Conductivity Distribution of the Earth Derived from Long Period Magnetic Data

Nils Olsen

Neustadt/Weinstrasse, 27th June 2009

DTU Space
National Space Institute
Long Period Transfer Functions
Derived from Magnetic Data

Nils Olsen

Neustadt/Weinstrasse, 27th June 2009

DTU Space
National Space Institute
Outline of Talk

1. Transfer functions
2. Source Fields
3. Transfer functions determinations before 1985
4. Transfer functions determinations 1985 - 1999
5. Latest attempts and new concepts
Outline of Talk

1. Transfer functions
2. Source Fields
3. Transfer functions determinations before 1985
4. Transfer functions determinations 1985 - 1999
5. Latest attempts and new concepts
Response functions for induction studies

Spherical Harmonic domain

\[ Q_n(\omega) = \frac{\iota_n^m(\omega)}{\epsilon_n^m(\omega)} \]

Spatial domain

\[ C(\omega) = \frac{B_z(\omega)}{\nabla_H \cdot B_H(\omega)} \]

\( \epsilon_n^m \) and \( \iota_n^m \) are spherical harmonic expansion coefficients of external (inducing), resp. internal (induced), sources.
Response functions for induction studies

Spherical Harmonic domain

\[ Q_n(\omega) = \frac{\iota_n^m(\omega)}{\epsilon_n^m(\omega)} \]

\[ Q_n = \frac{n}{n + 1} \frac{1 - \frac{n+1}{a} C_n}{1 + \frac{n}{a} C_n} \]

Spatial domain

\[ C(\omega) = \frac{B_z(\omega)}{\nabla_H \cdot \mathbf{B}_H(\omega)} \]

\[ C_n = \frac{a}{n + 1} \frac{1 - \frac{n+1}{n} Q_n}{1 + Q_n} \]

Earth’s radius \( r = a \)

\( \epsilon_n^m \) and \( \iota_n^m \) are spherical harmonic expansion coefficients of external (inducing), resp. internal (induced), sources

These responses are only valid for 1D Earth, \( |\nabla_H \sigma| \ll \sigma/|C| \)
Global vs. regional sounding

- **Spherical Harmonic Domain (global) sounding:**
  Spherical Harmonic analysis of $B_x$, $B_y$, $B_z$ yields $\epsilon_m^n(\omega), \iota_m^n(\omega)$

  $$Q_n(\omega) = \frac{\iota_m^n(\omega)}{\epsilon_m^n(\omega)}$$

- **Spatial domain (regional) sounding:**

  $$C(\omega) = \frac{B_z(\omega)}{\nabla_H \cdot B_H(\omega)}$$
Global vs. regional sounding

- **Spherical Harmonic Domain (global) sounding:**
  Spherical Harmonic analysis of $B_x$, $B_y$, $B_z$ yields $\epsilon_n^m(\omega), \iota_n^m(\omega)$

  $$Q_n(\omega) = \frac{\iota_n^m(\omega)}{\epsilon_n^m(\omega)}$$

  Both horizontal and vertical components are used *globally*

- **Spatial domain (regional) sounding:**

  $$C(\omega) = \frac{B_z(\omega)}{\nabla_H \cdot \mathbf{B}_H(\omega)}$$

  Vertical component is used *locally*, horizontal components are used *regionally*
Two methods for determining $\nabla_H \cdot \mathbf{B}_H(\omega)$

- $Z/H$:
  
  *Assumption* about source structure, e.g. $P_1^0$ for RC or $P_{m+1}^m$ for $m$th daily harmonic (Sq)

- Gradient sounding:
  *Estimation* of source structure from observed $\mathbf{B}_H = (B_x, B_y)$
  - (regional) polynomial fit yields
    \[ G = \nabla_H \cdot \mathbf{B}_H \]
  - $Z : \mathcal{Y}$
    - (global) spherical harmonic fit yields $\nu_n^m = \epsilon_n^m + \iota_n^m$
    \[ \mathcal{Y} = \nabla_H \cdot \mathbf{B}_H = \sum_{n,m} n(n+1) \nu_n^m P_n^m e^{im\phi} \]
Two methods for determining $\nabla_H \cdot \mathbf{B}_H(\omega)$

