnetic tension provides a restoring force on fluid parcels moving across field lines. The role played by magnetohydrodynamic waves, transporting disturbances in the flow and magnetic field and connecting disparate regions of the fluid, is crucial to our understanding of hydromagnetic systems. Magnetohydrodynamic waves in the Earth's liquid iron outer core have been proposed as the origin of changes of the Earth's magnetic field taking place on timescales of decades to centuries, and are thus of interest to both geomagnetists and paleomagnetists.

In the Earth's outer core, in addition to the magnetic forces acting on the electrically conducting fluid, we must also consider Coriolis forces resulting from planetary rotation, buoyancy forces due to gravity acting on density gradients and the constraints placed on flow by spherical shell geometry. Magnetohydrodynamic waves could be excited by convection-driven instabilities (Braginsky, 1964), topographically as flow is forced over bumps at the core-mantle boundary (Hide, 1966), by instabilities of the background magnetic field (Acheson, 1972), or even tidally due to deviations of rotating core geometry from exact sphericity (Kerswell, 1994).

This article focuses on the likely properties of magnetohydrodynamic waves in the Earth's outer core and provides a review of attempts to observe them. After a brief account of the history of investigations into magnetohydrodynamic waves in the section "Historical Review," the physics underpinning their existence will be described in the section "Force Balance and Waves in Rapidly Rotating Hydromagnetic Fluids." In particular, attention will focus on the emergence of a new characteristic timescale associated with such waves in rotating magnetohydrodynamic systems. Dispersion relations for magnetohydrodynamic waves when magnetic, buoyancy (Archimedes), and Coriolis forces are of equal importance (MAC waves) will be derived and interesting properties are noted in the section "Dispersion Relations for MC/MAC Waves in the Absence of Diffusion." The influence of diffusion, spherical geometry, and nonlinear effects on the waves will be discussed in the sections "Effects of Diffusion on MC/MAC Waves," "Influence of Spherical Geometry on MC/MAC Waves," and "Nonlinear Magnetohydrodynamic Waves," respectively. In the section "Magnetohydrodynamic Waves in a Stratified Ocean at the Core Surface," the suggestion that a stratified layer may exist at the top of the Earth's outer core is described and the type of waves that could be present there will be discussed. Finally, in the section "Magnetohydrodynamic Waves as a Mechanism for Geomagnetic Secular Variation," attempts to identify the presence of magnetohydrodynamic waves in the Earth's outer core through observations of the Earth's magnetic field will be reviewed and suggestions made as to how the wave hypothesis of geomagnetic secular variation could be tested using a combination of dedicated modeling and rapidly improving high-resolution observations.

For further details, the interested reader should consult the overviews by Hide and Stewartson (1972) and Braginsky (1989) or look in the textbooks by Moffatt (1978) or Davidson (2001). More technical reviews of the subject include Roberts and Soward (1972), Acheson and Hide (1973), Eltayeb (1981), Proctor (1994), and Zhang and Schubert (2000).

Historical review

Study of magnetohydrodynamic waves, especially with a focus on geophysical applications has a rich history and has captured the attention of some of the finest applied mathematicians and theoretical geophysicists over the past 50 years. Alfvén (1942) initiated the study of magnetohydrodynamic waves, investigating the simplest possible scenario where a balance of magnetic tension and inertia gives rise to waves, which became known as Alfvén waves in his honor (see *Alfvén Hannes* and *Alfvén waves*). Lehnert (1954) deduced that rapid rotation of the fluid system would lead to the splitting of plane Alfvén waves into two circularly polarized, transverse waves, one with period similar to inertial waves (a consequence of the intrinsic stability endowed to fluids by rotation) and a second with a much longer period. The latter represents a new, fundamental, timescale for rotating hydromagnetic systems which we shall refer to as the magnetic-Coriolis (MC) timescale. Chandrasekhar (1961) studied the effects of buoyancy on rotating magnetic systems, focusing primarily on axisymmetric motions invariant about the rotation axis. Braginsky (1964, 1967) realized the importance of nonaxisymmetric disturbances and showed that if magnetic, buoyancy, and Coriolis forces were equally important, fast inertial modes and slower magnetic modes would again result, but with periods also dependent on the strength of stratification. He christened these waves dependent on magnetic, buoyancy (Archimedes), and Coriolis forces as "MAC waves."

Hide (1966) was the first to consider the influence of spherical geometry on magnetohydromagnetic waves in a rotating fluid, studying the effects of the variation of Coriolis force with latitude. He showed that the resulting MC waves (commonly called MC Rossby waves) had the correct timescale to account for some parts of the geomagnetic secular variation, particularly its westward drift (see *Westward drift*). Malkus (1967) studied MC waves in a full sphere considering the special case when the background field increased in strength with distance from the rotation axis.

Elteyab (1972), Roberts and Stewartson (1974), Busse (1976), Roberts and Loper (1979), and Soward (1979) have demonstrated the importance of including magnetic and thermal diffusion in models of MAC waves, showing that the most unstable MAC waves in plane layer and annulus systems often occur on diffusive timescales. Elteyab and Kumar (1977) and Fearn (1979) carried out the first numerical studies of magnetohydrodynamic waves to include the effects of both buoyancy and diffusion in a rotating, spherical geometry. Fearn and Proctor (1983) went on to consider the effect of more geophysical physically plausible background magnetic fields and nonzero mean azimuthal flows. Most recently, Zhang and Gubbins (2002) have discussed the properties of convection-driven MAC and MC waves in a spherical shell geometry, studying a variety of background field configurations.

Force balance and waves in rapidly rotating hydromagnetic fluids

A physical understanding of MC waves can be achieved through consideration of the force balance in a rotating, electrically conducting, inviscid fluid that involves inertia, magnetic tension resisting flow across field lines (see *Alfvén waves* and *Magnetohydrodynamics*), and Coriolis forces acting normal to flows and to the axis of rotation. Coriolis forces are well known for causing circulating eddies in the atmosphere (e.g., hurricanes) and arise because, in a rotating reference frame, inertial motions follow curved trajectories rather than straight lines. It is useful to think about rotation imparting vorticity to a fluid, in the same way that magnetic fields impart tension perpendicular to magnetic field lines; vorticity imparts tension perpendicular to vortex lines (which lie parallel to the rotation axis) leading to a restoring force when fluid flows across them.

