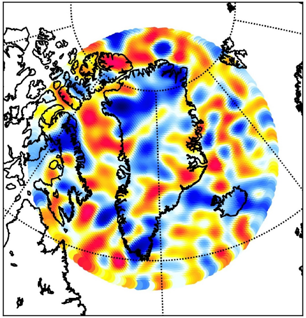
Regional modelling of the lithospheric magnetic field from satellite gradient data: A regularized Slepian function approach



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European Research Council
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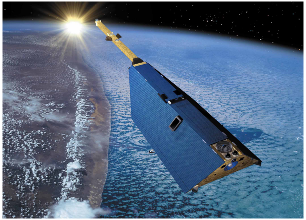
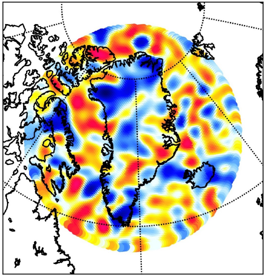
Motivation: Global vs regional lithospheric field models

- CHAMP: low altitude ~ 300 km, vector data, from solar minimum
- *Swarm*: 5 yrs, data from higher altitudes 440-510 km
- Magnetic anomalies can well determined from gradient estimates



How can we best exploit this data?

- High resolution maps, stable at Earth's surface
- Global maps convenient, but regional approach has advantages?



- (Revised) spherical cap analysis
  [e.g. Thébault, 2006; Thébault et al., 2013, 2016]
- Slepian functions on a sphere
  [Simons et al., 2006; Plattner and Simons, 2017]
  [Kim and Von Frese, 2017]

This study:

Satellite gradient data, altitude-cognizant gradient vector Slepian functions, L1 regularization of B_r at surface

Forward scheme: Slepian basis functions I

- Following *Plattner and Simons (2017)*, in potential field framework and considering only internal sources

$$\mathbf{B}(\boldsymbol{r}) = \nabla V(\boldsymbol{r}) \qquad \text{and} \qquad \nabla V(\boldsymbol{r}) = \sum_{l=1}^{L} \sum_{m=-l}^{l} \mathrm{g}_l^m \, A_l(r) \boldsymbol{E}_{lm}(\hat{\mathbf{r}})$$

$$\text{where} \qquad \boldsymbol{E}_{lm}(\hat{\boldsymbol{r}}) = \frac{1}{\sqrt{(l+1)(2l+1)}} \left[\hat{\boldsymbol{r}}(l+1) Y_{lm}(\hat{\boldsymbol{r}}) - \nabla_1 Y_{lm}(\hat{\boldsymbol{r}}) \right] \qquad \text{for } 0 \le l \le L$$

$$\text{and} \qquad A_l(r) = -a^{-1} \sqrt{(l+1)(2l+1)} \left(\frac{r}{a} \right)^{-(l+2)}, \qquad \text{for } 0 \le l \le L$$

- To find optimal basis functions with energy is concentrated a sub-region R, define

$$\mathbf{K} = \int_R \boldsymbol{\mathcal{E}}_L \cdot \boldsymbol{\mathcal{E}}_L^T \, \mathrm{d}\Omega, \tag{1}$$

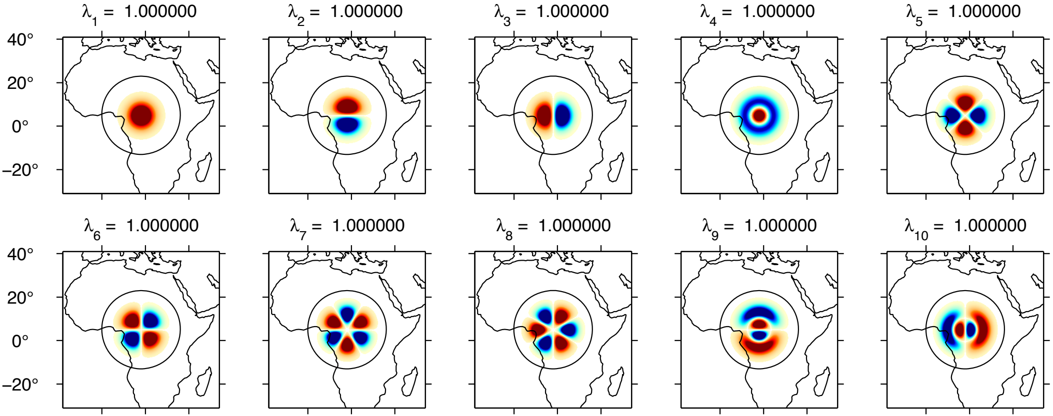
  where $\boldsymbol{\mathcal{E}}_L$ are column vectors collecting $\boldsymbol{E}_{lm}(\hat{\boldsymbol{r}})$.

