Use of Satellite Magnetic Field Observations in Data Assimilation Studies of Core Dynamics

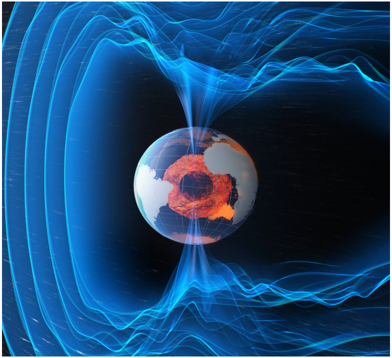
Chris Finlay[1], Nicolas Gillet[2], Magnus Hammer[1], and Olivier Barrois[2],

1: DTU Space, Technical University of Denmark

2: ISTerre, Université Grenoble 1, CNRS 1381, France

Introduction: The Earth's magnetic field

- Fundamental, but unseen, aspect of our planetary environment
- Produced primarily by dynamo in Earth's core
- Mediator between Earth and the solar wind
- Not steady; continuously evolving



[Image credit: ESA]

Outstanding scientific questions

- Structure of flow generating the field, and driving its evolution?
- Dynamical processes responsible for sub-decadal changes?

Use of magnetic observations to probe core dynamics

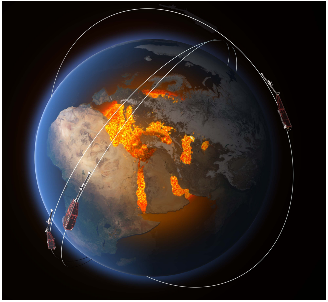
- Observed magnetic field changes reflect flow within the core:

$$\underbrace{\frac{\partial \mathbf{B}}{\partial t}}_{\text{Geomagnetic secular variation}} = \underbrace{\nabla \times (\mathbf{u} \times \mathbf{B})}_{\text{Advection \& stretching by core flow}} + \underbrace{\eta \nabla^2 \mathbf{B}}_{\text{Ohmic diffusion}}$$

- In **data assimilation**, we wish to combine physics-based forward models of the core flow and observations.

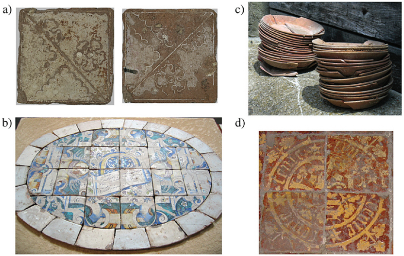
- Ideally using observations with uniform global coverage, spanning many decades

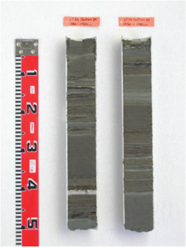
- But optimal combination of data and models requires realistic observation error covariances



[Image credit: ESA]

Field Observations I: Indirect records over long time scales





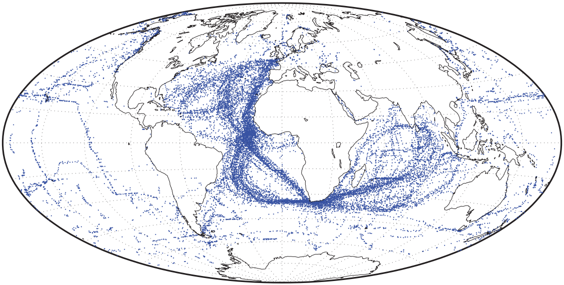
- Magnetization acquired by rocks during formation and artifacts during production records direction and intensity of the ancient field.

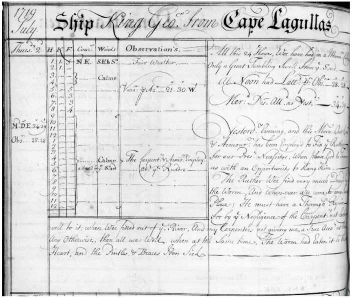
- Data is sparsely distributed in space (mostly from Europe and North America) and time [Constable, 2007; Hulot et al., 2010]

- Large dating errors are often large

- Suitable for monitoring only the low evolution of the largest-scale field e.g.dipole, perhaps quadrapole

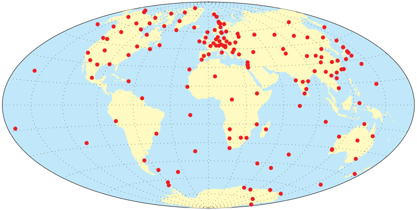
Field Observations II: Historical Data





- Mariners systematically recorded magnetic declination for navigational purposes
- Several hundred thousand data spanning 1500-1900 available
  [Jackson et al., 2000; Jonkers et al., 2003]
- Predominantly along trading routes, and in oceanic regions
- No field intensity data before 1840
- Suitable for studying decadal and centennial variations of large scale field

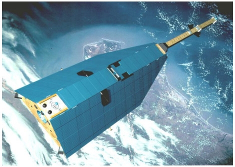
Field Observations III: Ground Observatories