In this system, four possible force balances are conceivable. The first three require rapid fluid motions while the final is only possible for slow fluid motions. They are as follows:

1. When magnetic forces are much stronger than Coriolis forces; magnetic tension alone balances inertia and disturbances are communicated by Alfvén waves (see *Alfvén waves*).
2. When Coriolis forces are much stronger than magnetic forces; vortex tension balances inertia and disturbances are communicated by inertial waves.
3. When magnetic and Coriolis forces are of similar strength; a combination of magnetic field tension and vortex tension balances inertia and disturbances are communicated by inertial magnetic Coriolis (inertial-MC) waves.
4. When fluid motions are slow so that inertia is unimportant in the leading order force balance but magnetic and Coriolis forces are of similar strength; in this scenario, magnetic and vortex tension are in balance and disturbances are communicated by MC waves.

Balance (4) thus permits the existence of a new class of slow wave in rotating hydromagnetic systems, which is absent in nonmagnetic and nonrotating systems. Time dependence in this case arises only through changes in the magnetic field that, via the Lorentz force, produces changes in the fluid flow.

An estimate of the MC timescale can be obtained by performing a scale analysis of the important terms in the equations for conservation of momentum and magnetic induction. In the momentum equation, Coriolis and magnetic (Lorentz) forces are in balance, so $2\Omega U = B^2/\mu\rho L_{MC}$, where Ω is the angular rotation rate, U is a typical velocity scale, B is a typical magnetic field strength, L_{MC} is a typical length scale over which changes associated with MC waves occur, ρ is the density of the fluid, and μ is the fluid's magnetic permeability. We also know that for a highly conducting fluid, changes in the magnetic field come primarily from advection, so scale analysis of the induction equation ignoring diffusion yields $B/T_{MC} = UB/L_{MC}$, where T_{MC} is a typical timescale over which changes associated with MC waves occur. Substituting this expression for U into the force balance leads to the relation $T_{MC} = 2\Omega L_{MC}^2\rho\mu/B^2$. The quantity $v_A = B/(\rho\mu)^{1/2}$ has units of velocity and is the phase speed of Alfvén waves (see *Alfvén waves*). Estimates for these quantities in the Earth's core are $\Omega = 7.3\times10^{-5}\,s^{-1}$, $\rho = 1\times10^4\,kg$, $\mu = 4\pi\times10^{-7}T^2\,m\,kg^{-1}\,s^2$, so that $T_{MC} = 10^{-6}L_{MC}^2/B^2$. Neither the magnetic field strength nor the length scale associated with its variation in the Earth's core is well known. Taking $B = 5\times10^{-4}$T as suggested by observations at the core surface and $L_{MC} = 3.5\times10^6$ m, the core radius, yields $T_{MC}\approx1.5\times10^6$ y. Equally plausibly, if we are consider a wave with azimuthal wave number 8, and assume that the field inside the outer core is 10 times the observed core surface field strength we find $T_{MC}\approx235$ years. The coincidence between the latter MC wave timescale and that of geomagnetic secular variation motivates attempts to link the two phenomena.

In the Earth's core, it is likely that buoyancy forces (either thermal or compositional) could also be important in the primary force balance (see *Core convection*). In the remainder of this article we shall therefore generalize our discussion to include buoyancy, which modifies the MC timescale to a MAC timescale because Archimedes forces are now present. In the next section we present an outline of the derivation of the dispersion relation for MAC waves.

Dispersion relations for MC/MAC waves in the absence of diffusion

To focus the discussion, while keeping mathematics to a minimum, we shall consider a rather basic model of a rapidly rotating, electrically conducting, incompressible fluid in an infinite three-dimensional domain, where there are no dissipative processes (viscous, magnetic, or thermal diffusion) operating. Buoyancy forces are included via the

Boussinesq model, with the degree of stratification depending on the magnitude of the background temperature gradient.

We shall work in Cartesian coordinates $(\widehat{x},\widehat{y},\widehat{z})$ with the axis of rotation along $\widehat{z}$, a uniform background field $\boldsymbol{B}_0 = (B_{0x}\widehat{x} + B_{0y}\widehat{y} + B_{0z}\widehat{z})$ and a uniform background temperature gradient of $-\beta\widehat{z}$. If α is the thermal expansivity of the fluid, then density is determined by the relation $\rho = \rho_0(1+\alpha\Theta)$ where Θ is the perturbation from the background temperature field, so that in a gravity field of $-g\widehat{z}$ there will be a buoyancy force of magnitude $g\alpha\Theta$ in the $\widehat{z}$ direction.

We shall consider small perturbations $(\boldsymbol{u},\boldsymbol{b},\Theta)$ about a state of no motion (so the background velocity field is zero) and consider only slow motions so that inertial terms can be neglected and attention can focus on the MC force balance. The linearized equations governing the evolution of small perturbations are then the momentum equation

$$\underbrace{2\boldsymbol{\Omega}\times\boldsymbol{u}}_{\substack{\text{Coriolis}\\\text{acceleration}\\\text{due to rotation}}} = \underbrace{-\frac{1}{\rho}\nabla p}_{\substack{\text{acceleration}\\\text{due to pressure}\\\text{gradient}}} + \underbrace{\frac{1}{\mu\rho}(\boldsymbol{B}_0\cdot\nabla)\boldsymbol{b}}_{\substack{\text{acceleration}\\\text{due to}\\\text{field tension}}} + \underbrace{g\alpha\Theta\widehat{z}}_{\substack{\text{buoyant}\\\text{acceleration}}}, \qquad \text{(Eq. 1)}$$

the induction equation

$$\underbrace{\frac{\partial\boldsymbol{b}}{\partial t}}_{\substack{\text{Change in the}\\\text{magnetic field}}} = \underbrace{(\boldsymbol{B}_0\cdot\nabla)\boldsymbol{u}}_{\substack{\text{Stretching of magnetic}\\\text{field by fluid motion}}}, \qquad \text{(Eq. 2)}$$

and temperature equation

$$\underbrace{\frac{\partial\Theta}{\partial t}}_{\substack{\text{Change in the}\\\text{temperature field}}} = \underbrace{\beta(\widehat{z}\cdot\boldsymbol{u})}_{\substack{\text{advection of temperature}\\\text{field by fluid motion}}}. \qquad \text{(Eq. 3)}$$

Taking $\partial/\partial t(\nabla\times)$ the momentum equation (1) to eliminate pressure gives

$$2(\boldsymbol{\Omega}\cdot\nabla)\frac{\partial\boldsymbol{u}}{\partial t} = \frac{(\boldsymbol{B}_0\cdot\nabla)}{\rho\mu}\frac{\partial}{\partial t}(\nabla\times\boldsymbol{b}) + g\alpha\frac{\partial}{\partial t}(\nabla\times\Theta\widehat{z}), \qquad \text{(Eq. 4)}$$

while taking the curl $(\nabla\times)$ of the induction equation (2) we find,

$$\frac{\partial}{\partial t}(\nabla\times\boldsymbol{b}) = (\boldsymbol{B}_0\cdot\nabla)(\nabla\times\boldsymbol{u}). \qquad \text{(Eq. 5)}$$

