Stochastic process priors and satellite era field modelling

Chris Finlay, Nils Olsen, Nicolas Gillet, Dominique Jault & Lars Tøffner-Clausen

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- What else is there?
- Good old Ørsted is still providing some scalar data!
- And the observatories (some now reporting quasi-definitive data)

Ørsted scalar data availability 2011-2013

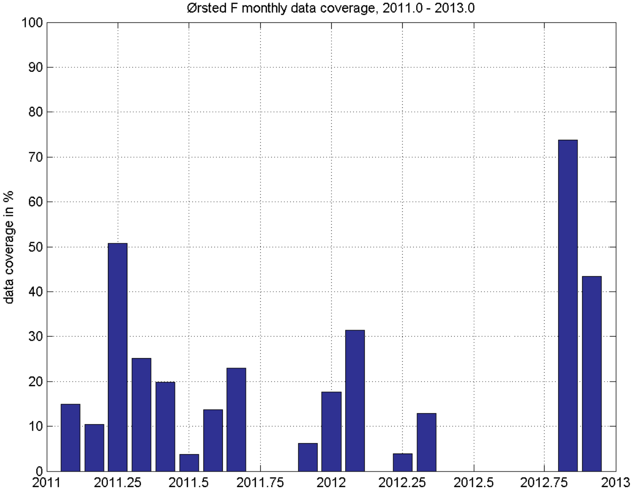

Fig 1:Availability of OER scalar data 2011-2013.

A simple test field model

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- 6th order splines for SH up to degree 16, 0.5yr knot spacing.
- Third time derivative regularization.
- Static field to SH degree 50. External field to degree 2.

Ørsted scalar quiet-time data used

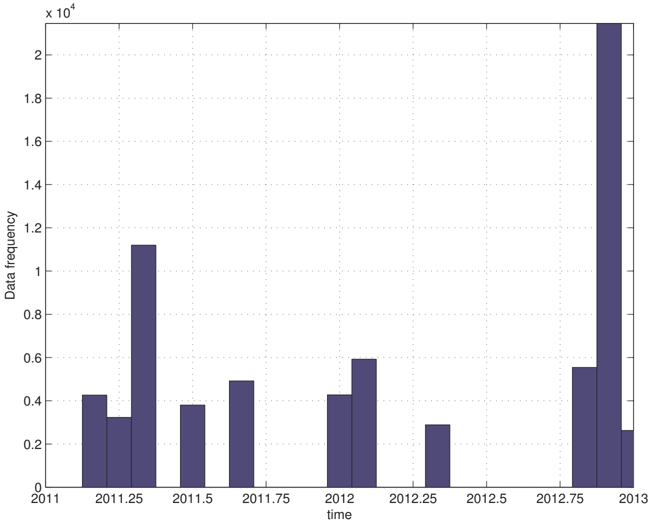

Fig 2:Radial field at the core suface, truncated at degree 13. model.

Core surface field in 2012.9

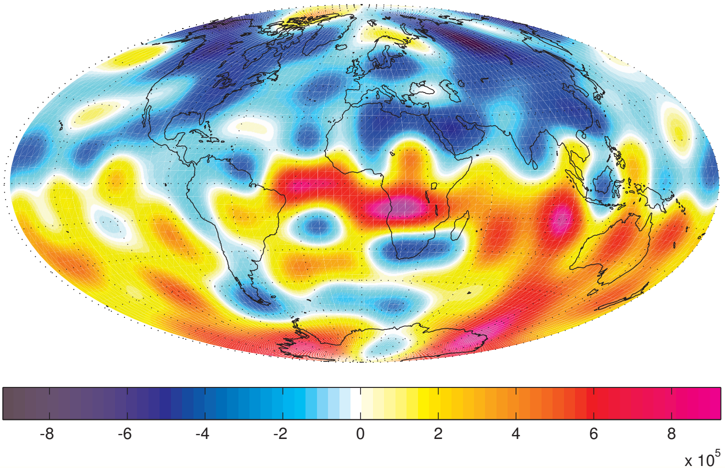

Fig 3:OER scalar data quiet time data used for 2011-2013 model.