- Then performing eigenanalysis and retaining only the J largest eigenvalues one can write

$$\mathbf{K}\mathbf{G}_J = \mathbf{G}_J \boldsymbol{\Lambda}_J, \quad \text{with} \quad \mathbf{G}_J = (\mathrm{g}_1, \ldots, \mathrm{g}_\alpha, \ldots, \mathrm{g}_J) \quad \text{with } 1 \le J \le (L+1)^2 \tag{2}$$

  and the vectors g_α define the Slepian basis functions.

Forward scheme: Slepian basis functions II



[From Simons et al., 2009: 10 largest bandlimited eigenfunctions optimally concentrated within a circularly symmetric domain on sphere]

Forward scheme: Slepian basis functions III

- Altitude-dependence can then be re-introduced via the vector $\mathbf{A}(r)$ that involves the functions $A_l(r)$

$$\boldsymbol{\mathcal{G}}_{\uparrow J} = \mathbf{G}_J^T \mathbf{A}(r) \boldsymbol{\mathcal{E}}_L, \qquad \text{with } 1 \leq J \leq (L+1)^2. \tag{3}$$

- Forward model relating vector field components at satellite altitude to the Slepian model coefficients is then

$$\widehat{\mathbf{d}^{sat}} = \mathbf{G}_J^{sat}\, \mathbf{m}_J. \tag{4}$$

  where $\mathbf{G}_J^{sat}$ involves rows of $\boldsymbol{\mathcal{G}}_{\uparrow J}$ evaluated using the relevant data locations and components

- Approximate satellite data gradients can also be included by via suitable differences of (4)

- For the results present here we used $L = 200$, cap half-widths of 15 to 25 degrees and J=1500 to 2500

- Numerical implementation:
  Python, based on/tested against SLEPIAN hotel Matlab routines [Plattner and Simons, 2017]

Inversion: L_1-norm regularization

- We wish to estimate Slepian coefficients at Earth's surface. Such downward continuation is unstable.
- Here we choose a truncation degree J high enough that the majority of the eigenspectrum is captured, discarding only very small eigenvalues.
- But regularize, also seeking models minimizing the L_1 norm of B_r the Earth's surface

$$\Phi = \left(\mathbf{d}^{sat} - \mathbf{G}^{sat}\mathbf{m}_J\right)^T \mathbf{W}_h \left(\mathrm{d}^{sat} - \mathbf{G}_J^{sat}\mathbf{m}_J\right) + \alpha^2 \|\mathbf{R}\mathbf{m}_J\|_1 \tag{5}$$

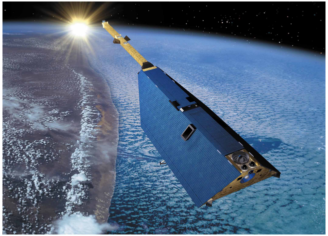
where $\mathbf{W}_h$ is a weighting matrix, with QD latitude dependent data error estimates, and iteratively updated Huber weights and $\mathbf{R}$ evaluates B_r on an equal area grid at Earth's surface from $\mathbf{m}_J$

- Implemented via an IRLS scheme [Farquharson and Oldenburg, 1998] involving $\mathbf{W}_m$

$$\mathbf{m}_{J,k+1} = \left(\left(\mathbf{G}_J^{sat}\right)^T \mathbf{W}_{h,k}\mathbf{G}_J^{sat} + \alpha^2 \mathbf{R}^T \mathbf{W}_{m,k}\mathbf{R}\right)^{-1} \mathbf{G}_J^{sat}\mathbf{W}_{h,k}\mathbf{d}^{sat} \tag{6}$$

- Tested with synthetic data, with and without noise added, using LS, L_2 and L_1 regularizations

CHAMP and Swarm satellite data



- Extended version of LCS-1 dataset [Olsen et al., 2017]
- Geomagnetic quiet conditions
- Prediction of CHAOS-6-x2 internal field model to degree 14 and magnetospheric model removed

- **CHAMP** vector field along-track gradients, 2006 to 2010.
- Altitudes below 350km, solar minimum conditions

- **Swarm** vector field along & cross-track gradients, 2013 to 2018
- Satellites A, C now at 440 km, longitudinal sep. $\sim$150 km