- **$Z/H$:**
  
  *Assumption* about source structure,
  e.g. $P_1^0$ for RC or $P_{m+1}^m$ for $m$th daily harmonic (Sq)
  allows determination of response from data of a *single* site

- **Gradient sounding:**
  
  *Estimation* of source structure from observed $\mathbf{B}_H = (B_x, B_y)$
  
  - (regional) polynomial fit yields
    
    $$\mathcal{G} = \nabla_H \cdot \mathbf{B}_H$$

  - **$Z : \mathcal{Y}$**
    
    (global) spherical harmonic fit yields $\nu_n^m = \epsilon_n^m + \iota_n^m$
    
    $$\mathcal{Y} = \nabla_H \cdot \mathbf{B}_H = \sum_{n,m} n(n+1)\nu_n^m P_n^m e^{im\phi}$$

  Vertical component is used *locally*, horizontal components are used *regionally*
The C-Response

Transfer functions

The C-Response

spherical harmonic degree $n$

limit of no induction, $|C_n| = \frac{2\pi a}{d(n+1)}$ [km]

$pulsations$  $Sq$

$DP, EEJ$  $Dst$

$|C_n| [km]$
The C-Response

Transfer functions

spherical harmonic degree $n$

$|C_n| [\text{km}]$

$\lambda_n = 2\pi a/(n+1) [\text{km}]$

Zero wavenumber response, $|C_0|$

Limit of no induction, $|C_n| = \frac{2\pi}{\gamma(n+1)}$

Not suitable for induction studies

Pulsations $\sqrt{S_Q}$

DP, EEJ $\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$

$\Delta$ $\Delta$
The C-Response

spherical harmonic degree $n$

[Diagram showing $|C_0|$ vs. $\lambda_n$ with labels for $1$ km, $100$ km, $400$ km, $250$ km, and $0.42$ km conductivity values.]

- $T = 10$ min
- $T = 2$ hrs
- $T = 1$ day
- $T = 1$ month
- $T = 6$ months
- $T = 1$ year
- $T = 11$ yrs

Zero wavenumber response, $|C_0|$.
The C-Response

Transfer functions

The C-Response

spherical harmonic degree $n$

<table>
<thead>
<tr>
<th>$C_n$</th>
<th>km</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 km</td>
<td>0.4 Ωm</td>
</tr>
<tr>
<td>100 km</td>
<td>10$^4$ Ωm</td>
</tr>
<tr>
<td>400 km</td>
<td>70 Ωm</td>
</tr>
<tr>
<td>250 km</td>
<td>16 Ωm</td>
</tr>
<tr>
<td>0.42 Ωm</td>
<td></td>
</tr>
</tbody>
</table>

Zero wavenumber response, $|C_0|$
The C-Response

Transfer functions

spherical harmonic degree $n$

$|C_n| = \lim_{n \to \infty} \frac{\text{induction}}{2^n}$

C = \begin{cases} 1 \text{ km} & 0.4 \Omega \text{m} \\ 100 \text{ km} & 10^4 \Omega \text{m} \\ 400 \text{ km} & 70 \Omega \text{m} \\ 250 \text{ km} & 16 \Omega \text{m} \\ 0.42 \Omega \text{m} \end{cases}$

$\text{Dst}$

$\text{DP, EEJ}$

$\text{Sq}$

$pulsations$

$\text{Sq source}$

$\text{Zero wavenumber response, } |C_0|$

$\lambda_n = \frac{2\pi a}{n+1}$ [km]

$T = \frac{1}{2\pi} \lambda_n$

$T = 10 \text{ min}$

$T = 1 \text{ min}$

$T = 1 \text{ day}$

$T = 1 \text{ month}$

$T = 6 \text{ months}$

$T = 1 \text{ year}$

$T = 11 \text{ yrs}$

$T = 1 \text{ year}$

$T = 2 \text{ hrs}$

$T = 1 \text{ day}$

$T = 1 \text{ month}$

$T = 6 \text{ months}$

$T = 1 \text{ year}$

$T = 11 \text{ yrs}$
Outline of Talk

1. Transfer functions
2. Source Fields
3. Transfer functions determinations before 1985
4. Transfer functions determinations 1985 - 1999
5. Latest attempts and new concepts
RC, the magnetospheric ring-current