SV and SA spectrum at core surface in 2012.9

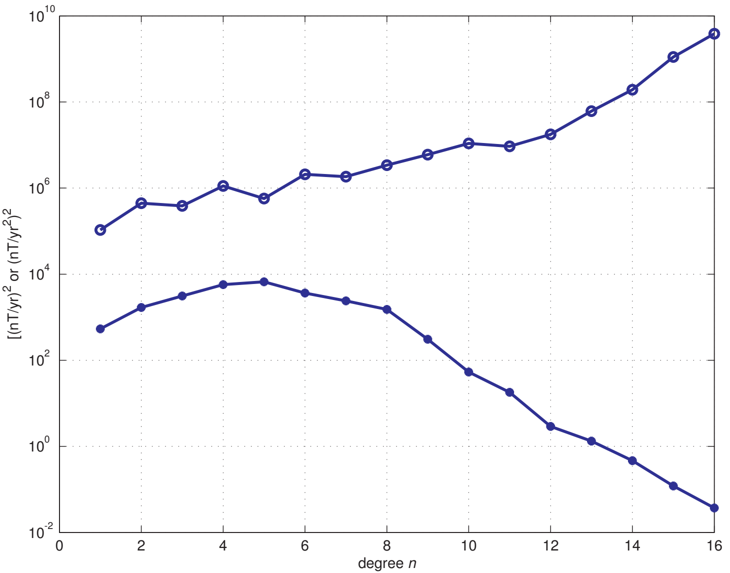

Fig 4:SV and SA spectrum for 2012.9.

SV of selected coefficients

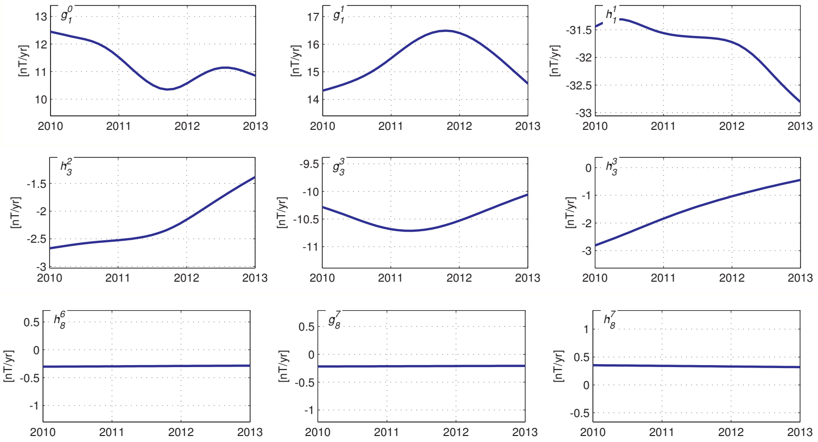

Fig 5: SV for selected Gauss coefficients

Fit to observatory monthly means

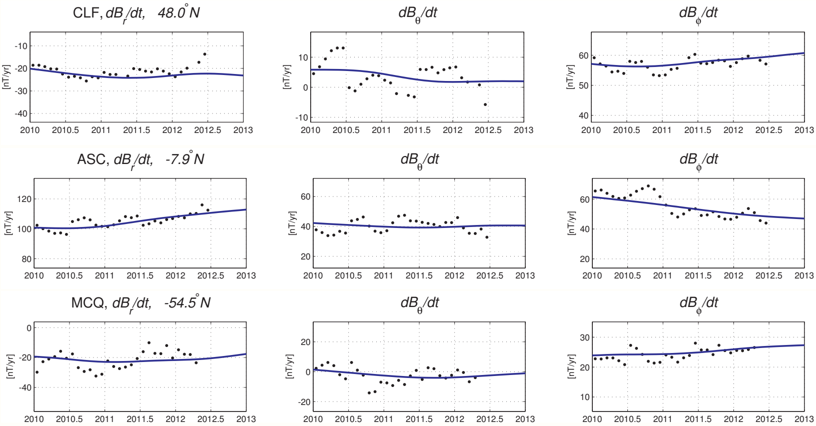

Fig 6: Fit of time-dep model to revised observatory monthly means. Components are plotted in dipole co-ordinates.