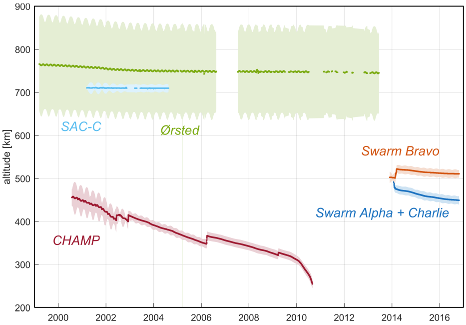


- Modern global network now consists of almost 200 geomagnetic observatories [Matzka et al., 2010; Chulliat et al., 2017]
- Measure full vector field with absolute accuracy typically less than 5 nT
- Little data from large oceanic regions (except for islands)
- Long times series are 'gold standard' for studying field change over past 100 yrs
- Vital for selection, calibration and validation of satellite data

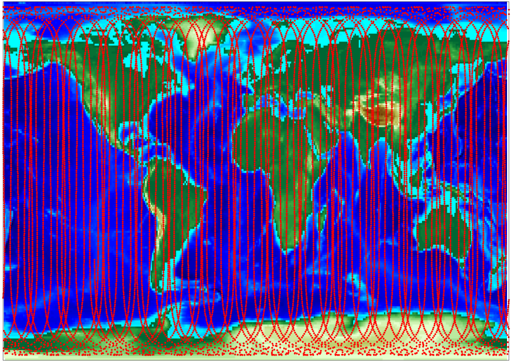
Field Observations IV: Satellite data

- Since 1999, we now almost continuous monitoring of the geomagnetic field from space [e.g. Olsen and Stolle, 2012]
- Low-Earth, polar orbiting satellites, with altitudes between 250 and 850 km
- Instruments with absolute accuracy $<$ 0.5 nT in intensity, full vector field determined using attitude informations from star trackers
- Require magnetically clean satellites and careful in-flight data calibration





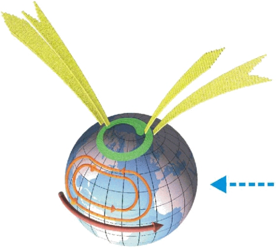
Field Observations IV: Satellite data, regular global coverage



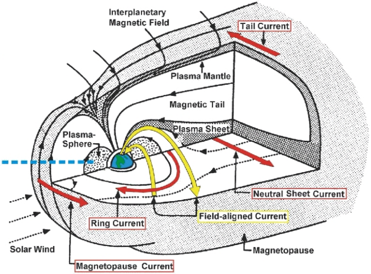
- Global coverage in a few days: above are ground tracks of 3 days from single satellite
- Time series are short - longest span of a single mission is 14 yrs
- Need to combine data from different satellite missions
- Flying through ionosphere, currents there perturb data, especially in polar region
- Possible to use field gradients along-track and across track (with *Swarm* constellation)

High accuracy observations reveal multiple field sources

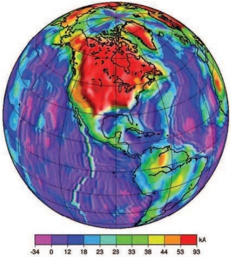
Ionospheric currents



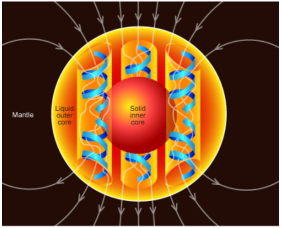
Magnetospheric currents



Lithospheric Magnetization



Geodynamo in the Earth's core



Representation of observations via spherical harmonic model

- Assume measurements made in a current free region, so $\mathbf{B}$ is a potential field

$$\mathbf{B} = -\nabla V \qquad \text{and} \qquad \nabla \cdot \mathbf{B} = 0$$

  Potential is a superposition of field from internal and external sources

$$V = V_{int} + V_{ext}$$

- Internal sources are the internal solution to Laplace's equation

where $$V_{int}(r,\theta,\phi,t) = a \sum_{n=1}^{N} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} \left[g_n^m(t)\cos m\phi + h_n^m(t)\cos m\phi\right] P_n^m(\cos\theta)$$

- With time-dependence accounted for by a B-spline expansion of model coefficients

$$g_n^m(t) = \sum_p g_n^{mp} M_p(t).$$

The COV-OBS field model

- Based on ground observatory annual means 1840-2015 (latest update, COV-OBS-x1, [Gillet et al., 2015])
- And satellite data from Magsat (1980), Ørsted (1999-2013), SAC-C (2001-2004), CHAMP (2000-2010) and *Swarm* (post-2013)
- Model truncated at spherical harmonic degree n=14.
- Model parameters determined by minimizing a cost function: data misfit norm & norm based on a-priori estimates of model covariances,

$$\Theta = [\mathbf{d} - \mathbf{f}(\mathbf{m})]^T \mathbf{C}_e^{-1} [\mathbf{d} - \mathbf{f}(\mathbf{m})] + \mathbf{m}^T \mathbf{C}_m^{-1} \mathbf{m}$$