Substituting from Eq. (5) into Eq. (4) for $\partial/\partial t(\nabla\times\boldsymbol{b})$ gives a vorticity equation quantifying the MAC balance with terms arising from Coriolis forces on the left-hand side and terms arising from the magnetic and buoyancy forces on the right-hand side

$$2(\boldsymbol{\Omega}\cdot\nabla)\frac{\partial\boldsymbol{u}}{\partial t} = \frac{(\boldsymbol{B}_0\cdot\nabla)^2}{\rho\mu}(\nabla\times\boldsymbol{u}) + g\alpha\frac{\partial}{\partial t}(\nabla\times\Theta\widehat{z}). \qquad \text{(Eq. 6)}$$

Operating on Eq. (6) with $((B_0\cdot\nabla)^2\nabla\times)/\rho\mu$ we can then eliminate $((\boldsymbol{B}_0\cdot\nabla)^2\nabla\times\boldsymbol{u})/\rho\mu$ from the term on the left-hand side by using Eq. (6) once again. By utilizing the well-known relation for incompressible fluids that $\nabla\times\nabla\times\boldsymbol{u} = -\nabla^2\boldsymbol{u}$, and then taking the dot product with $\widehat{z}$ while noting that $\widehat{z}\cdot(\nabla\times\Theta\widehat{z}) = 0$ and $\widehat{z}\cdot(\nabla\times\nabla\times\Theta\widehat{z}) = -(\partial^2/\partial x^2+\partial^2/\partial y^2)\Theta = -\nabla_H^2\Theta$ leaves

$$4(\boldsymbol{\Omega}\cdot\nabla)^2\frac{\partial^2}{\partial t^2}u_z = -\left[\frac{(\boldsymbol{B}_0\cdot\nabla)^2}{\rho\mu}\right]^2\nabla^2 u_z - g\alpha\frac{(\boldsymbol{B}_0\cdot\nabla)^2}{\rho\mu}\nabla_{\mathrm{H}}^2\frac{\partial\Theta}{\partial t}.$$

Finally, we make use of the temperature Eq. (3) to eliminate $\partial\Theta/\partial t$ and obtain a sixth-order equation in u_z, which we shall refer to as the diffusionless MAC wave equation

$$\left(4(\boldsymbol{\Omega}\cdot\nabla)^2\frac{\partial^2}{\partial t^2} + \left[\frac{(\boldsymbol{B}_0\cdot\nabla)^2}{\rho\mu}\right]^2\nabla^2 - g\alpha\beta\frac{(\boldsymbol{B}_0\cdot\nabla)^2}{\rho\mu}\nabla_{\mathrm{H}}^2\right)u_z = 0.$$

Properties of diffusionless MAC waves can now be deduced by substitution of plane traveling wave solutions of the form $u_z = \mathrm{Re}\{\widehat{u}_z e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\}$, where $\boldsymbol{k}$ is the wavevector and ω is the angular frequency

$$4(\boldsymbol{\Omega}\cdot\boldsymbol{k})^2\omega^2 - \left[\frac{(\boldsymbol{B}_0\cdot\boldsymbol{k})^2}{\rho\mu}\right]^2 k^2 - \frac{(\boldsymbol{B}_0\cdot\boldsymbol{k})^2}{\rho\mu}g\alpha\beta(k_x^2+k_y^2) = 0. \qquad \text{(Eq. 7)}$$

This expression can be written more concisely by observing that terms in it correspond to characteristic natural frequencies for magnetic-inertial (Alfvén) waves, gravity waves, and inertial waves in rotating fluids, respectively

$$\omega_{\mathrm{M}}^2 = \frac{(\boldsymbol{B}_0\cdot\boldsymbol{k})^2}{\rho\mu},\quad \omega_{\mathrm{A}}^2 = \frac{g\alpha\beta(k_x^2+k_y^2)}{k^2},\quad \omega_{\mathrm{C}}^2 = \frac{4(\boldsymbol{\Omega}\cdot\boldsymbol{k})^2}{k^2}, \qquad \text{(Eq. 8)}$$

so Eq. (7) simplifies to

$$\omega_{\mathrm{C}}^2\omega^2 - \omega_{\mathrm{M}}^4 - \omega_{\mathrm{M}}^2\omega_{\mathrm{A}}^2 = 0. \qquad \text{(Eq. 9)}$$

Solving for ω gives the necessary condition (or dispersion relation) that must be satisfied by the angular frequency and wavevectors of plane MAC waves,

$$\omega = \pm\frac{\omega_{\mathrm{M}}^2}{\omega_{\mathrm{C}}}\left(1+\frac{\omega_{\mathrm{A}}^2}{\omega_{\mathrm{M}}^2}\right)^{1/2} = \pm\frac{k(\boldsymbol{B}_0\cdot\boldsymbol{k})^2}{2\rho\mu(\boldsymbol{\Omega}\cdot\boldsymbol{k})}\left(1+\frac{g\alpha\beta\rho\mu(k_x^2+k_y^2)}{k^2(\boldsymbol{B}_0\cdot\boldsymbol{k})^2}\right)^{1/2}. \qquad \text{(Eq. 10)}$$

Note that this is singular if $\boldsymbol{B}_0\cdot\boldsymbol{k} = 0$ or if $\boldsymbol{\Omega}\cdot\boldsymbol{k} = 0$, so diffusionless MAC waves cannot propagate normal to magnetic field lines or the rotation axis. Their frequency depends strongly on their wavelength (i.e., they are highly dispersive) and on their direction (i.e., they are anisotropic). In the special case when the background magnetic field and the direction of the rotation axis are parallel to the direction of wave propagation, and when buoyancy forces are absent ($\alpha = 0$), the dispersion relation simplifies to $\omega = B_0^2k^2/2\Omega\rho\mu$ or $T_{\mathrm{MC}} = 2\Omega\rho\mu L_{\mathrm{MC}}/B_0^2$ as was deduced from scaling arguments in the previous section. The phase speed of the waves is then $c = \omega/k = B_0^2k/2\Omega\rho\mu$ and it is seen that waves with shorter wavelengths travel faster.

Effects of diffusion on MC/MAC waves

So far we have neglected the influence of any source of dissipation (viscous, magnetic, or thermal diffusion) on the system in order to simplify both the mathematical analysis and the physical picture. It is now necessary to consider their effects. Naively, we might expect the presence of dissipation should merely damp disturbances and irreversibly transform energy to an unusable form. Although such processes undoubtedly occur, they are not the only effects of the presence of diffusion. Perhaps more importantly, diffusion adds extra degrees of freedom to the system and facilitates the destabilization of waves that are stable in the absence of diffusion (see Roberts and Loper, 1979). This rather counterintuitive effect means that instability of MAC/MC waves can occur for smaller unstable density or magnetic field gradients than if no diffusion were present. In fact, such diffusive instability turns out to be possible, even in the presence of a stable density gradient.