- Error budget dependent on QD-latitude and field component, based on residuals to CHAOS-6-x2 model

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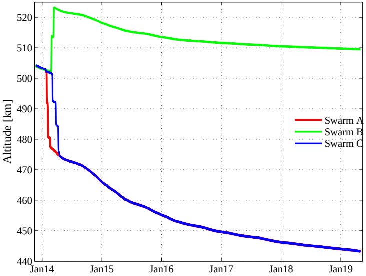
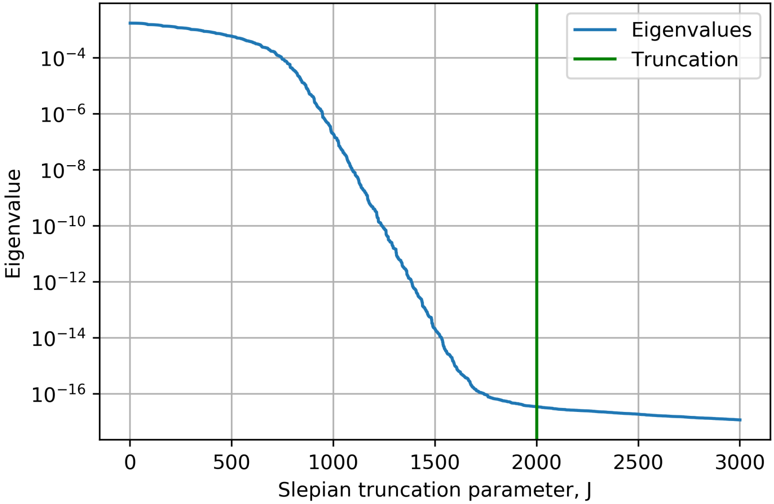


Example 1: West central Africa - Bangui region, Eigenspectrum



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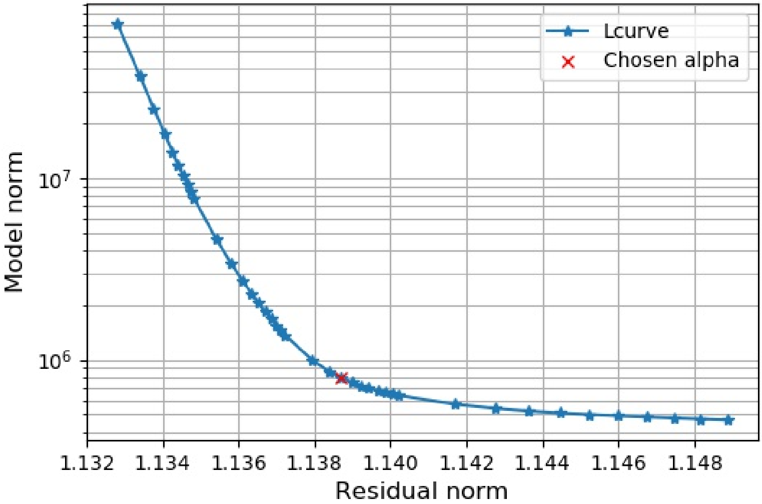

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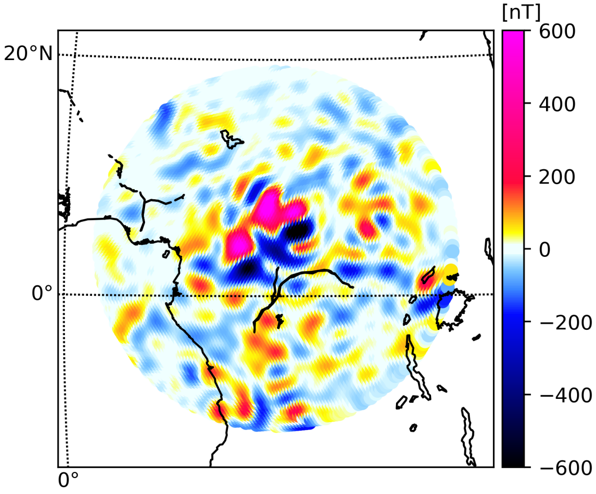

- LCS1 (sat, global) prediction of B_r on equal area grid at Earth's surface.