- Assume a-priori **zero mean, stationary, random process**, no spatial cross-covariances, identical variances for coefficients with same degree n.

$$C_n(\tau) = \sigma_n^2 \, \rho_n(\tau)$$

- Temporal prior correlation $\rho_n(\tau)$ is assumed to follow an AR(2) process

$$\rho_n(\tau) = \left[1 + \sqrt{3} \frac{|\tau|}{\tau_c} \right] exp \left(- \frac{\sqrt{3}|\tau|}{\tau_c} \right)$$

- This allows discontinuities in d^2B/dt^2 (i.e. 'jerks') & spectral slope f^{-4}

The COV-OBS field model

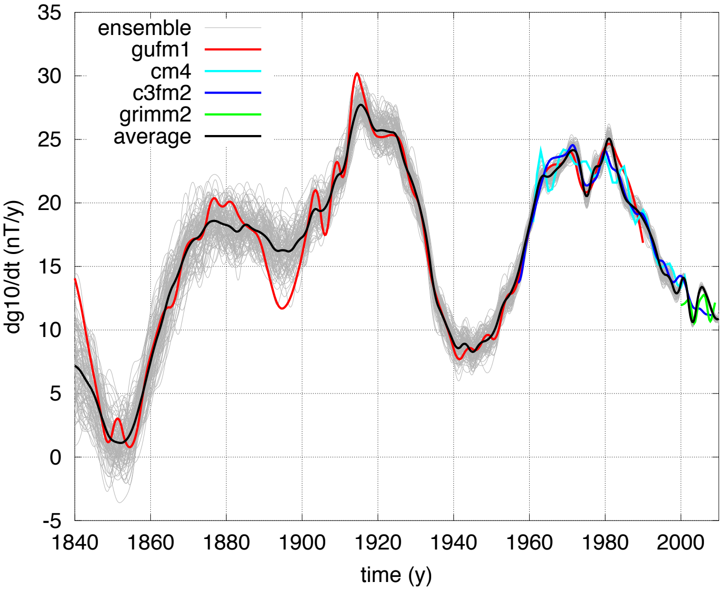
- Directional and scalar intensity observations used, so minimization problem is nonlinear
- Solution obtained using an iterative Newton-type algorithm of the form

$$\mathbf{m_{i+1}} = \mathbf{m_i} + \mathbf{C}\left\{\nabla f(\mathbf{m_i})\mathbf{C}_e^{-1}\left[\mathbf{d} - \mathbf{f}(\mathbf{m_i})\right] - \mathbf{C}_m^{-1}\mathbf{m_i}\right\}$$

$$\text{where} \quad \mathbf{C} = \left[\nabla f(\mathbf{m_i})^T \mathbf{C_e}^{-1} \nabla f(\mathbf{m_i}) + \mathbf{C_m}^{-1}\right]^{-1}$$

- **A probabilistic solution** is obtained using both $\mathbf{m}$ and $\mathbf{C}$, and generating **an ensemble of models** that sample the posterior pdf of the model parameters

COV-OBS model: time dependence of coefficients



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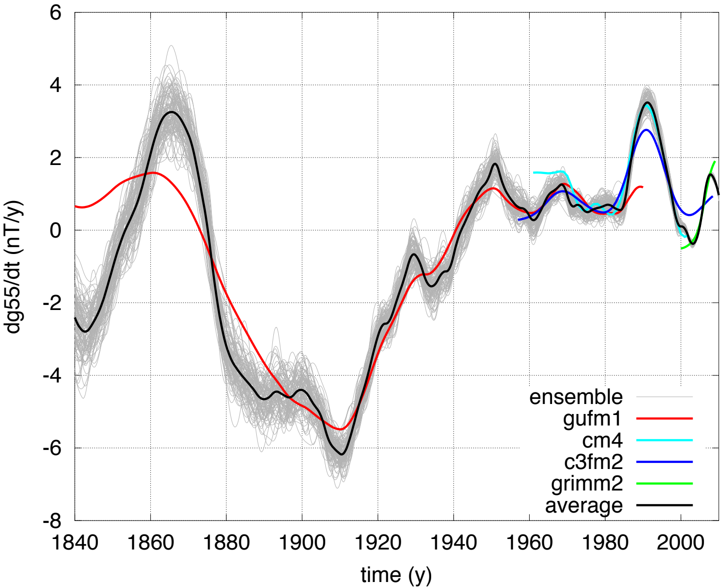
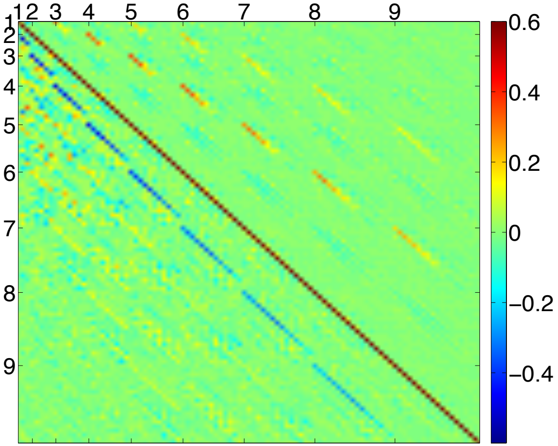