The diffusive instability mechanism works most effectively when the oscillation frequency matches the rate of diffusion, so the timescale of the most unstable MC/MAC waves will be that of the diffusion process that is facilitating the instability. Diffusion thus introduces new preferred timescales into the MC/MAC wave problem.

To include diffusion in the mathematical description of MAC waves, we must replace the operator $\partial/\partial t$ by $(\partial/\partial t - \nu\nabla^2)$ in the momentum equation, by $(\partial/\partial t - \eta\nabla^2)$ in the induction equation and by $(\partial/\partial t - \kappa\nabla^2)$ in the heat equation. Retaining the acceleration term from the momentum equation and including the Laplacian (diffusion) terms before the substitution of plane wave solutions results in a more complicated dispersion relation for diffusive MAC waves

$$\left(\omega_C^2(\omega + i\eta k^2)^2 - \left[(\omega + i\nu k^2)(\omega + i\eta k^2) - \omega_M^2\right]^2\right)(\omega + i\kappa k^2) + \omega_A^2(\omega + i\eta k^2)\left[(\omega + i\nu k^2)(\omega + i\eta k^2) - \omega_M^2\right] = 0. \qquad \text{(Eq. 11)}$$

Restricting ourselves to the conditions present in the Earth's outer core, where we expect ohmic diffusion to dominate viscous and thermal diffusion ($\eta \gg \nu, \kappa$) and where we can again neglect inertial accelerations when considering slow oscillations, this expression simplifies to

$$\left(\omega_C^2(\omega + i\eta k^2)^2 - \omega_M^4\right)\omega - \omega_M^2\omega_A^2(\omega + i\eta k^2) = 0. \qquad \text{(Eq. 12)}$$

The link to diffusionless MC waves becomes apparent if buoyancy forces are negligible ($\omega_A = 0$) when Eq. (12) reduces to,

$$\omega = \frac{\omega_M^2}{\omega_C} - i\eta k^2. \qquad \text{(Eq. 13)}$$

Here the classical damping role of magnetic diffusion is obvious, causing MC waves with shorter wavelengths to decay in amplitude more quickly than MC waves with longer wavelengths. More detail on diffusive MC waves and their consequences for on geodynamo simulations can be found in Walker *et al.* (1998).

Influence of spherical geometry on MC/MAC waves

The Earth's outer core is not an infinite plane layer, but a thick spherical shell with an inner radius approximately one-third of its outer radius. How does spherical shell geometry influence propagation properties, stability, and the planform of magnetohydrodynamic waves? It appears that when the magnetic field is strong enough, and the Lorentz force dominates the force balance in the momentum equation, then the spherical boundaries play a secondary role. On the other hand, when the magnetic field is weak, the influence of the Coriolis force and its latitudinal variations caused by the spherical geometry are crucial. In the absence of any certain knowledge of the strength of the magnetic field in the Earth's core it is unclear if spherical shell geometry has a controlling influence on magnetohydrodynamic waves there, so the safest course is to use spherical geometry and study a variety of magnetic field strengths.

Hide (1966) was the first to appreciate the importance of the latitudinal dependence of the Coriolis force for waves in the Earth's core. He developed a simple analytical model of MC waves retaining only the linear variation of Coriolis force with latitude (this is known to meteorologists and oceanographers as a β plane model and is the necessary ingredient for the restoring force responsible for Rossby waves). We shall refer to Hide's waves as magnetic-Coriolis (MC) Rossby waves. He showed they would propagate westward in a thick spherical shell and could have a timescale similar to that of geomagnetic secular variation.

Eltayeb and Kumar (1977) and Fearn (1979) included both thermal buoyancy and diffusion and worked in spherical geometry. They were confronted by a rather complex scene, with several different mechanisms giving rise to different types of magnetohydrodynamic waves, any of which could potentially be important in the Earth's core. They identified four distinct regimes where different waves were favored. Only a brief overview of the four possible regimes is presented here.

Type I: Magnetically modified, buoyancy-driven Rossby waves

When the magnetic field is very weak, wave motion is essentially that produced by convection in a rapidly rotating sphere (i.e., thermal Rossby waves or Busse rolls—see *Core convection*). The flow consists of columnar rolls parallel to the rotation axis, arranged on a cylindrical shell that intersects the outer boundary at midlatitudes. At the onset of convection and in the absence of any mean azimuthal flow these waves drift eastward on a thermal diffusion timescale. The planform of the waves is columnar because the Coriolis force promotes invariance along the rotation axis (see *Proudmann-Taylor theorem*). The magnetic field acts only as a small perturbation and actually stabilizes the system, increasing the critical Rayleigh number compared to the nonmagnetic systems. Instability is driven by the component of buoyancy perpendicular to the rotation axis and is balanced primarily by the Coriolis force (varying with latitude due to the spherical geometry) and viscous diffusion.

Type II: Buoyancy-driven magneto-Rossby waves

With stronger magnetic fields, buoyancy is balanced by the magnetic (Lorentz) force as well as the Coriolis force. The planform of Type II waves is similar to those of Type I waves and they too propagate on the thermal diffusion timescale. Both westward and eastward propagation of these waves is possible, depending on the relative magnitudes of magnetic and thermal diffusion. It should be noted that the magnetic field now plays a destabilizing roll, catalyzing the onset of convection. The dominant role of the uniform imposed magnetic field causes an increase in the length scale so that the number of waves fitting around a cylindrical shell decreases, while the latitude of the rolls moves toward where the magnetic field is strongest. Figure M148 shows the form of the radial magnetic field disturbance produced by a magneto-Rossby wave in spherical geometry when the imposed magnetic field increases linearly with distance from the rotation axis (the force-free field of Malkus, 1967). This figure is the result of an eigenvalue calculation (using the code of Jones *et al.*, 2003) used to determine the most unstable wave in a regime when magnetic and Coriolis forces are approximately of equal magnitude.

Type III: Buoyancy-driven MAC waves

When magnetic forces are much stronger than the Coriolis forces, boundary curvature associated with spherical geometry plays a less important role and the most unstable wave is of diffusive MAC-type, again propagating on the thermal diffusion timescale. The planform of the waves is no longer that of columnar rolls because the strong magnetic fields permit departures from z independence. Both westward- and eastward-propagating waves are possible in this regime.