- Source structure is mainly given by $P_1^0 = \cos \theta_d$ ($\theta_d$ is dipole co-latitude)
- Period range between a few hours and 100 days
- Depth range 300 km to 1500 km
- Can be used for induction studies with satellite data (source is external to observation point!)
**Sq**, the ionospheric regular daily variation

- Source structure of $m$th daily harmonic ($m = 1, 2 \ldots 6$) dominated by $P_m^m e^{imT}$ with local time $T$
- Period range between 4 hrs (for $m = 6$) and 24 hours ($m = 1$)
- Depth range 300 to 600 km
Outline of Talk

1. Transfer functions
2. Source Fields
3. Transfer functions determinations before 1985
4. Transfer functions determinations 1985 - 1999
5. Latest attempts and new concepts
Transfer functions determinations before 1985

Important publications by Ulrich Schmucker:


- *Magnetic and electric fields due to electromagnetic induction by external sources*, Landolt-Börnstein, Springer Verlag, 1985

Development of $Z: \nabla$ method (spherical version of gradient method)
Application of the various methods to observatory hourly mean values
Ulrich Schmucker’s Responses as of 1985

Transfer functions determinations before 1985

12 hours 1 day 1 week 1 month 3 months

Z/H, Sq
Z:Y, Sq
Z/H’, Sq
Z:Y, RC

Im{C}
Re{C}

period [secs]
[km]

Nils Olsen (DTU Space)
Ulrich Schmucker’s Responses as of 1985

\[ \rho^*(\omega) = 2\mu_0\omega \text{Im}\{C(\omega)\}^2 \]
\[ z^*(\omega) = \text{Re}\{C(\omega)\} \]

after U. Schmucker, *Landolt-Börnstein*, 1985
Outline of Talk

1. Transfer functions

2. Source Fields

3. Transfer functions determinations before 1985

4. Transfer functions determinations 1985 - 1999

5. Latest attempts and new concepts
Data mining: recovery of “old” data sets

- Hourly mean values of IGY/C (“Chapman-Gupta collection”)
- Throughout quality check of data from 100-130 observatories from 1957.5-1960.0 (IGY/C), 1964-65 (IQSY), and 1979-80 (IMS)
- These hourly mean values became basis for digital data collection at WDC-C (Copenhagen, now Edinburgh)
Data mining: recovery of “old” data sets

- Hourly mean values of IGY/C ("Chapman-Gupta collection")
- Throughout quality check of data from 100-130 observatories from 1957.5-1960.0 (IGY/C), 1964-65 (IQSY), and 1979-80 (IMS)
- These hourly mean values became basis for digital data collection at WDC-C (Copenhagen, now Edinburgh)

Accounting for day-to-day variability when estimating transfer functions

- $Sq$ analysis using potential and $Z : \mathcal{Y}$ method (Schmucker 1999)
Responses as of 1999

- Potential method, Sq (Schmucker 1999)
- Z:Y, Sq (Schmucker 1999)
- Z:Y, Sq+RC (Olsen 1999)
Responses as of 1999

\[ \rho^*(\omega) = 2\mu_0 \omega \Im\{C(\omega)\}^2 \]
\[ z^*(\omega) = \Re\{C(\omega)\} \]
Outline of Talk

1. Transfer functions
2. Source Fields
3. Transfer functions determinations before 1985
4. Transfer functions determinations 1985 - 1999
5. Latest attempts and new concepts
Two incompatible approaches

- **$Z/H$** method, assumption about source structure (here: $P_1^0 = \cos \theta$)

\[
B_z(\omega) = + C(\omega) \cdot G(\omega) = - C(\omega) \frac{a \tan \theta}{2} B_x(\omega)
\]

with $G = \nabla_H \cdot B_H = - \frac{a \tan \theta}{2} B_x$

Assumption: 1D Earth (no lateral variation of conductivity)