COV-OBS model: model covariance matrix

- Solution characterized not only by $\bar{\mathbf{m}}$ but also by $\mathbf{C}$
- $\mathbf{C}$ encapsulates formal model variances and their cross-covariances btw coeff.
- Example: model correlation matrix in 2005 (top triangle) and 1925 (bottom triangle)



Some remarks

- Spherical harmonic field models are a very efficient way to encapsulate information from a large number of diverse data including ground observatories and satellites
- Allow a formal separation of field into internal and external components
- Resulting SH model coefficients can be considered as 'observations' and input to a data assimilation framework to determine core flow dynamics
(talk of N. Gillet, this session)

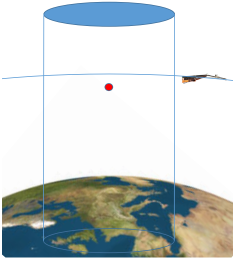
Limitations of data assimilation based on spherical harmonic field models

- Accounting for data covariances is crude; resulting model covariances are too optimistic
- Core dynamics and data are never directly confronted, so prior information from the former cannot be used to help separate out the signal of interest in the data

Is there another way we could use satellite data more directly, without dealing with millions of instantaneous data?

Alternative: Point estimates as 'Virtual observatories'

- **Time series of monthly point estimates at satellite altitude** [Mandea and Olsen, 2006; Olsen and Mandea, 2007; Beggan et al., 2009; Whaler and Beggan, 2015]
- Take all data within cylinder of chosen radius
- Choose selection criteria e.g. only dark, quiet time data
- Remove estimates of crustal, magnetospheric and S_q fields
- Work with sums and differences of data, along and across track
- Threshold for minimum number of data

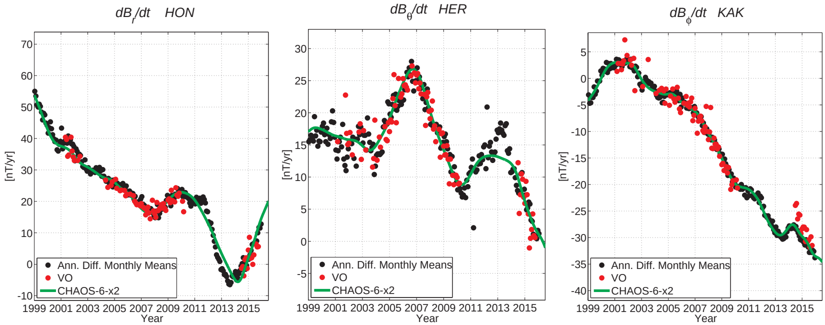


  - Robust (Huber weighted) fit of **local cubic potential** to all data in cylinder

$$\begin{aligned} V(x,y,z) = & v_x x + v_y y + v_z z + v_{xx} x^2 + v_{yy} y^2 - (v_{xx} + v_{yy}) z^2 \\ & + 2v_{xy} xy + 2v_{xz} xz + 2v_{yz} yz - (v_{xyy} + v_{xzz}) x^3 \\ & + 3v_{xxy} x^2 y + 3v_{xxz} x^2 z + 3v_{xyy} xy^2 + 3v_{xzz} xz^2 + 6v_{xyz} xyz \\ & - (v_{xxy} - v_{yzz}) y^3 + 3v_{yyz} y^2 z + 3v_{yzz} yz^2 - (v_{xxz} + v_{yyz}) z^3 \end{aligned} \tag{1}$$

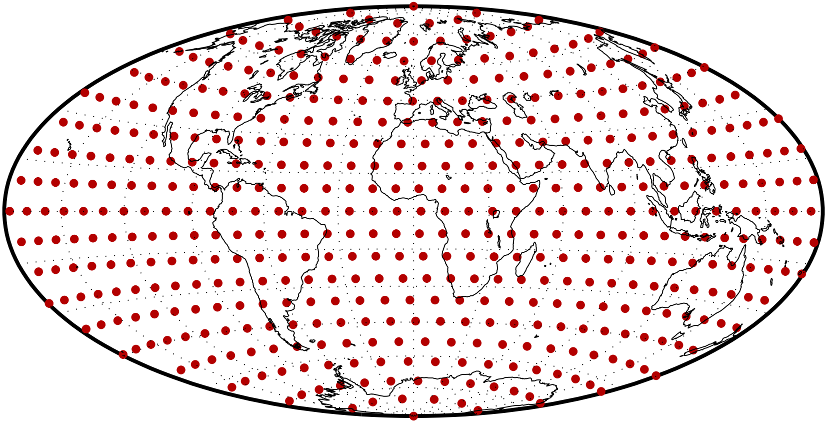
  - Then calculate prediction at chosen reference point using $\mathbf{B} = -\nabla V$