Residuals - histogram

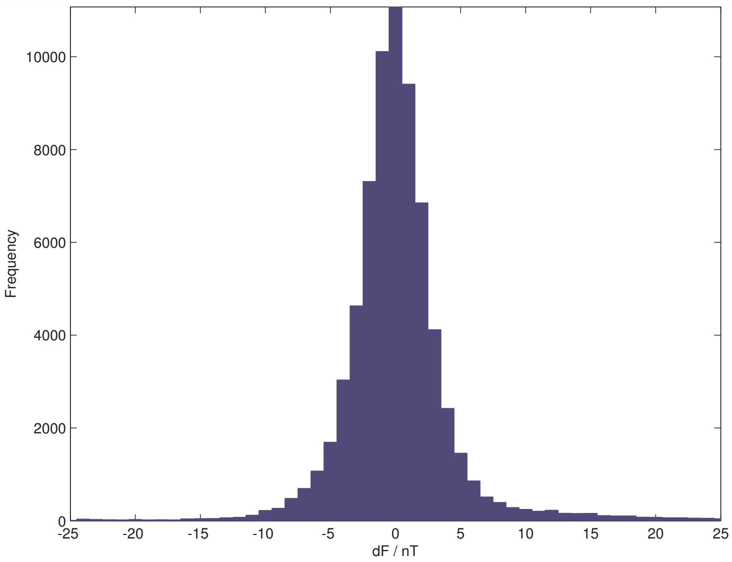

Fig 7:Histogram of residuals.

Residuals - geographical distribution

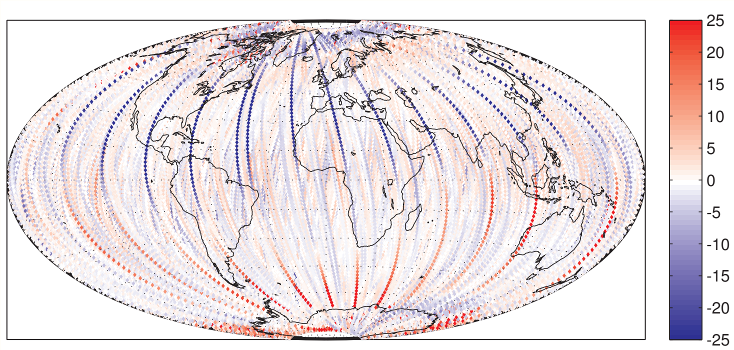

Fig 8:Residuals as a function of geographical location.

Residuals vs time

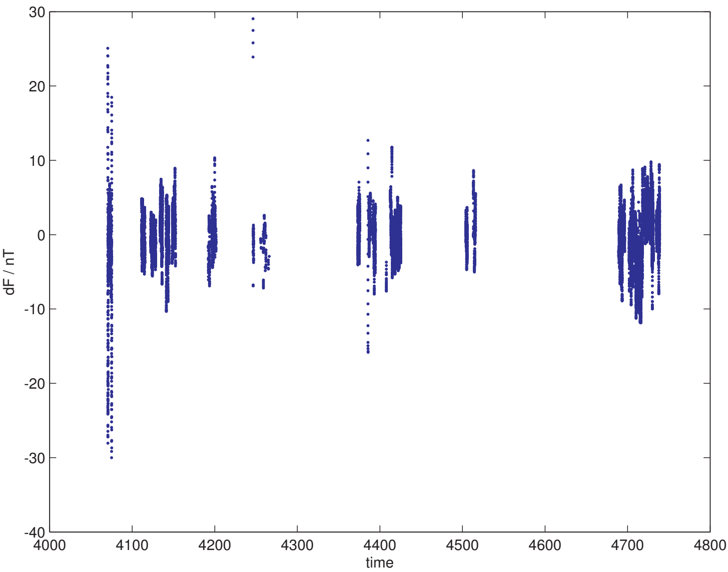

Fig 9:OER F residuals vs time (mjd).

Summary

- Ørsted scalar data are available 2011-2013
- Now online at
  ftp://ftp.space.dtu.dk/data/magnetic-satellites/Oersted/mag-f/
- RMS residual at non-polar latitudes 3.59 nT, Huber-weighted RMS resdiual at non-polar latitudes 2.64nT.
- Together with observatory data, Ørsted data can be used to produce reasonable regularized field models during the gap between CHAMP and Swarm.

Can we better quantify uncertainties in the observed field during the satellite era?

Probabilistic (Bayesian) approach to field modelling

- Recall the geomagnetic forward problem: $\mathbf{d} = f(\mathbf{m}) + \mathbf{e}$
  Statistics of $\mathbf{e}$ described by $\mathbf{C_e}$. Prior knowledge on $\mathbf{m}$ by $\mathbf{C_m}$.