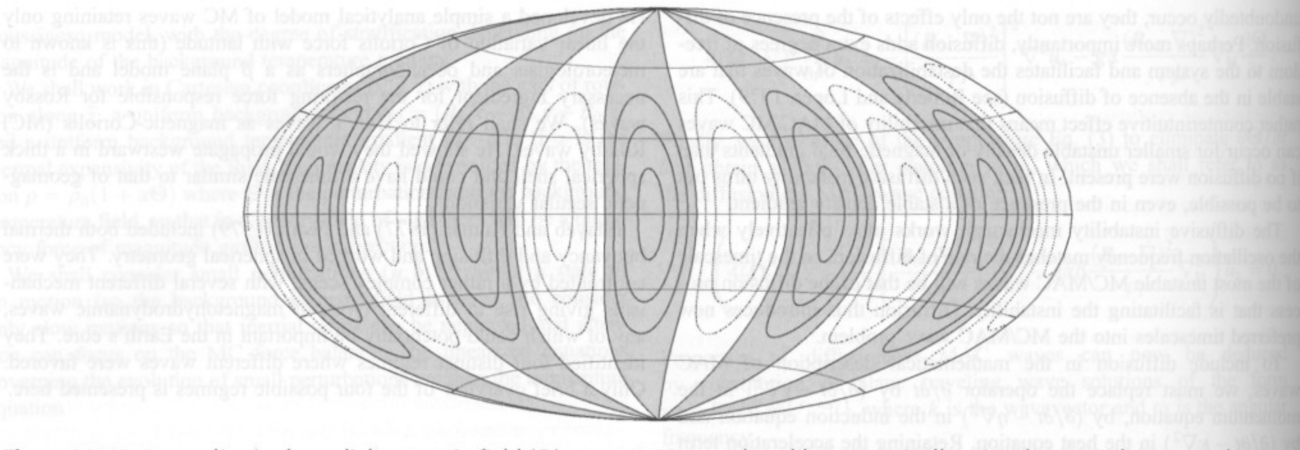

Figure M148 Anomalies in the radial magnetic field (B_r) at $r = 0.95r_0$ produced by a marginally critical, $m = 5$, buoyancy-driven magneto-Rossby wave. The imposed magnetic field is purely toroidal and increases linearly in magnitude with distance from the rotation axis. Units are arbitrary because no nonlinear saturation mechanisms are included in this model. Gray regions with solid contours indicate negative field anomalies, and white regions with dotted contours indicate positive field anomalies.

Type IV: Magnetically driven MAC waves

When the magnetic field becomes sufficiently strong or complex, then MC/MAC waves can be produced by either diffusive (resistive) or ideal instability of the background magnetic field. The resulting waves are of diffusive MC/MAC or diffusionless MC/MAC type and propagate on either the magnetic diffusion or on the MC/MAC timescale. They do not require the presence of buoyancy for their existence, and can even occur when the background density field is stable.

Fearn and Proctor (1983) have considered the additional effect of the presence of a background azimuthal flow (including shear) and found that this tends to stabilize diffusive MAC waves. They observed that such waves are localised at the extrema of the shear, moving with an azimuthal speed equal to the fluid velocity at that point. This indicates MC/MAC waves could perhaps be preferentially excited in zonal jets and would drift by advection rather than propagation, which could perhaps be of relevance at low latitudes in the Earth's outer core (see the section "Magnetohydrodynamic Waves as a Mechanism for Geomagnetic Secular Variation").

Nonlinear magnetohydrodynamic waves

All the magnetohydrodynamic waves discussed up to now have been linear in nature. This implies that (i) waves can simply be superposed without considering any mutual interaction and (ii) there is no feedback between the waves and the rest of the system. This scenario is unphysical because unstable waves can grow without limit, but is nonetheless useful for determining the types of waves most easily excited in a particular regime of interest. Early studies by Braginsky (1967) and Roberts and Soward (1972), though deriving linear equations for MAC waves riding on general background states, emphasized the importance of understanding nonlinear feedback processes. They noted that waves are determined by the background state, but the background state is itself altered by the waves. In seeking to interpret geophysical observations indicative of hydromagnetic waves in the Earth's core, we should remember that linear analysis is only formally valid for small perturbations to an artificial, steady background state and cannot tell us how waves will evolve, saturate, interact with each other, or what flow structures might result from nonlinear bifurcations of the waves. Attempts to understand such processes deserve a concerted theoretical and numerical modeling effort in the future.

Some progress in understanding nonlinear MAC waves has already been made. El Sawi and Eltayeb (1981) have derived higher order equations for MAC waves in a plane layer in the presence of a slowly varying background mean flow. Their equations describe the evolution of diffusionless MAC wave amplitude via the conservation of wave action (wave energy divided by wave frequency per unit volume). This conservation law tells us that wave energy increases, at the expense of the energy of the background state, whenever a wave moves into a region where its frequency is higher. More recently Ewen and Soward (1994) have derived equations describing the evolution of the amplitude of diffusive MAC waves in the limit of a weak magnetic field. They find that an azimuthal (geostrophic) mean flow is driven by magnetic forces resulting from the MAC waves and is linearly damped by viscous diffusion at the boundary.

A start has also been made at numerically investigating the nonlinear evolution of magnetohydrodynamic waves in rapidly rotating, convecting spherical shells. As the unstable density gradient is increased, it is observed that the system undergoes bifurcations from steadily traveling magnetohydrodynamic waves to vacillating wave motions for which both temporal and spatially symmetries have been broken (see *Magnetoconvection*).

Study of nonlinear MC and MAC waves is still in its infancy and may yet yield important and exciting insights that will help us to better understand how magnetohydrodynamic waves might manifest themselves in the Earth's core.

Magnetohydrodynamic waves in a stratified ocean at the core surface?

There has been some debate over the possibility of a stratified layer or "inner ocean" at the top of the Earth's outer core and the MAC waves that would be supported there (Braginsky, 1999). This stratified layer has yet to be observed seismically, though its existence seems plausible on thermodynamic grounds with light fluid released during the solidification of the inner core expected to pond below the core-mantle boundary. Oscillations of such a layer would be of shorter period than the MC/MAC waves expected in the body of the outer core due to the presence of an additional restoring force due to density stradification.

The dynamics of a stably stratified layer would be dominated by its thin spherical shell geometry. There would undoubtedly be many similarities with the water ocean on the Earth's surface, but with the additional complications caused by the presence of magnetic forces. In particular, MC Rossby waves, which rely on the change in the Coriolis force with latitude for their existence, are likely to be present within such an ocean. Braginsky has developed models of both axisymmetric and nonaxisymmetric disturbances of such a stably stratified layer and has suggested they could be responsible for short period geomagnetic secular variation.