Comparisons with selected ground observatories



A global grid of point estimates

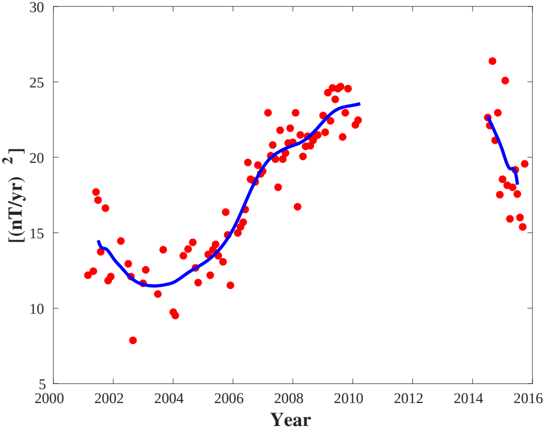
- Approx equal area grid, based on Recursive Zonal Equal Area Sphere Partitioning (Leopardi, 2006)



Derivation of the covariance matrix $\mathrm{C_e}$

- Time series of 3 components (e.g. dB_r/dt,dB_θ/dt,dB_ϕ/dt) at P locations, -> $3P$ series in all, each of length N_T
- Detrend each times series using cubic smoothing spline and Generalized Cross Validation

$$\mathbf{x}_i = \mathbf{d}B_i/dt - \widetilde{\mathbf{d}B_i/dt}$$



Derivation of the covariance matrix $\mathrm{C_e}$

- Place these $3P$ time series into columns of N_T x $3P$ matrix

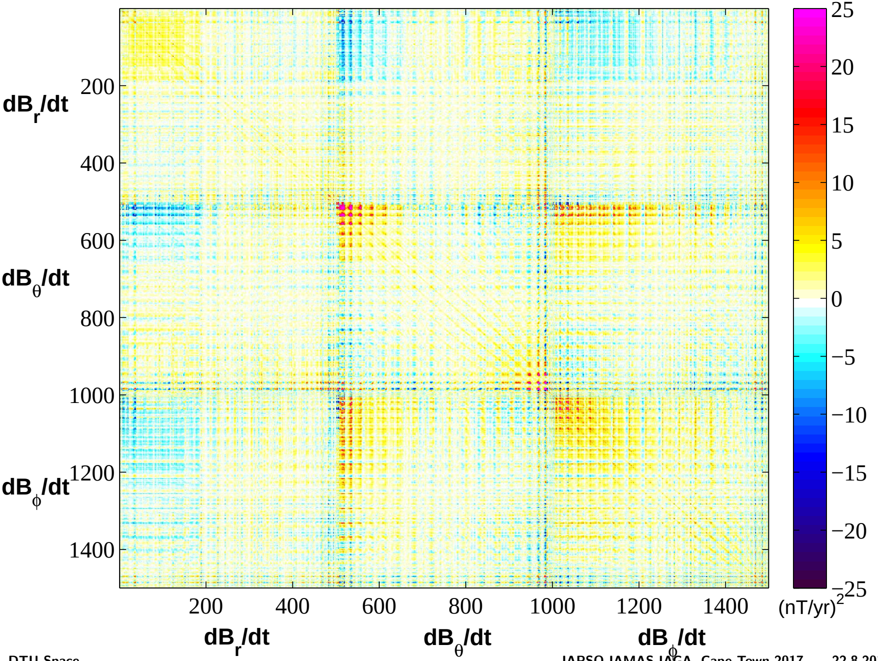
$$\left(\begin{array}{c|c|c|c} \cdot & \cdot & \cdots & \cdot \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_{3P} \\ \cdot & \cdot & \cdots & \cdot \end{array} \right)$$

- Compute covariances between columns of this matrix

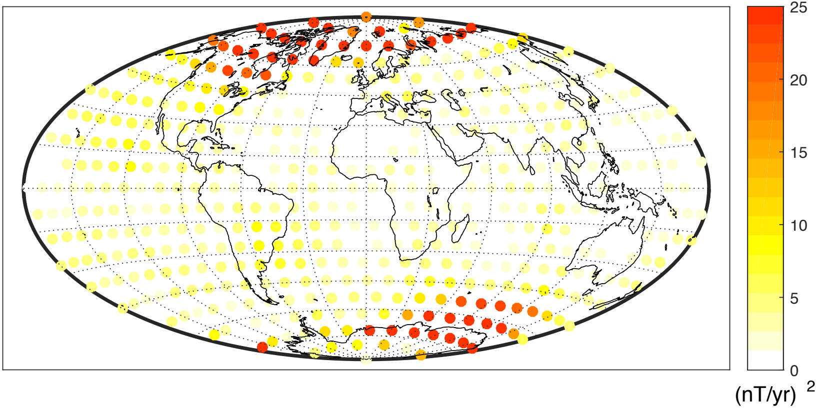
$$\mathbf{C}_e = \mathrm{Cov}(\mathbf{x}_i, \mathbf{x}_j) = \frac{1}{N_T} \sum_{k=1}^{N_T} x_{i,k}\, x_{j,k}$$