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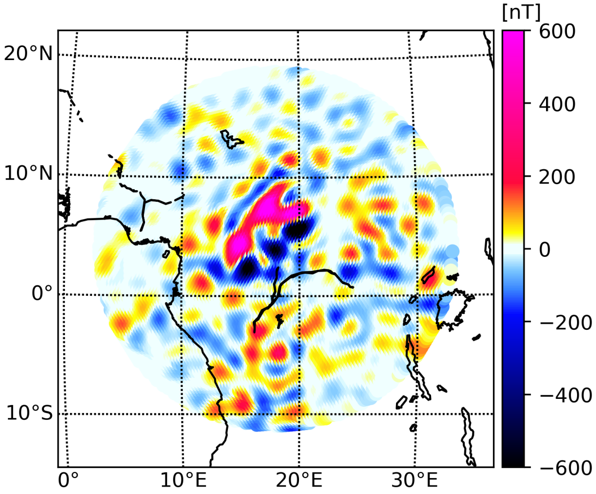
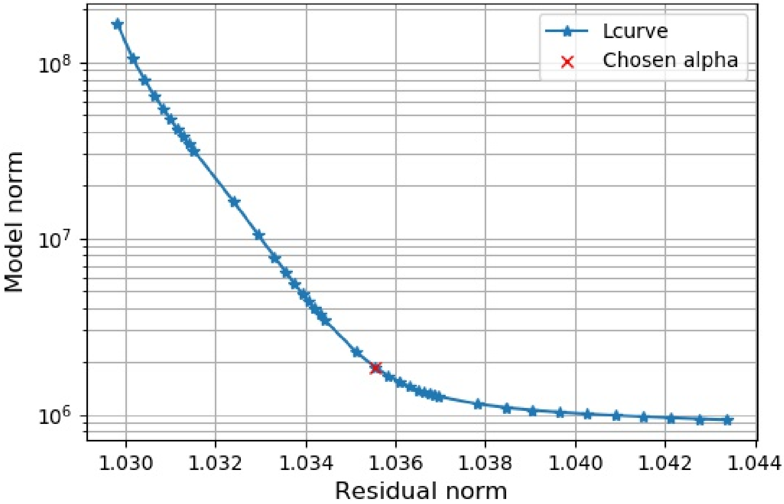


- 15 deg cap, L1 regularization of B_r on equal area grid at Earth's surface.

Example 2: Greenland, L-curve



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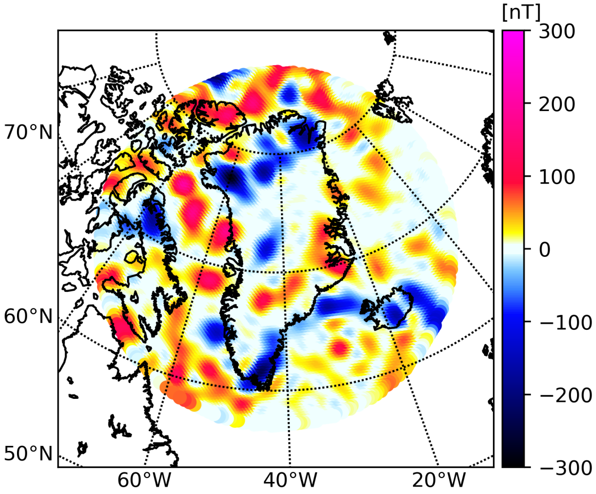

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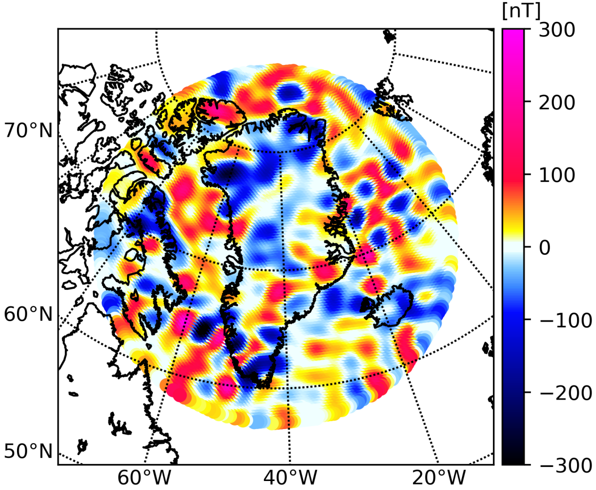