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$$\Theta = [\mathbf{d} - \mathbf{f}(\mathbf{m})]^T \mathbf{C}_e^{-1} [\mathbf{d} - \mathbf{f}(\mathbf{m})] + \mathbf{m}^T \mathbf{C}_m^{-1} \mathbf{m}$$

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- Do this using an iterative Newton-type algorithm

$$\mathbf{m_{i+1}} = \mathbf{m_i} + \mathbf{C} \left\{ \nabla f(\mathbf{m_i}) \mathbf{C}_e^{-1} \left[\mathbf{d} - \mathbf{f}(\mathbf{m_i}) \right] - \mathbf{C}_m^{-1} \mathbf{m_i} \right\}$$

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- **Sample posterior pdf** (defined by both $\bar{\mathbf{m}}$ and $\mathbf{C}$) to generate **an ensemble of models** characterising the solution.
- When no obs, ensemble has statistics specified by prior $\mathbf{C_m}$.

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- Our prior on correlation: $\rho_n(\tau)$ is that of an AR(2) process:

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- Algorithm familiar except $\mathbf{C_m}$ **is dense** and **no damping parameter**.

Spectrum of centennial to decadal time scales

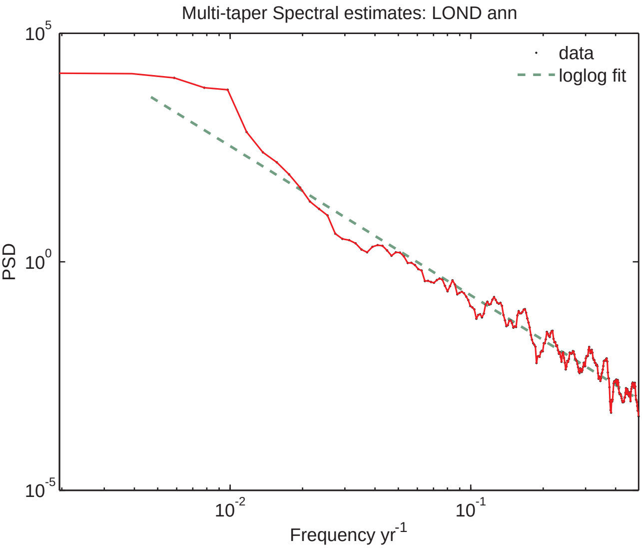

Fig 10: Spectrum of spot measurements or annual means of D near London 1570-2012. Best fitting slope 200yrs-2 yrs: -3.27.

Spectrum of decadal to sub-annual time scales

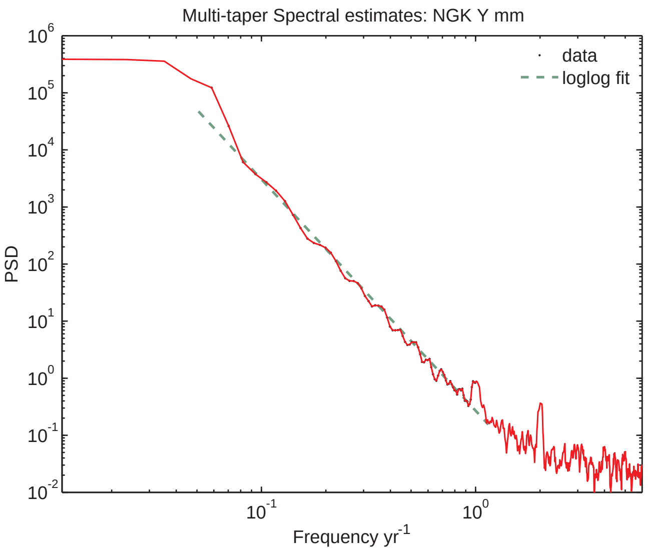

Fig 11: Spectrum of monthly means of Y from NGK 1932-2005.
Best fitting slope 20yrs-1yr: -4.05.

Spectrum of sub-decadal to sub-annual time scales

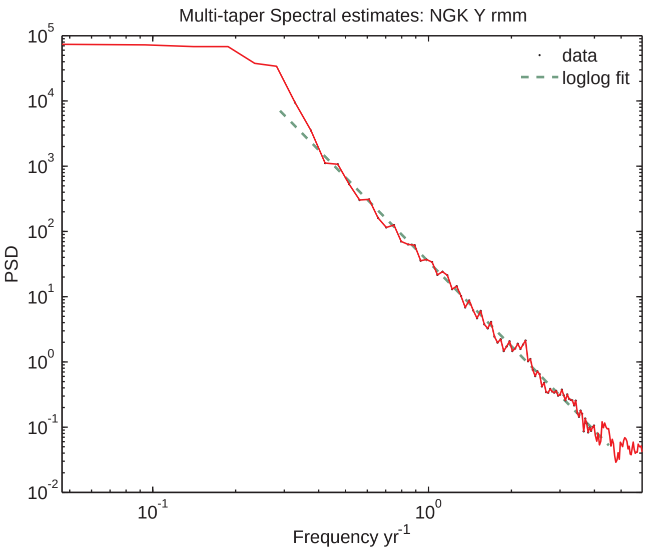

Fig 12: Spectrum of revised monthly means of Y from NGK 1997-2011.
Best fitting slope 3yrs-0.25yrs: -4.30.