- $\mathbf{C}_e$ has size $3P$ x $3P$=1500 x 1500 (manageable in inversions)

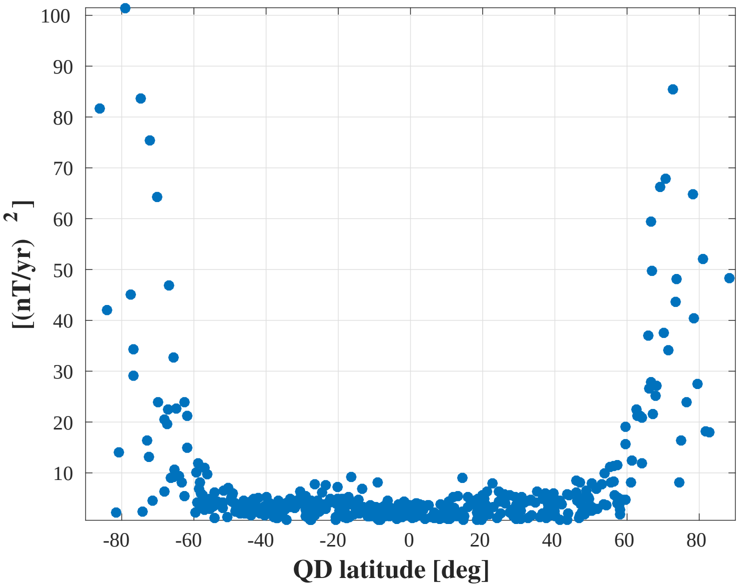
The (full) covariance matrix C_e



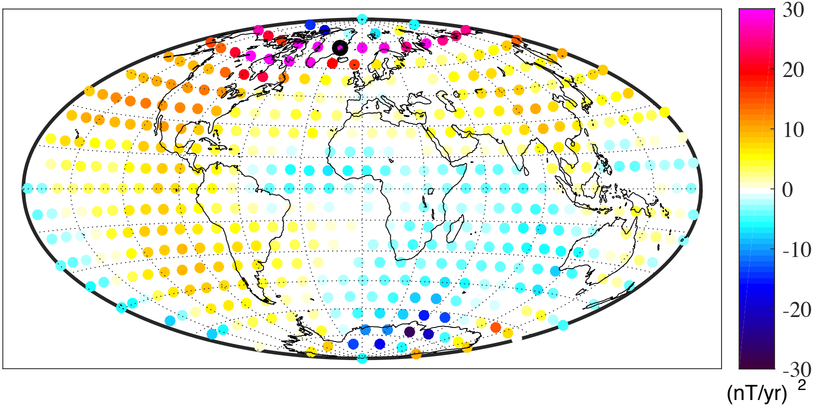
Data error variances for each location: dB_θ/dt



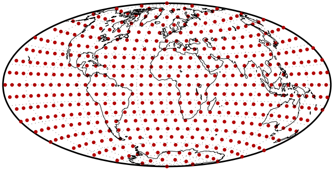
Data error variances for dB_θ/dt, dependence on QD latitude



Spatial covariances: btw dB_θ/dt and dB_θ/dt



Example application: Satellite-based point estimate & dynamo simulation statistics



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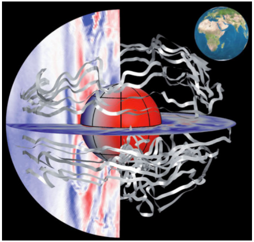


[Image courtesy of J. Aubert]

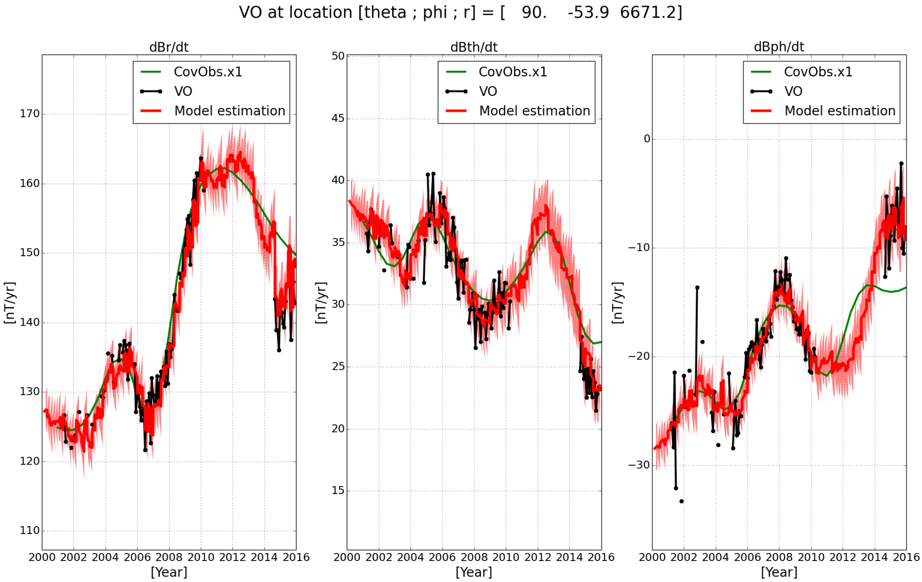
- Data at time t_k, $\mathbf{d}_k = \{B_r; B_\theta; B_\phi; dB_r/dt; dB_\theta/dt; dB_\phi/dt\}$
- Model at time t_k: $\mathbf{m}_k = \{\mathbf{g}_k, \mathbf{u}_k\}$
- Prior model covariances $\mathbf{C_m}$ from CE dynamo model [Aubert et al., 2013]