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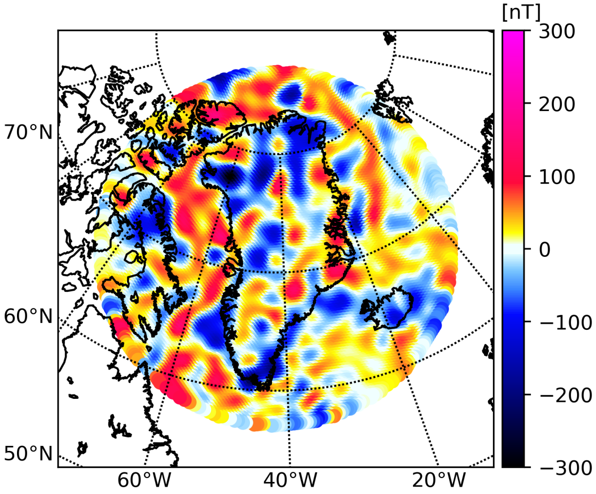

- EMM2015 prediction (sat + aero) of B_r on equal area grid at Earth's surface.

Discussion

Advantages

- Can easily handle satellite data with varying altitudes, no extra BC to apply
- **Inversions manageable**: regional datasets and relatively small number of model parameters
- Can fine-tune regularization level appropriate for specific region (i.e. L-curve for each region)

Challenges

- Need to choose a truncation level for the Slepian functions
- Choice of model norm and regularization parameter - more objective prior information?
- Care still needed to avoid edge effects

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- Initial results promising:

  (i) Mid and low latitudes: High amplitude yet stable near strong anomalies, e.g. Bangui
  (ii) Polar latitudes: More realistic structures than in similar global models e.g. Greenland.

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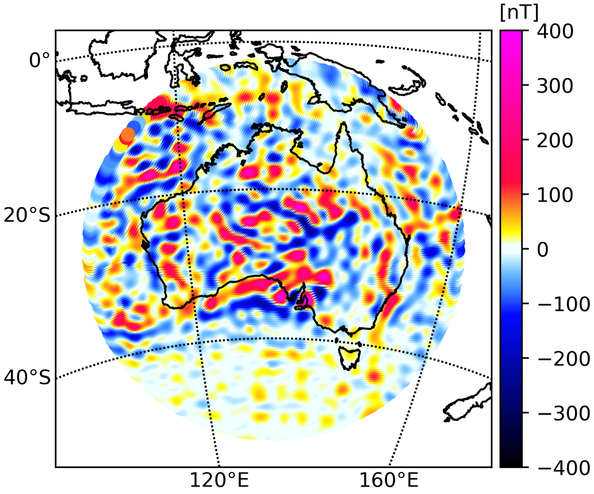
- Straightforward to apply to other regions of geological interest
  e.g. Antarctica, Subduction zones etc.

Another example: Australia



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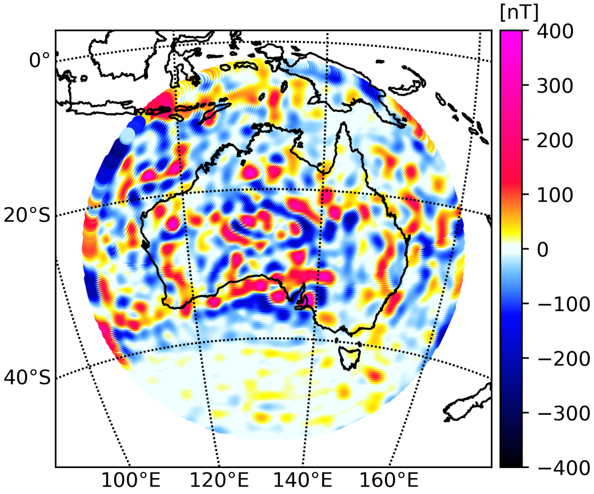

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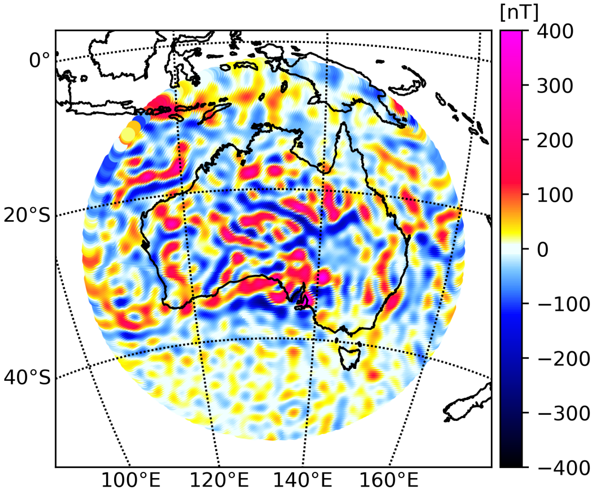

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how many pages to expect for this document.