Spectrum of sub-decadal to sub-annual time scales

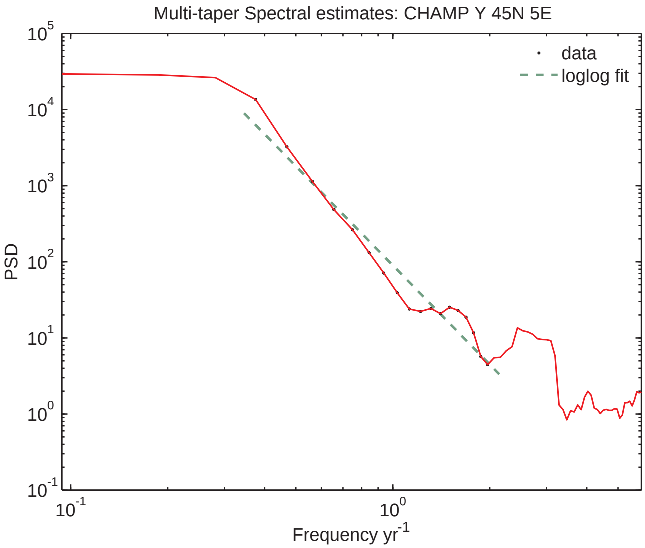

Fig 13: Spectrum monthly means of Y at virtual obs from CHAMP data at 400km, 2000.6-2010.3. Best fitting slope 3yrs-0.5yrs: -4.33.

Spectrum of external field (Dst) variations

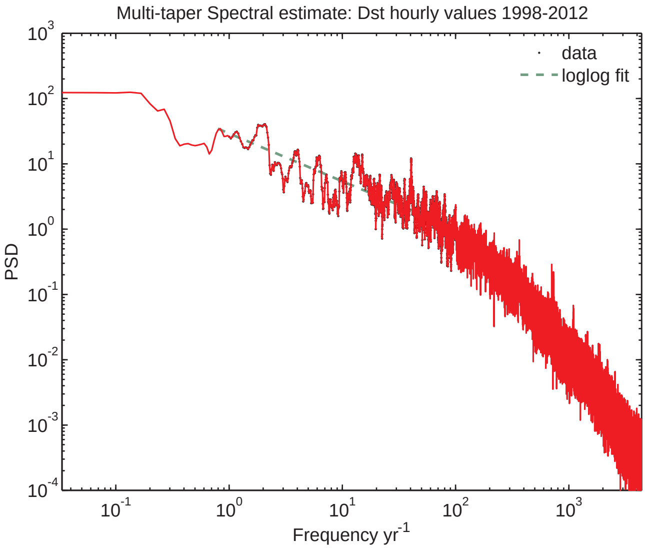

Fig 14: Spectrum of Dst hourly means between 1998 and 2012.
Best fitting slope 1yr - 0.01yrs: -0.73.

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- So is SV just all rotating convection?
- What spectral slope would QG or magnetostrophic models predict? Do MC waves or TO change the slope?

Convection-driven dynamo composite spectrum

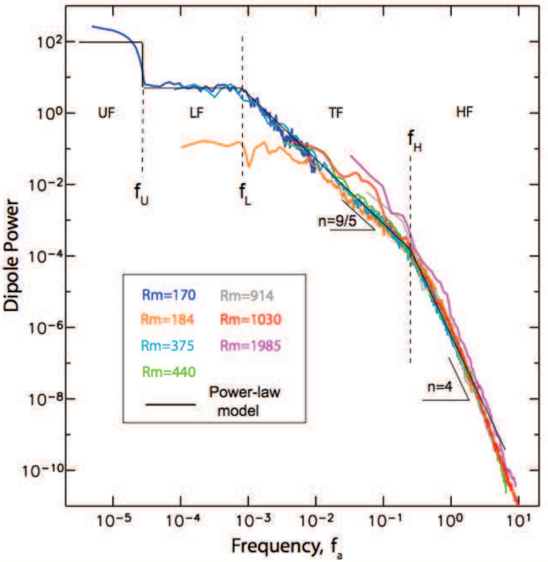


Fig 15: Spectrum from dynamo simulations by Olson et al., 2012.