Unfortunately, MAC waves and MC Rossby waves in a hidden ocean at the top of the outer core are not the only possible source of short period geomagnetic secular variation-torsional oscillations within the body of the core (see *Oscillations, torsional*) are an equally plausible explanation. Until the existence of the hidden ocean of the core can be confirmed, study of magnetohydrodynamic waves that may exist there will remain of primarily theoretical interest.

Magnetohydrodynamic waves as a mechanism for geomagnetic secular variation

It has been common knowledge since the time of Halley that the Earth's magnetic field changes significantly over decades to centuries. Perhaps the most striking aspect of this geomagnetic secular variation is the westward motion of field features (see Westward drift). Several explanations have been proposed for this Westward drift, but today there are two widely accepted candidate mechanisms. The first involves bulk fluid motion at the surface of the outer core that advects magnetic field features. Bullard *et al.* (1950) originally envisaged this involving westward flow of all the fluid close to the core surface, but modern core flow inversions (see *Core motions*) have refined this suggestion—it now appears that a westward equatorial jet under the Atlantic hemisphere is sufficient to explain much of the westward drift of the geomagnetic field observed at the surface. The source of this proposed equatorial jet is still debated, but geodynamo models indicate that it could be produced by nonlinear inertial forces that are a by-product of columnar convection in a sphere, or by thermal winds due to an inhomogeneous heat flux into the mantle (see *Inhomogeneous boundary conditions and the dynamo*). The second possible mechanism is that motivating the inclusion of this article in an *Encyclopedia of Geomagnetism and Paleomagnetism*—propagation of magnetohydrodynamic waves in the Earth's outer core. We shall henceforth refer to this mechanism as the wave hypothesis.

Hide (1966) and Braginsky (1967) proposed the wave hypothesis on theoretical grounds and each attempted to test it through the consideration of available records of the geomagnetic secular variation. It is informative to review these pioneering attempts before considering other possible ways to search for the presence of magnetohydrodynamic waves in the Earth's outer core.

Hide (1966) identified three major factors in favor of the wave hypothesis: (i) MC waves had periods comparable with the timescale of geomagnetic secular variation; (ii) MC waves had dispersion times comparable with the timescale of geomagnetic secular variation; and (iii) shorter wavelength MC waves had larger phase velocities and similarly higher order spherical harmonic components of the geomagnetic field drifted faster. His observational analysis was based on the mean westward drift rates of spherical harmonics up to degree 4, from seven previous publications, spanning 135 years from 1830 to 1965. Despite the failure of detailed comparisons between the predicted and observed drift rates for individual spherical harmonics, this study was instrumental in persuading many geophysicists that a wave origin for geomagnetic secular variation was worth serious consideration.

Braginsky (1967, 1972, 1974) sought to confirm the wave hypothesis by comparing his own theoretical predictions of the spectrum of diffusionless MAC waves to an observationally inferred spectrum of geomagnetic secular variation. Despite the poor quality of data available in the early

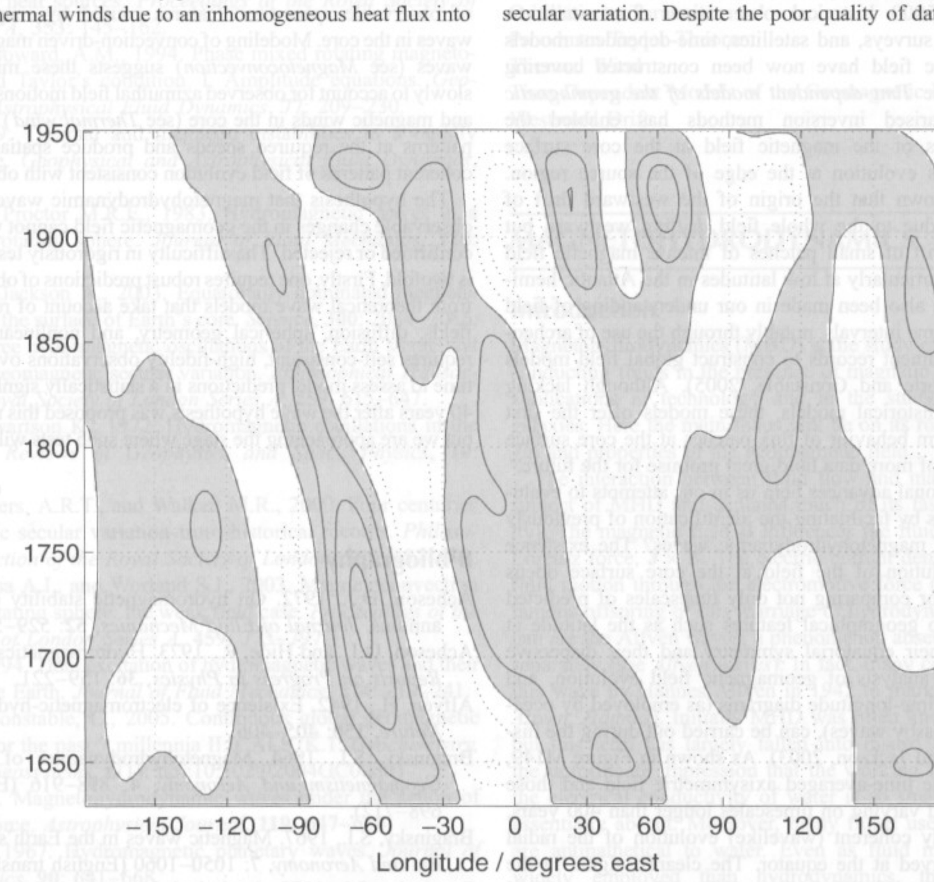

Figure M149 Equatorial time-longitude plot of processed radial magnetic field B_r (with time-averaged axisymmetric field and field variations with timescales longer than 400 years removed) from the historical field model *gufm1*, (Jackson *et al.*, 2000). Field evolution in the azimuthal direction at the equator is shown and consists of spatially and temporally coherent wavelike anomalies, with dominant azimuthal wave number $m = 5$ and moving consistently westward. The contour lines are at intervals of 5×10^4 nT. Gray regions with solid contours indicate negative field anomalies, and white regions with dotted contours indicate positive field anomalies.