- **Induction arrows**, to detect lateral conductivity variations

\[
B_z(\omega) = T_x(\omega) \cdot B_x(\omega) + T_y(\omega) \cdot B_y(\omega)
\]

Assumption: homogeneous source ($|k|, n \to 0$)
Two incompatible approaches

- **Z/H method**, assumption about source structure (here: $P_1^0 = \cos \theta$)

  \[
  B_z(\omega) = +C(\omega) \cdot \mathcal{G}(\omega)
  \]

  \[
  = -C(\omega) \frac{\tan \theta}{2} B_x(\omega)
  \]

  with $\mathcal{G} = \nabla_H \cdot \mathbf{B}_H = -\frac{\tan \theta}{2} B_x$

  Assumption: 1D Earth (no lateral variation of conductivity)

- **Induction arrows**, to detect lateral conductivity variations

  \[
  B_z(\omega) = T_x(\omega) \cdot B_x(\omega) + T_y(\omega) \cdot B_y(\omega)
  \]

  Assumption: homogeneous source ($|\mathbf{k}|$, $n \to 0$)

  In both approaches $B_z \propto B_x$

  Interpretation depends on chosen assumptions
How to make them compatible ...

Observed vertical magnetic field $B_z = B_{nz} + B_{az}$ consists of normal part $B_{nz}$ and anomalous part $B_{az}$

$$B_{nz} = C \cdot G, \quad G = \nabla_{H} \cdot B_{nH}$$

$$B_{az} = T_{x} \cdot B_{nx} + T_{y} \cdot B_{ny}$$

$$B_z = B_{nz} + B_{az} = C \cdot G + T_{x} \cdot B_{nx} + T_{y} \cdot B_{ny}$$
Generalized Gradient Sounding 1/3


\[ \mathbf{E}_H = \mathbb{Z} \mathbf{B}_{nH} \]

\[ [\nabla \times \mathbf{E}]_z = -i\omega B_z \]

Lateral variation of conductivity:

\[ B_z = C_{xx} \frac{\partial B_{nx}}{\partial y} + \frac{\partial C_{xx}}{\partial y} B_{nx} + \ldots \]

with \( C \)-response tensor \( \underline{\underline{C}} = i\omega \underline{\mathbb{Z}} \)

Eight terms on right side
Generalized Gradient Sounding 2/3

Using $\nabla \times \mathbf{B} = 0$ and rearranging reduces this to five terms:

$$
B_z = C_1 \left( \frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right) + C_2 \left( \frac{\partial B_{ny}}{\partial y} - \frac{\partial B_{nx}}{\partial x} \right) + C_3 \frac{\partial B_{nx}}{\partial y} + T_x B_{nx} + T_y B_{ny}
$$

with

$$
C_1 = \frac{C_{xy} - C_{yx}}{2}, \\
C_2 = \frac{C_{xy} + C_{yx}}{2}, \\
C_3 = \frac{C_{xx} - C_{yy}}{2}, \\
T_x = \frac{\partial C_{xx}}{\partial y} - \frac{\partial C_{yx}}{\partial x}, \\
T_y = \frac{\partial C_{xy}}{\partial y} - \frac{\partial C_{yy}}{\partial x}
$$
Generalized Gradient Sounding 2/3

Using $\nabla \times \mathbf{B} = 0$ and rearranging reduces this to five terms:

$$B_z = C_1 \left( \frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right) + C_2 \left( \frac{\partial B_{ny}}{\partial y} - \frac{\partial B_{nx}}{\partial x} \right) + C_3 \frac{\partial B_{nx}}{\partial y} + T_x B_{nx} + T_y B_{ny}$$

with

$$C_1 = \frac{C_{xy} - C_{yx}}{2}$$
$$C_2 = \frac{C_{xy} + C_{yx}}{2}$$
$$C_3 = \frac{C_{xx} - C_{yy}}{2}$$
$$T_x = \frac{\partial C_{xx}}{\partial y} - \frac{\partial C_{yx}}{\partial x}$$
$$T_y = \frac{\partial C_{xy}}{\partial y} - \frac{\partial C_{yy}}{\partial x}$$