A first application to the satellite era: 2007-2013

- Using the same approach as the COV-OBS model of Gillet et al., 2013.
- But co-estimating external and high degree static field.
- Cubic splines to degree 14, 0.5 year knot spacing.
- Static internal field to degree 60. External field to degree 2.
- No regularization: using a-prior model covariance matrix.
- Based on Matérn function of order 3/2 (i.e. AR(2): the -4 slope.
- Variance and time scales given by previous satellite model.

Ensemble fit to observatory monthly means

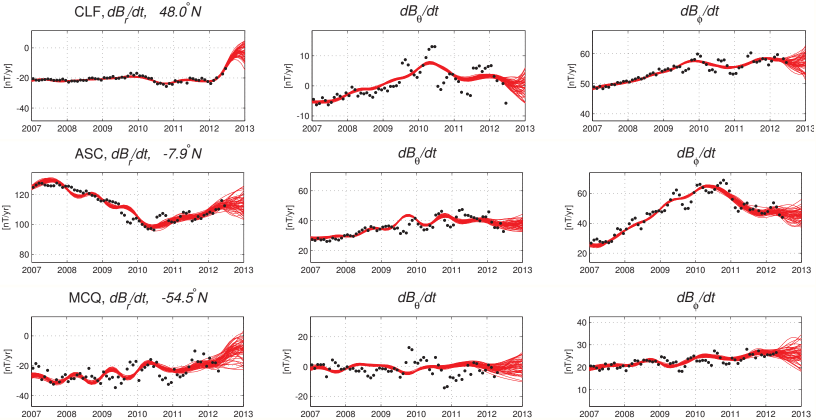


Fig 16: Fit of time-dep model to revised observatory monthly means. Components are plotted in dipole co-ordinates.

Ensemble SV for selected coefficients

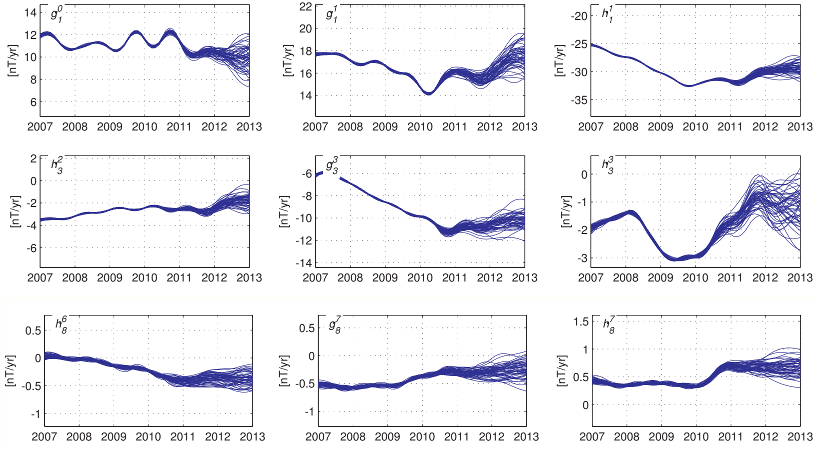
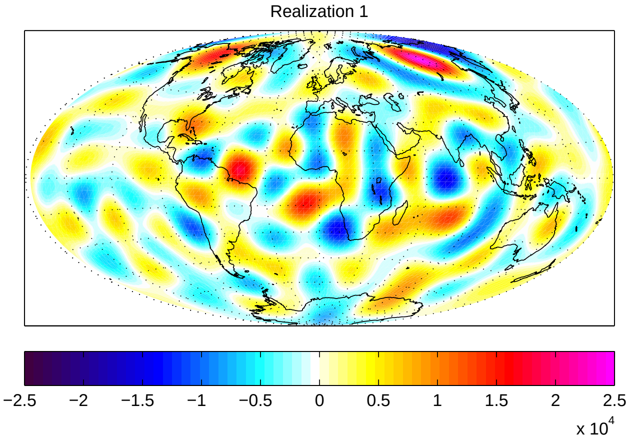


Fig 17: SV for selected Gauss coefficients

Realizations of core surface radial SV in 2012.9



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- How to couple this to core dynamics models to assimilation of "real" satellite data?