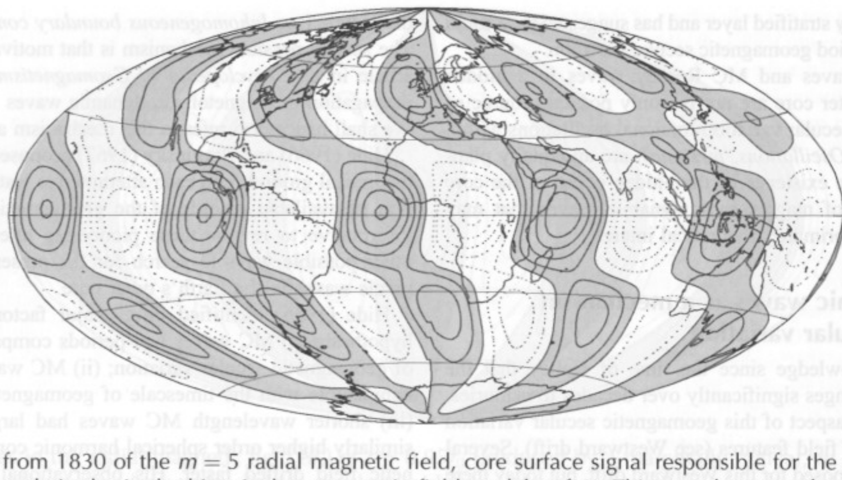

Figure M150 Snapshot from 1830 of the $m = 5$ radial magnetic field, core surface signal responsible for the wavelike pattern of field evolution observed at low latitudes in the historical geomagnetic field model *gufm1*. This snapshot was obtained by restricting the period of field variations to between 125 and 333 years and the wave number of field variations to $m = 5$ (i.e., FK filtering). The contour lines are at intervals of 4×10^3 nT. Gray regions with solid contours indicate negative field anomalies, and white regions with dotted contours indicate positive field anomalies.

1970s, Braginsky's efforts were important in demonstrating that the wave hypothesis was at least compatible with observations.

In the last 20 years there have been significant advances in our observational knowledge of the Earth's magnetic field and its evolution. Using over 365000 historical observations from maritime records, observatories, surveys, and satellites, time-dependent models of the global magnetic field have now been constructed covering the past 400 years (see *Time-dependent models of the geomagnetic field*). Use of regularised inversion methods has enabled the construction of images of the magnetic field at the core surface allowing us to map its evolution at the edge of the source region. This technique has shown that the origin of the westward drift of magnetic field is not due to the whole field drifting westward but rather is due to the drift of small patches of intense magnetic field (see *Westward drift*), particularly at low latitudes in the Atlantic hemisphere. Advances have also been made in our understanding of field evolution over longer time intervals, notably through the use of archeomagnetic and lake sediment records to construct global field models for the past 3 ka (Korte and Constable, 2005). Although lacking the resolution of the historical models, these models offer the first glimpse of the long-term behavior of flux patches at the core surface and with the inclusion of more data hold great promise for the future.

Can recent observational advances help us in our attempts to evaluate the wave hypothesis by facilitating the identification of previously obscured signatures of magnetohydrodynamic waves? The existence of images of the evolution of the field at the core surface opens up fresh possibilities for comparing not only timescales of predicted wave motions, but also geographical features such as the latitude at which waves occur, their equatorial symmetry and their dispersive properties. Space-time analysis of geomagnetic field evolution, and particularly the use of time-longitude diagrams (as employed by oceanographers to study Rossby waves), can be carried out during the historical epoch (Finlay and Jackson, 2003). As shown in Figure M149, after the removal of the time-averaged axisymmetric field and those components of the field varying on timescales longer than 400 years, spatially and temporally coherent (wavelike) evolution of the radial magnetic field is observed at the equator. The clearest signal in the time-longitude plot has an azimuthal wave number of $m = 5$, a period of around 250 years and travels at $\sim$17 km yr^{-1} westwards. Figure M150 shows the result of frequency-wave number filtering to recover the spatial structure of this wave at the core surface. The domination of aspects of geomagnetic secular variation by a single wave number disturbance suggests that a magnetohydrodynamic wave (perhaps driven by an instability) might currently be present at low latitudes in Earth's outer core.

The major challenge for theoreticians is to keep pace with improving observations and construct models accurate enough to predict the structure of observable space-time features caused by magnetohydrodynamic waves in the core. Modeling of convection-driven magnetohydrodynamic waves (see *Magnetoconvection*) suggests these might propagate too slowly to account for observed azimuthal field motions. However, thermal and magnetic winds in the core (see *Thermal wind*) could advect wave patterns at the required speeds and produce spatially and temporally coherent patterns of field evolution consistent with observations.

The hypothesis that magnetohydrodynamic waves produce directly observable changes in the geomagnetic field cannot yet be conclusively confirmed or rejected. The difficulty in rigorously testing the hypothesis is twofold. Firstly, one requires robust predictions of observable signatures from theoretical wave models that take account of realistic background fields, diffusion, spherical geometry, and nonlinearity. Secondly, one requires self-consistent, high-fidelity observations over a long period of time to assess model predictions in a statistically significant way. Almost 40 years after the wave hypothesis was proposed this remains a tall order, but we are approaching the stage where such tests will be feasible.

Christopher Finlay

Bibliography

Acheson, D.J., 1972. On hydromagnetic stability of a rotating fluid annulus. *Journal of Fluid Mechanics*, **52**: 529–541.

Acheson, D.J., and Hide, R., 1973. Hydromagnetics of rotating fluids. *Reports on Progress in Physics*, **36**: 159–221.

Alfvén, H., 1942. Existence of electromagnetic-hydrodynamic waves. *Nature*, **150**: 405–406.

Braginsky, S.I., 1964. Magnetohydrodynamics of the Earth's core. *Geomagnetism and Aeronomy*, **4**: 898–916 (English translation, 698–712).

Braginsky, S.I., 1967. Magnetic waves in the Earth's core. *Geomagnetism and Aeronomy*, **7**: 1050–1060 (English translation, 851–859).

Braginsky, S.I., 1972. Analytical description of the geomagnetic field of past epochs and determination of the spectrum of magnetic waves in the core of the Earth. *Geomagnetism and Aeronomy*, **12**: 1092–1105 (English translation, 947–957).

Braginsky, S.I., 1974. Analytical description of the geomagnetic field of past epochs and determination of the spectrum of magnetic