5 transfer functions, collected in $C$ and $T$
Generalized Gradient Sounding 3/3

- $B_z = C_0 \cdot G$
  - univariate
  - corresponds to 1D isotropic conductivity

\[ G = G_1 = \left( \frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right) \]
Generalized Gradient Sounding 3/3

- $B_z = C_0 \cdot G$
  - univariate
  - corresponds to 1D isotropic conductivity

- $B_z = C_1 \cdot G_1 + C_2 \cdot G_2 + C_3 \cdot G_3 + T_x \cdot B_{nx} + T_y \cdot B_{ny}$
  - 5-variate
  - general 3D anisotropic conductivity

\[
G = G_1 = \left( \frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right)
\]
\[
G_2 = \left( \frac{\partial B_{nx}}{\partial x} - \frac{\partial B_{ny}}{\partial y} \right)
\]
\[
G_3 = \frac{\partial B_{nx}}{\partial y}
\]
Generalized Gradient Sounding 3/3

- $B_z = C_0 \cdot G$
  - univariate
  - corresponds to 1D isotropic conductivity
- $B_z = C_1 \cdot G_1 + C_2 \cdot G_2 + C_3 \cdot G_3$
  - tri-variate
  - corresponds to 1D anisotropic conductivity
- $B_z = C_1 \cdot G_1 + C_2 \cdot G_2 + C_3 \cdot G_3 + T_x \cdot B_{nx} + T_y \cdot B_{ny}$
  - 5-variate
  - general 3D anisotropic conductivity

$G = G_1 = \left( \frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right)$

$G_2 = \left( \frac{\partial B_{nx}}{\partial x} - \frac{\partial B_{ny}}{\partial y} \right)$

$G_3 = \frac{\partial B_{nx}}{\partial y}$
Generalized Gradient Sounding 3/3

- \( B_z = C_0 \cdot G \)
  - univariate
  - corresponds to 1D isotropic conductivity
- \( B_z = C_1 \cdot G_1 + C_2 \cdot G_2 + C_3 \cdot G_3 \)
  - tri-variate
  - corresponds to 1D anisotropic conductivity
- \( B_z = C_0 \cdot G + T_x \cdot B_{nx} + T_y \cdot B_{ny} \)
  - tri-variate
  - corresponds to “moderate deviation” from 1D conductivity
- \( B_z = C_1 \cdot G_1 + C_2 \cdot G_2 + C_3 \cdot G_3 + T_x \cdot B_{nx} + T_y \cdot B_{ny} \)
  - 5-variate
  - general 3D anisotropic conductivity

\[
G = G_1 = \left( \frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right)
\]
\[
G_2 = \left( \frac{\partial B_{nx}}{\partial x} - \frac{\partial B_{ny}}{\partial y} \right)
\]
\[
G_3 = \frac{\partial B_{nx}}{\partial y}
\]
Application to European Hourly Mean Values 1964-65
Observatory Wingst (near Hamburg/Germany)

$C_2$ show systematic non-zero values, increasing with frequency indicate anisotropy of the $C$-response at Wingst.

$C_2$ and $C_3$ are shown in the diagrams:

- **tri-variate**
- **5-variate**

Long-period induction arrows

7.5 cpd, $T = 3.3$ hrs, $z^* = 260$ km

Long-period induction arrows

6.5 cpd, $T = 3.7$ hrs, $z^* = 280$ km

Long-period induction arrows

$5.5 \text{ cpd, } T = 4.4 \text{ hrs, } z^* = 315 \text{ km}$

Long-period induction arrows

4.5 cpd, $T = 5.3$ hrs, $z^* = 360$ km

Long-period induction arrows

3.5 cpd, $T = 6.9$ hrs, $z^* = 415$ km

Long-period induction arrows

2.5 cpd, $T = 9.6$ hrs, $z^* = 500$ km

Latest attempts and new concepts

Long-period induction arrows

1.5 cpd, $T = 16$ hrs, $z^* = 610$ km