SV and SA spectrum at core surface in 2011: 02i

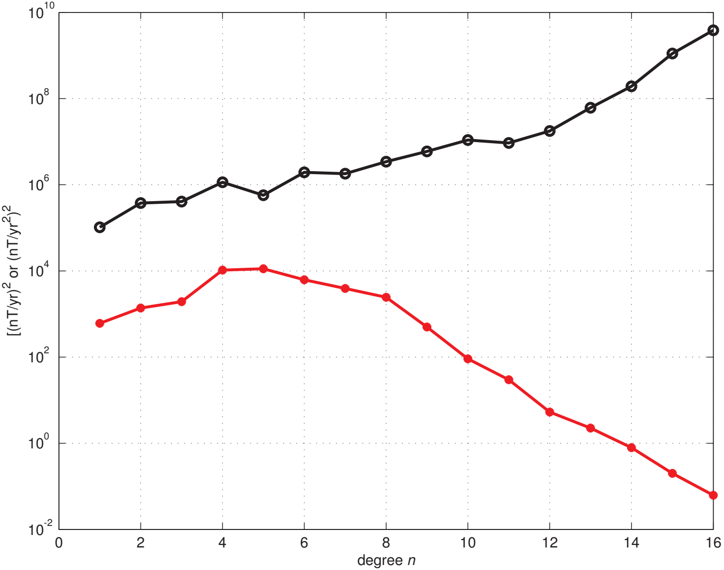

Fig :SV and SA spectrum for 2011.0: conventional model.

SV and SA spectrum at core surface in 2011:02j

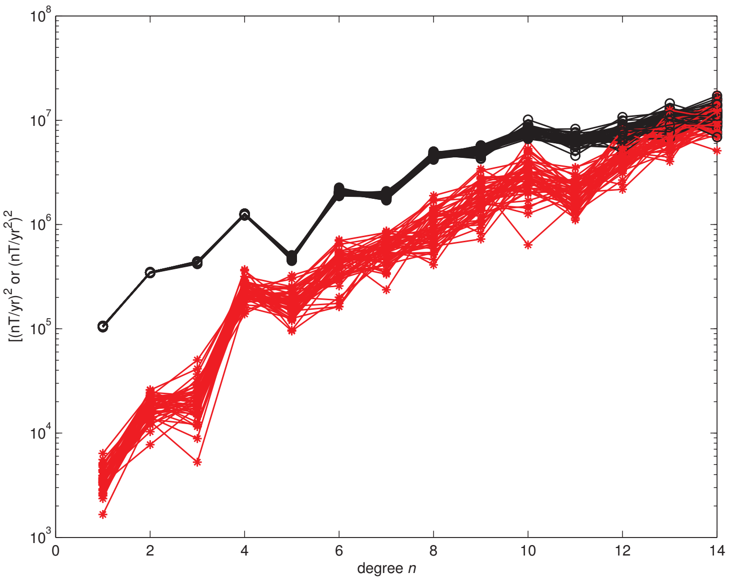

Fig :SV and SA spectrum for 2011.0: ensemble mean model.

Fit of ensemble of model to obsy annual means

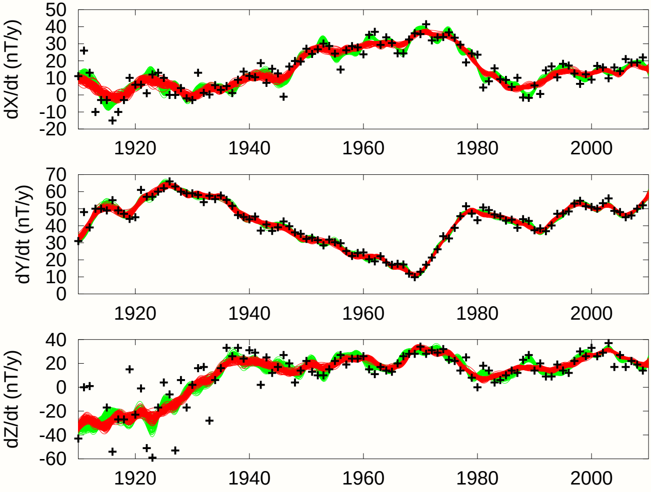

Fig : Fit of ensemble of COV-OBS field models to observatory annual means from Eskdalemuir (UK). Red are internal field models only, green includes ext. dipole.