- Model estimation using a Kalman Filter algorithm [Fournier et al., 2013, Gillet et al., 2015]
- Forecast carried out using a stochastic equation using an ensemble approach => posterior model pdf available

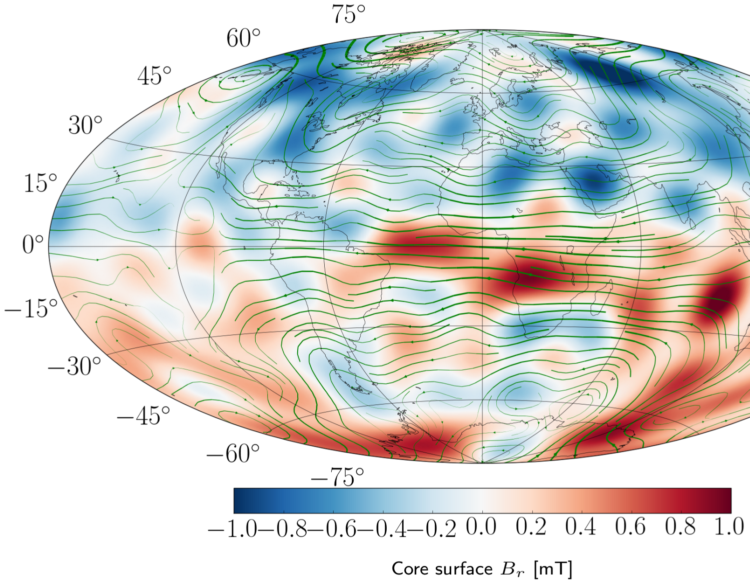
$$\mathbf{m}_{k+1} = \mathbf{A}\mathbf{m}_k + \mathbf{C}_m\mathbf{G}^T\left(\mathbf{G}\mathbf{C}_m\mathbf{G}^T + \mathbf{C}_e\right)^{-1}(\mathbf{d}_{k+1} - \mathbf{G}\mathbf{A}\mathbf{m}_k) \qquad (2)$$

- Prelim. expts: diagonal $\mathbf{C}_e$, frozen model covariances

Example application: Fit to point estimate time series



Example application: Field and flow at core surface in 2008



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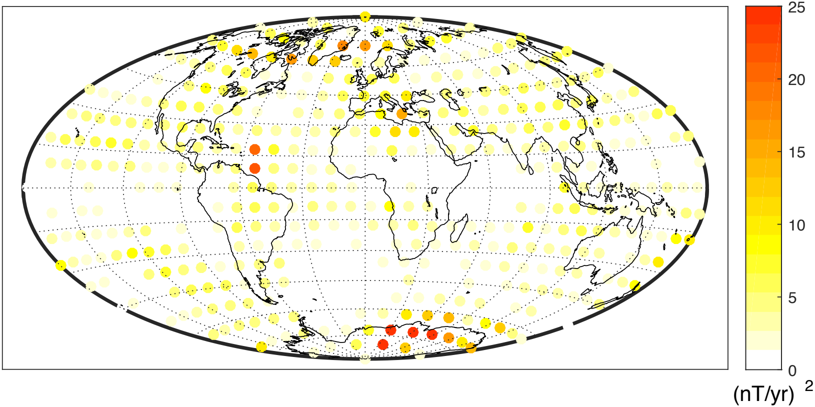
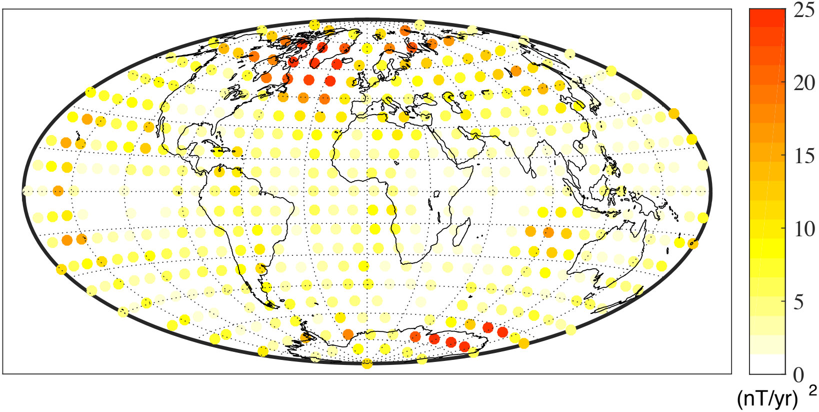
More detailed results and discussion of assimilation algorithm: Gillet et al., this session!