waves in the core of the Earth. II. *Geomagnetism and Aeronomy*, **14**: 522–529 (English translation, 441–447).
Braginsky, S.I., 1989. Magnetohydrodynamic waves within the Earth. In James, D. E. (eds.), *Encyclopedia of Solid Earth Geophysics*. Kluwer Academic Publishers.
Braginsky, S.I., 1999. Dynamics of the stably stratified ocean at the top of the core. *Physics of the Earth and Planetary Interiors*, **111**: 21–34.
Bullard, E.C., Freedman, C., Gellman, H., and Nixon, J., 1950. The westward drift of the Earth's magnetic field. *Philosophical Transactions of the Royal Society of London*, **243**: 67–92.
Busse, F.H., 1976. Generation of planetary magnetism by convection. *Physics of the Earth and Planetary Interiors*, **12**: 350–358.
Chandrasekhar, S., 1961. *Hydrodynamic and Hydromagnetic Stability*. Oxford: Oxford University Press.
Davidson, P.A., 2001. *An Introduction to Magnetohydrodynamics*. Cambridge: Cambridge University Press.
El Sawi, M., and Eltayeb, I.A., 1981. Wave action and critical surfaces for hydromagnetic-inertial-gravity waves. *Quarterly Journal of Mechanics and Applied Mathematics*, **34**: 187–202.
Eltayeb, I.A., 1972. Hydromagnetic convection in a rapidly rotating fluid layer. *Proceedings of the Royal Society of London Series A*, **326**: 229–254.
Eltayeb, I.A., 1981. Propagation and stability of wave motions in rotating magnetic systems. *Physics of the Earth and Planetary Interiors*, **24**: 259–271.
Eltayeb, I.A., and Kumar S., 1977. Hydromagnetic convective instability of a rotating self-gravitating fluid sphere containing a uniform distribution of heat sources. *Proceedings of the Royal Society of London Series A*, **353**: 145–162.
Ewen, S.A., and Soward, A.M., 1994. Phase mixed rotating magnetoconvection and Taylor's condition. I. Amplitude equations. *Geophysical and Astrophysical Fluid Dynamics*, **77**: 209–230.
Fearn, D.R., 1979. Thermal and magnetic instabilities in a rapidly rotating sphere. *Geophysical and Astrophysical Fluid Dynamics*, **14**: 102–126.
Fearn, D.R., and Proctor M.R.E., 1983. Hydromagnetic waves in a differentially rotating sphere. *Journal of Fluid Mechanics*, **128**: 1–20.
Finlay, C.C., and Jackson A., 2003. Equatorially dominated magnetic field change at the surface of Earth's core. *Science*, **300**: 2084–2086.
Hide, R., 1966. Free hydromagnetic oscillations of the Earth's core and the theory of geomagnetic secular variation. *Philosophical Transactions of the Royal Society of London Series A*, **259**: 615–647.
Hide R., and Stewartson K., 1972. Hydromagnetic oscillations in the Earth's core. *Reviews of Geophysics and Space Physics*, **10**: 579–598.
Jackson, A., Jonkers, A.R.T., and Walker, M.R., 2000. Four centuries of geomagnetic secular variation from historical records. *Philosophical Transaction of the Royal Society of London*, **358**: 957–990.
Jones, C.A., Mussa A.I., and Worland S.J., 2003. Magnetoconvection in a rapidly rotating sphere: the weak field case. *Proceedings of the Royal Society of London Series A*, **459**: 773–797.
Kerswell, R.R., 1994. Tidal excitation of hydromagnetic waves and their damping in the Earth. *Journal of Fluid Mechanics*, **274**: 219–241.
Korte, M., and Constable, C., 2005. Continuous global geomagnetic field models for the past 7 millennia II: CALS7K.1. *Geochemistry, Geophysics, Geosystems*, **6**(1): doi:10.1029/2004GC00801
Lehnert B., 1954. Magnetohydrodynamic waves under the action of the Coriolis force. *Astrophysical Journal*, **119**: 647–654.
Malkus, W.V.R., 1967. Hydromagnetic planetary waves. *Journal of Fluid Mechanics*, **90**: 641–668.
Moffatt H.K., 1978. *Magnetic Field Generation in Electrically Conducting Fluids*. Cambridge: Cambridge University Press.
Proctor, M.R.E., 1994. Convection and magnetoconvection. In Proctor, M.R.E., and Gilbert, A.D. (eds.), *Lectures on Solar and Planetary Dynamos*. Cambridge: Cambridge University Press, 97–115.
Roberts, P.H., and Loper, D.E., 1979. On the diffusive instability of some simple steady magnetohydrodynamic flows. *Journal of Fluid Mechanics*, **90**: 641–668.
Roberts, P.H., and Soward, A.M., 1972. Magnetohydrodynamics of the Earth's core. *Annual Review of Fluid Mechanics*, **4**: 117–153.
Roberts, P.H., and Stewartson, K., 1974. On finite amplitude convection in a rotating magnetic system. *Philosophical Transactions of the Royal Society of London*, **277**: 287–315.
Soward, A.M., 1979. Convection-driven dynamos. *Physics of the Earth and Planetary Interiors*, **20**: 281–301.
Walker, M.R., Barenghi, C.F., and Jones C.A., 1998. A note on dynamo action at asymptotically small Ekman number. *Geophysical and Astrophysical Fluid Dynamics*, **88**: 261–275.
Zhang, K., and Gubbins, D., 2002. Convection-driven hydromagnetic waves in planetary fluid cores. *Mathematical and Computer Modelling*, **36**: 389–401.
Zhang, K., and Schubert, G., 2000. Magnetohydrodynamics in rapidly rotating spherical systems. *Annual Review of Fluid Mechanics*, **32**: 409–443.

Cross-references

Alfvén Waves
Alfvén, Hannes Olof Gösta
Core Convection
Core Motions
Inhomogeneous Boundary Conditions and the Dynamo
Magnetoconvection
Magnetohydrodynamics
Oscillations, Torsional
Proudman-Taylor Theorem
Thermal Wind
Time-Dependent Models of the Geomagnetic Field
Westward Drift

MAGNETOHYDRODYNAMICS

Introduction

Magnetohydrodynamics (MHD) is the study of the flow of electrically conducting fluids in the presence of magnetic fields. It has significant applications in technology and in the study of planets, stars, and galaxies. Here the main focus will be on its role in explaining the origin and properties of the geomagnetic field.

The interaction between fluid flow and magnetic field defines the subject of MHD and explains much of its fascination (and complexity). The magnetic field **B** influences the fluid motion **V** through the Lorentz force, **J** × **B**. The electric current density **J** is affected by the fluid motion through the electromotive force (emf), **V** × **B**. The most famous offspring of this marriage of hydrodynamics to electromagnetism are the Alfvén waves, a phenomenon absent from the two subjects separately (see *Alfvén waves*). In fact, many consider the discovery of this wave by Hannes Alfvén in 1942 to mark the birth of MHD (see *Alfvén, Hannes*). Initially MHD was often known as hydromagnetics, but this term has largely fallen into disuse. Like MHD, it conveys the unfortunate impression that the working fluid is water. In reality, the electrical conductivity of water is so small that MHD effects are essentially absent. Moreover, many fluids used in MHD experiments are antipathetical to water. Even as fluid mechanics is now more widely employed than hydrodynamics, the terms magnetofluid mechanics or magnetofluid dynamics, which are already sometimes employed, may ultimately displace MHD.

Since electric and magnetic fields are on an equal footing in electromagnetism (EM), it may seem strange that the acronym EMHD is not preferred over MHD. In many systems, however, including the Earth's