Secular variation of axial dipole

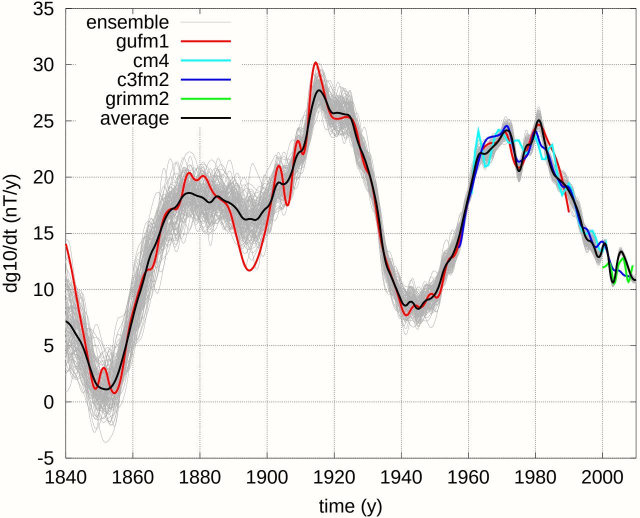

Fig : Evolution of axial dipole $g_1^0(t)$ in COV-OBS models.

Secular variation of higher sectorial coefficient

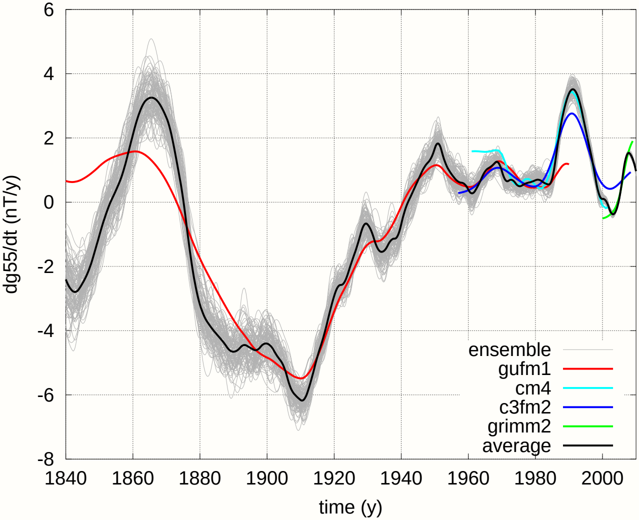

Fig:: Time evolution of the $g_5^5(t)$ sectoral coefficient in COV-OBS models.

Realizations of core surface field in 1920

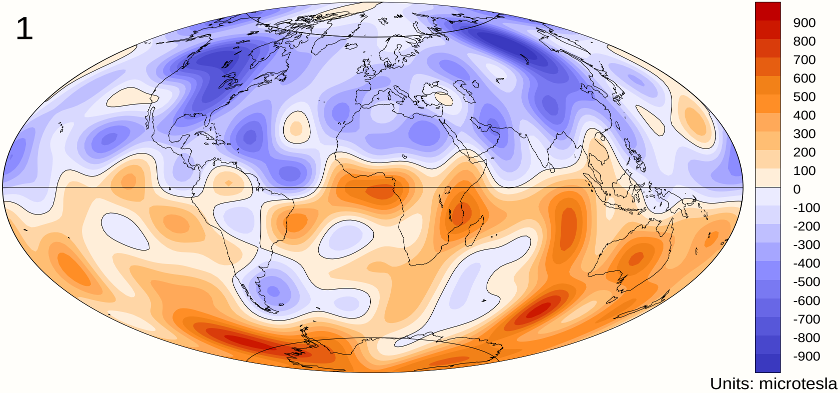

Fig: B_r at core surface in 1920.0 from the *COV-OBS model : units* μT

- Some features are persistently present, others not.

Model covariance matrix at one epoch

- Solution characterized not only by $\bar{\mathbf{m}}$ but also by $\mathbf{C}$
- $\mathbf{C}$ encapsulates model uncertainties and their correlations btw coeff.

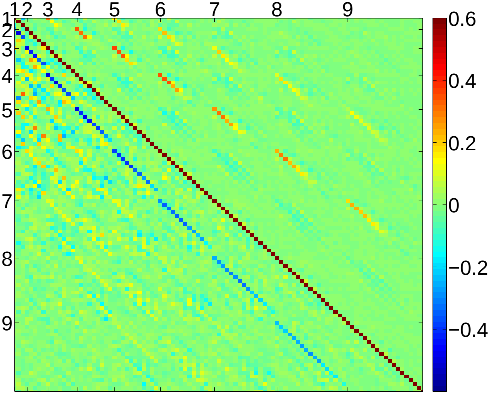

Fig : Model covariance matrix in 1925 (bottom) and 2005 (top) from COV-OBS model.