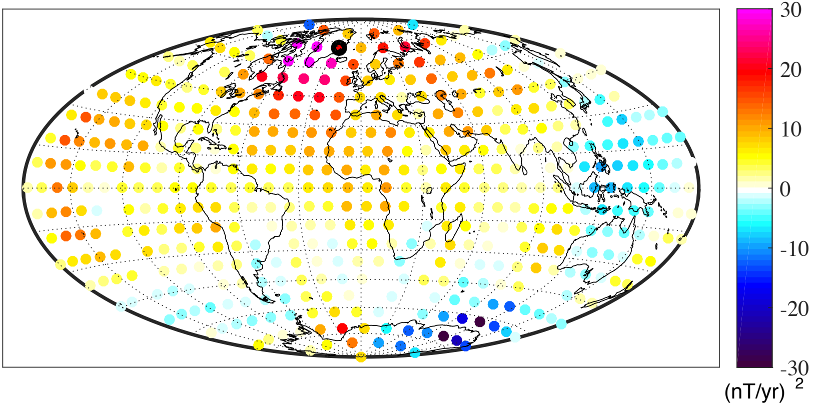
Data error variances for each location: dB_r/dt



Data error variances for each location: dB_ϕ/dt



Spatial covariances: btw dB_θ/dt and dB_ϕ/dt



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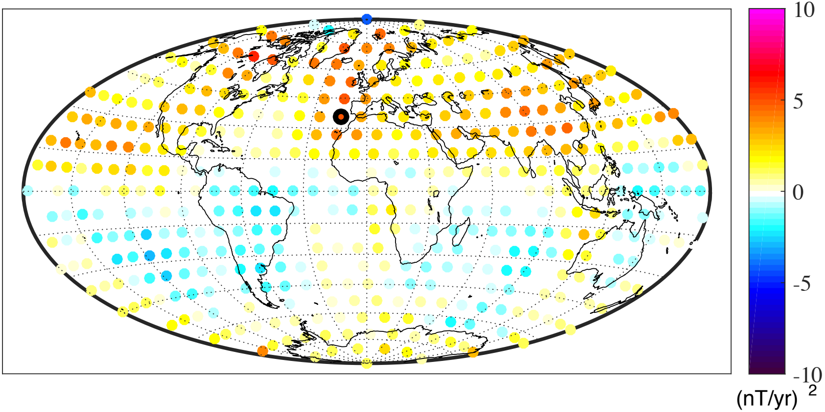

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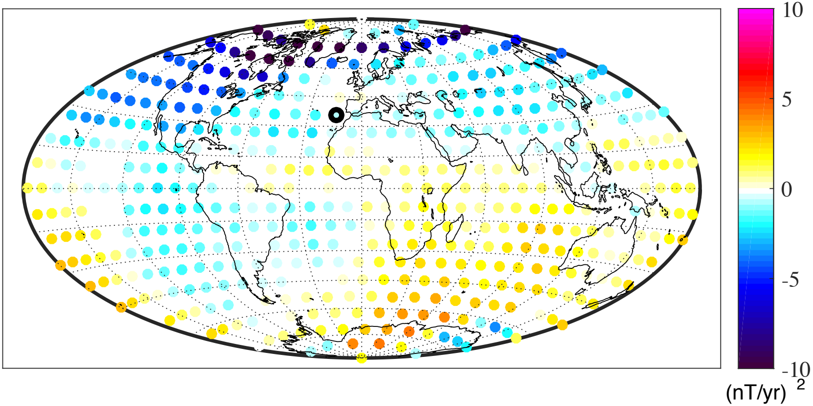

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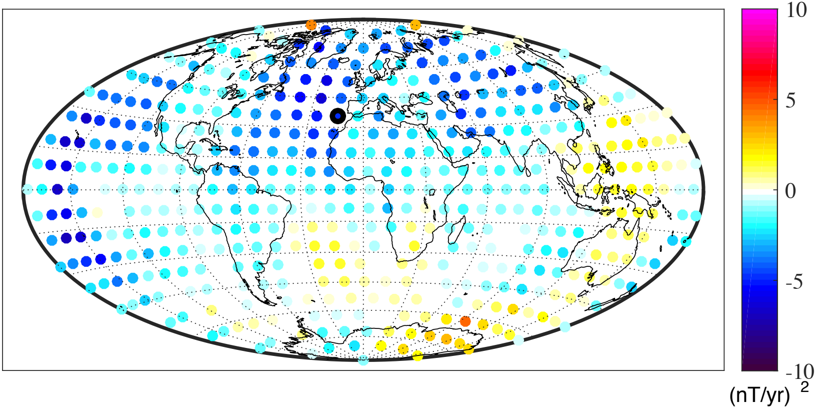

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